AN ROINN OIDEACHAIS AGUS EOLAÍOCHTA | DEPARTMENT OF EDUCATION AND SCIENCE

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Marking Schemes — *Leaving Certificate Examinations, 1999*

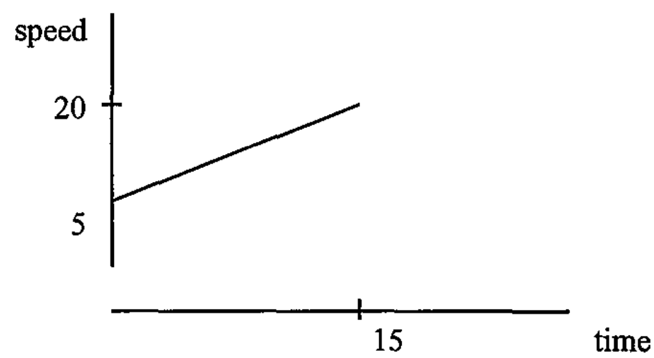
Applied Mathematics — *Ordinary Level*

LEAVING CERTIFICATE 1999

APPLIED MATHS - ORDINARY LEVEL

MARKING SCHEME

1	A car A, accelerating uniformly, passes a point p with a speed of 5 m/s. 15 seconds later it reaches point q and passes q with a speed of 20 m/s. Draw an accurate speed-time graph for the motion of car A between p and q. Find A's acceleration and find, also, |pq|. At the same time as A passes p, a second car B also passes p but with an initial speed of 6 m/s and accelerates uniformly to arrive at q at the same time as A. What is B's acceleration? Find, also, B's velocity at the point q.



15

$v = u + ft$ or $f = \text{slope}$

$20 = 5 + f(15)$

$f = 1$ $\quad$ $f = \dfrac{15}{15} = 1$

10

$|pq| = ut + \frac{1}{2}ft^2$ or $|pq| = \text{area}$

$= 5(15) + \frac{1}{2}(1)(15^2)$ $\quad$ $= 5(15) + \frac{1}{2}(15)(15)$

$= 187.5$ $\quad$ $= 187.5$

10

Car B $\quad$ $u = 6,\ s = 187.5,\ t = 15$

$s = ut + \frac{1}{2}ft^2$

$187.5 = 6(15) + \frac{1}{2}f(225)$

$f = \dfrac{13}{15}$ or 0.86

10

$v = u + ft$

$= 6 + \left(\dfrac{13}{15}\right)(15)$

$= 19$ m/s

5

LEAVING CERTIFICATE MENU

2 A ship D travelling with a velocity of $3\vec{i} - 4\vec{j}$ m/s is 5 km due west of ship C. Ship D sends out a distress signal which is picked up by C and C travels immediately with a velocity of $2\vec{i} - 4\vec{j}$ m/s so as to come to the assistance of D.

(i) What is D's speed and direction?

(ii) Find the magnitude and direction of the velocity of C relative to D.

(iii) How long, to the nearest minute, will it take C to reach D?

(i)

speed $= \sqrt{(3)^2 + (-4)^2}$

$= 5$ 10

direction: $\tan\theta = \dfrac{4}{3}$

$\theta = 53.13°$ or $53° 7'$ south of east 10

(ii)

$V_{CD} = V_C - V_D$

$= (2\vec{i} - 4\vec{j}) - (3\vec{i} - 4\vec{j})$ 5

$= -\vec{i}$ 5

magnitude $= 1$ 5

direction west 5

(iii)

time = distance ÷ relative velocity

= 5000 ÷ 1

= 5000 seconds

= 83 minutes 10

3.	A projectile is fired upwards from the top of a cliff of height 50 m with a velocity of 25 m/s at an angle A to the horizontal. Tan A = $\frac{3}{4}$. The projectile strikes the sea at a distance d from the foot of the cliff. Calculate d.

$$\vec{r} = (25\cos A\,.t)\,\vec{i}$$ 10

$$+\ (25\sin A\,.t - \tfrac{1}{2}g\,t^2)\,\vec{j}$$ 10

$$r\vec{j} = -50$$ 10

$$\Rightarrow \quad 25\left(\frac{3}{5}\right)t - 5t^2 = -50$$

$$15t - 5t^2 = -50$$

$$t^2 - 3t - 10 = 0$$

$$(t-5)(t+2) = 0$$

$$t = 5$$ 10

$$d = r\vec{i} \quad \text{when } t = 5$$

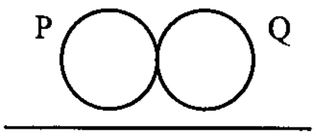
$$= 25\left(\frac{4}{5}\right)5$$

$$= 100\text{ m}$$ 10

4 Two smooth spheres P and Q collide directly on a smooth horizontal table. The mass of P is twice that of Q. Before collision, P and Q are moving in opposite directions with equal speeds. As a result of the collision sphere P comes to rest.

Find the coefficient of restitution for the collision.

Find, also, the fraction of the total kinetic energy lost as a result of the collision.



	mass	velocity before	velocity after
P	2m	u	0
Q	m	- u	v_2

PCM $\quad 2m(u) + m(-u) = 2m(0) + m(v_2)$ 10

$$m u = m(v_2)$$

$$u = v_2$$ 10

NEL $\quad v_1 - v_2 = -e(u_1 - u_2)$ 10

$$0 - u = -e(u + u)$$

$$u = 2eu$$

$$e = \tfrac{1}{2}$$ 10

K.E. before $= \tfrac{1}{2}(2m)u^2 + \tfrac{1}{2}m(-u)^2 = \tfrac{3}{2}mu^2$

K.E. after $= \tfrac{1}{2}mu^2$

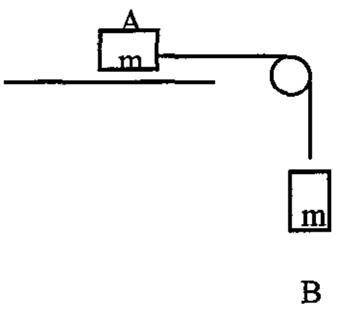
K.E. lost $= \tfrac{3}{2}mu^2 - \tfrac{1}{2}mu^2$

Fraction of K.E. lost $= \dfrac{mu^2}{\tfrac{3}{2}mu^2} = \dfrac{2}{3}$ 10

5 Two blocks, A and B, each of mass m, are connected by a light inextensible string passing over a smooth pulley at the edge of a smooth table. B hangs freely under gravity.

(i) When the system is released from rest, find the common acceleration of the blocks.

(ii) If the smooth table is replaced by a rough table and the system is released from rest, as before, the common acceleration of the blocks is half what it was in the first case. Find this coefficient of friction between A and the rough table.



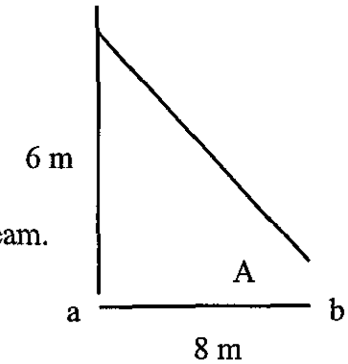
(i)

$T = mf$	10
$mg - T = mf$	10
$mg = 2mf$	
$f = \frac{g}{2}$ or 5	10

(ii)

$T - \mu R = mf$	5
$mg - T = mf$	5
$mg - \mu R = 2mf$	
$f = \frac{g}{4}$ and $R = mg$	5
$mg - \mu(mg) = \frac{mg}{2}$	
$\frac{mg}{2} = \mu mg$	
$\mu = \frac{1}{2}$	5

6 The diagram shows a uniform beam [ab], which is 8 m long and has a mass of 20 kg. The beam is freely hinged at a to a vertical wall and held in a horizontal position by a light, inextensible string attached to b and to a point on the wall 6 m vertically above a. The angle between the string and the beam is A, as shown.

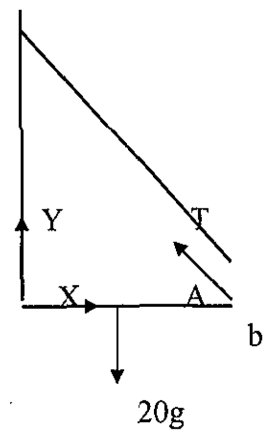


Show in a diagram, all the forces acting on the beam.

(i) Find sin A and cos A.

(ii) Find the vertical and horizontal components of the reaction of the hinge.

(iii) Find the value of the tension in the string.



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(i) $\tan A = \frac{6}{8} = \frac{3}{4}$

$\sin A = \frac{3}{5}$ 5

$\cos A = \frac{4}{5}$ 5

(ii) + (iii) Take moments about b

Y (8) = 20 g (4)

Y = 10 g or 100 10

Resolve vertically Y + T sin A = 20 g

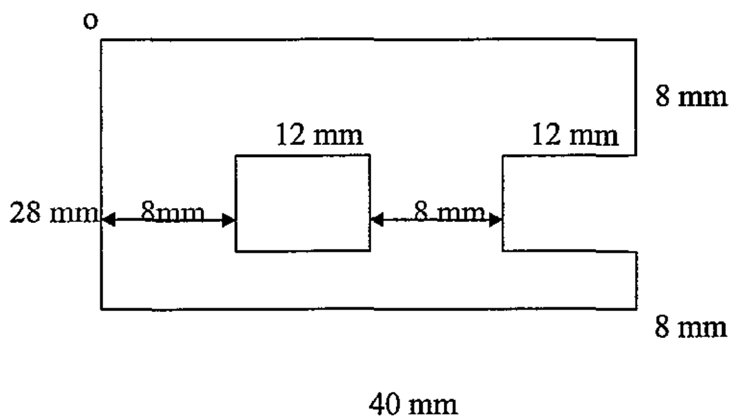
10 g + T (0.6) = 20 g

$T = \frac{50g}{3}$ or 166.7 10

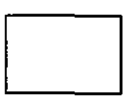
Resolve horizontally X = T cos A

$X = \frac{40g}{3}$ or 133.3 5

7 A uniform rectangular piece of metal measures 28 mm x 40 mm.
Two square pieces each of side 12 mm are removed, as in the diagram:



Calculate the distance from the point o to the centre of gravity of the remaining piece, correct to one place of decimals.

Shape	Area	c.g. co-ords	
	144	(34 , - 14)	5
	144	(14 , - 14)	5
	1120	(20 , - 14)	5
	832	(x , y)	5

$832x = 1120(20) - 144(14) - 144(34)$ 10

$= 22400 - 2016 - 4896$

$= 15488$

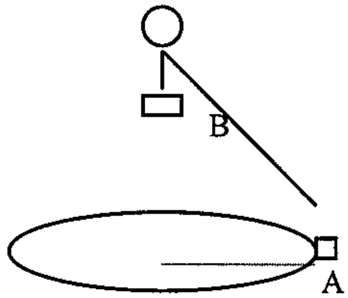
$x = \frac{15488}{832}$ or 18.615 5

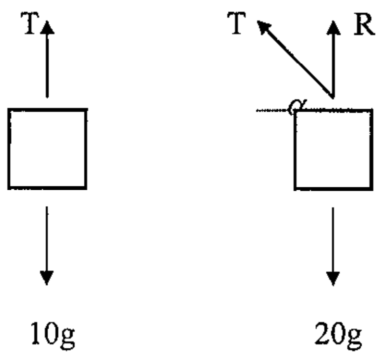
$y = -14$ 5

$\text{distance} = \sqrt{(18.615)^2 + (-14)^2}$

$= 23.3$ 10

8 The diagram shows two particles A and B of mass 20 kg and 10 kg respectively. Particle A is on a smooth horizontal table and is connected to particle B by a light inextensible string passing through a smooth ring fixed at a distance of 0.75 m above the table, so that B hangs freely under gravity and remains at rest. Particle A describes a horizontal ciecle of radius 1 m, whose centre is vertrically below the ring, with constant angular velocity ω radians/s.





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(i) $T = 10g$ or 100 — 10

(ii) $T\cos\alpha = m r \omega^2$ — 5

$100(0.8) = 20(1)\omega^2$ — 5

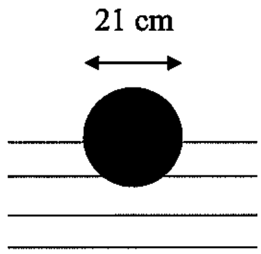
$\omega = 2$ — 5

(iii) $T\sin\alpha + R = 20g$ — 5

$10g(0.6) + R = 20g$

$R = 14g$ or 140 — 5

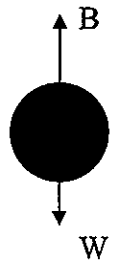
9 A solid sphere, of diameter 21 cm and of density ρ kg/m^3, floats in water with half its volume submerged.



Draw a diagram showing all the forces acting on the sphere and find the value of ρ.

The same sphere is then pulled under the surface and is held, totally submerged, by a light inextensible string attached to the sphere and to the bottom of the tank.

Show, in a diagram, all the forces acting on the sphere in this case, and find the tension in the string.



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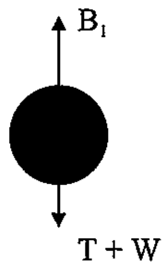
$$W = B$$ 5

$$W = \frac{\frac{1}{2}W(1)}{\rho}$$ 5

$$\rho = \tfrac{1}{2}$$ 5



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$$T + W = B_1$$ 5

$$T + W = \frac{W(1)}{\rho}$$ 5

$$T + W = 2W$$

$$T = W$$

$$T = \rho V g$$

$$T = (500)\left\{\tfrac{4}{3}\pi\left(\tfrac{0.21}{2}\right)^3\right\}(10)$$

$$T = 24.255 \text{ N}$$ 5