

**AN ROINN OIDEACHAIS AGUS EOLAÍOCHTA**

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**LEAVING CERTIFICATE EXAMINATION , 2002**

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**APPLIED MATHEMATICS – HIGHER LEVEL**

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**FRIDAY, 21 JUNE – AFTERNOON, 2.00 to 4.30**

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Six questions to be answered. All questions carry equal marks.

Mathematics Tables may be obtained from the Superintendent.

Take the value of  $g$  to be  $9.8 \text{ m/s}^2$ .

**Marks may be lost if necessary work is not clearly shown or you do not indicate where a calculator has been used.**

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1. (a) A stone is thrown vertically upwards under gravity with a speed of  $u$  m/s from a point 30 metres above the horizontal ground. The stone hits the ground 5 seconds later.
- (i) Find the value of  $u$ .
- (ii) Find the speed with which the stone hits the ground.
- (b) A particle, with initial speed  $u$ , moves in a straight line with constant acceleration. During the time interval from 0 to  $t$ , the particle travels a distance  $p$ . During the time interval from  $t$  to  $2t$ , the particle travels a distance  $q$ . During the time interval from  $2t$  to  $3t$ , the particle travels a distance  $r$ .
- (i) Show that  $2q = p + r$ .
- (ii) Show that the particle travels a further distance  $2r - q$  in the time interval from  $3t$  to  $4t$ .
2. (a) Two boats, B and C, are each moving with constant velocity. At a certain instant, boat B is 10 km due west of boat C. The speed and direction of boat B relative to boat C is 2.5 m/s in the direction  $60^\circ$  south of east.
- (i) Calculate the shortest distance between the boats, to the nearest metre.
- (ii) Calculate the length of time, to the nearest second, for which the boats are less than or equal to 9 km apart.
- (b) The velocity of ship P relative to a steady wind is 20 km/hr in the direction  $80^\circ$  north of east. The velocity of ship Q relative to the same steady wind is 10 km/hr in the direction  $20^\circ$  south of west.
- Calculate the magnitude and direction of the velocity of ship P relative to ship Q.  
Give your answers to the nearest km and the nearest degree, respectively.

3. (a) A particle is projected from a point on the horizontal ground with a speed of 39.2 m/s inclined at an angle  $\alpha$  to the horizontal ground. The particle is at a height of 14.7 m above the horizontal ground at times  $t_1$  and  $t_2$  seconds, respectively.

(i) Show that  $t_2 - t_1 = \sqrt{64 \sin^2 \alpha - 12}$ .

(ii) Find the value of  $\alpha$  for which  $t_2 - t_1 = \sqrt{20}$ .

- (b) A particle is projected with velocity  $u$  m/s at an angle  $\theta$  to the horizontal, up a plane inclined at an angle  $\beta$  to the horizontal. (The plane of projection is vertical and contains the line of greatest slope). The particle strikes the plane at right angles.

(i) Show that  $2 \tan \beta \tan(\theta - \beta) = 1$ .

(ii) Hence, or otherwise, show that if  $\theta = 2\beta$ , the range of the particle up the inclined plane is  $\frac{u^2}{g\sqrt{3}}$ .

4. (a) A particle is describing simple harmonic motion with period  $\frac{\pi}{4}$  seconds about a point  $o$ . When the particle is 6 cm from the point  $o$ , its speed is  $8\sqrt{13}$  cm/s.

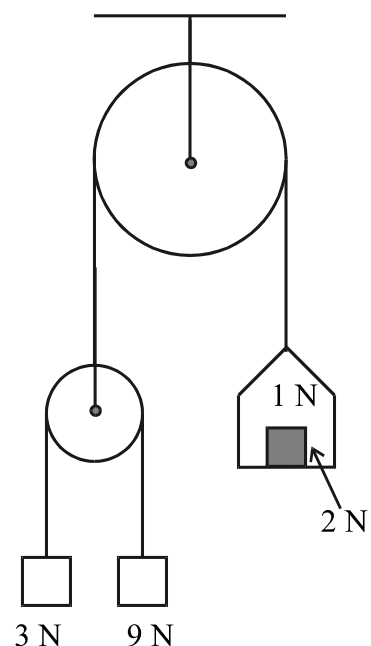
Find the amplitude of the motion.

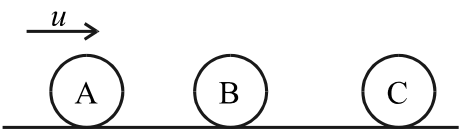
- (b) A smooth light pulley is connected by a light inextensible string passing over a smooth light fixed pulley to a scale pan of weight 1 N. A particle of weight 2 N is placed symmetrically on the centre of the scale pan. Two particles of weight 3 N and 9 N are connected by a light inextensible string passing over the smooth light pulley. The system is released from rest.

The acceleration of the scale pan is  $\frac{g}{2}$  m/s<sup>2</sup> vertically upwards.

- (i) Find the acceleration, in terms of  $g$ , of the particle of weight 9 N.

- (ii) Find the normal reaction (force) between the particle of weight 2 N and the scale pan.



5. (a) Three identical smooth spheres, A, B and C, lie at rest on a smooth horizontal table with their centres in a straight line.  Sphere A is projected towards B with speed  $u$ . Sphere A collides directly with B and then B collides directly with C. Sphere C moves, after the collision, with a speed of  $\frac{5u}{8}$ .

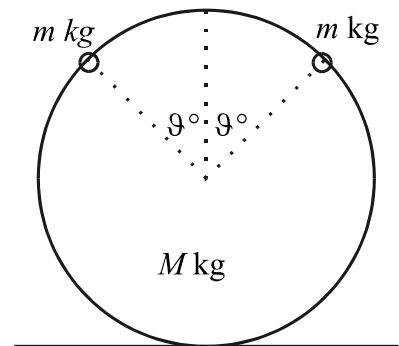
The coefficient of restitution for each of the two collisions is  $e$ .

Find  $e$ , correct to two places of decimals.

- (b) A smooth sphere P collides with an identical smooth sphere Q which is at rest. The velocity of P before impact makes an angle  $\alpha$  with the line of centres at impact, where  $0^\circ \leq \alpha < 90^\circ$ . The velocity of P is deflected through an angle  $\mathcal{G}$  by the collision, so that its velocity after impact makes an angle  $\mathcal{G} + \alpha$  with the line of centres at impact. The coefficient of restitution between the spheres is  $\frac{1}{4}$ .

Show that  $\tan \mathcal{G} = \frac{5 \tan \alpha}{3 + 8 \tan^2 \alpha}$ .

6. A smooth uniform vertical hoop of radius  $r$  and mass  $M$  kg stands in a vertical plane on a horizontal surface. The hoop threads two small rings, each of mass  $m$  kg. The rings are released from rest at the top of the hoop.



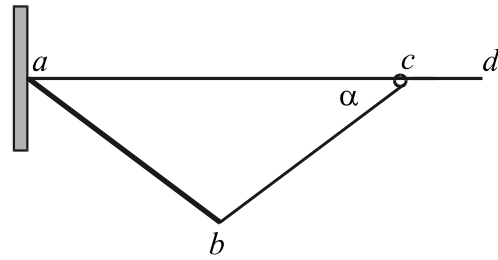
- (i) When the two rings have each fallen through an angle of  $\mathcal{G}^\circ$  on opposite sides of the hoop, show that the normal force of reaction exerted by the hoop on each ring is

$$mg(3 \cos \mathcal{G} - 2) \text{ N,}$$

where this force is taken to act in the outward direction from the centre of the hoop.

- (ii) Show that the hoop will rise from the table if  $m > \frac{3M}{2}$ .

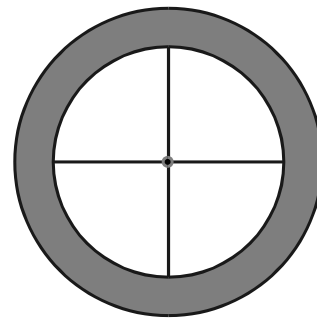
7. A uniform rod,  $[ab]$ , of weight  $4W$  and length  $2l$ , is free to rotate smoothly about the fixed point  $a$ . A fixed wire,  $[ad]$ , extends horizontally from  $a$ . The end  $b$  of the rod is attached by a light inelastic string,  $[bc]$ , of length  $2l$ , to a ring, of weight  $W$  and negligible diameter, which can slide on the wire. The coefficient of friction between the ring and the wire is  $\mu$ . The string makes an angle  $\alpha$  with the horizontal when the system is in limiting equilibrium (that is, just on the point of slipping).



- (i) Show that  $\tan \alpha = \frac{1}{2\mu}$ .
- (ii) Show that the tension in the string is  $W\sqrt{1+4\mu^2}$ .

8. (a) Prove that the moment of inertia of a uniform rod of mass  $m$  and length  $2l$  about an axis through its centre perpendicular to the rod is  $\frac{1}{3}ml^2$ .

- (b) The diagram shows a two-dimensional wheel (shaded area) and four spokes arranged inside the wheel as shown. The inner and outer radii of the wheel are  $6a$  and  $8a$ , respectively. Each spoke is of mass  $m$  and length  $6a$ . The total mass of the wheel and four spokes is  $18m$ .



- (i) Show that the mass per unit area of the wheel (shaded area) is  $\frac{m}{2\pi a^2}$ .
- (ii) Show that the total moment of inertia of the wheel and four spokes about an axis through the centre and perpendicular to the plane of the wheel is  $748ma^2$ .
- (iii) If  $m = 100$  grammes and  $a = 10$  cm, how much work is done in bringing this wheel and spokes to rest from 6000 revolutions per minute?

9. (a) When placed in liquid A, a uniform solid cylinder floats upright with  $\frac{2}{5}$  of its axis immersed in the liquid.

When placed in liquid B, the uniform solid cylinder floats upright with  $\frac{4}{7}$  of its axis immersed in the liquid.

What fraction of the cylinder's axis is immersed when the cylinder floats upright in a uniform mixture of equal volumes of liquid A and liquid B?

- (b) A hollow spherical shell of external radius 0.5 m and uniform thickness 0.1 m floats in a liquid of relative density 0.9.

The relative density of the material of the shell is  $\frac{36}{61}$ .

What fraction of the volume enclosed by the external surface of the shell is immersed in the liquid?

10. (a) Solve the differential equation

$$\frac{dy}{dx} = e^{x-y}$$

given that  $y = \ln 4$  when  $x = 0$ .

- (b) A particle starts from rest and moves in a horizontal line. Its speed  $v$  at time  $t$  is given by the equation

$$\frac{dv}{dt} = 100 - v.$$

- (i) Find the time taken for the speed of the particle to increase from 25 m/s to 75 m/s.
- (ii) How far does the particle travel in going from rest to a speed of 75 m/s?
- (iii) Determine the limiting speed,  $v_1$ , of the particle.  
(that is,  $v \rightarrow v_1$  as  $t \rightarrow \infty$ ).