AN ROINN OIDEACHAIS AGUS EOLAÍOCHTA

MATHEMATICS

Junior Certificate

GUIDELINES FOR TEACHERS

THESE GUIDELINES

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changes and clarifications,
and much more…

National Council for Curriculum and Assessment
An Chomhairle Náisiúnta Curaclaim agus Measúnachta
NCCA

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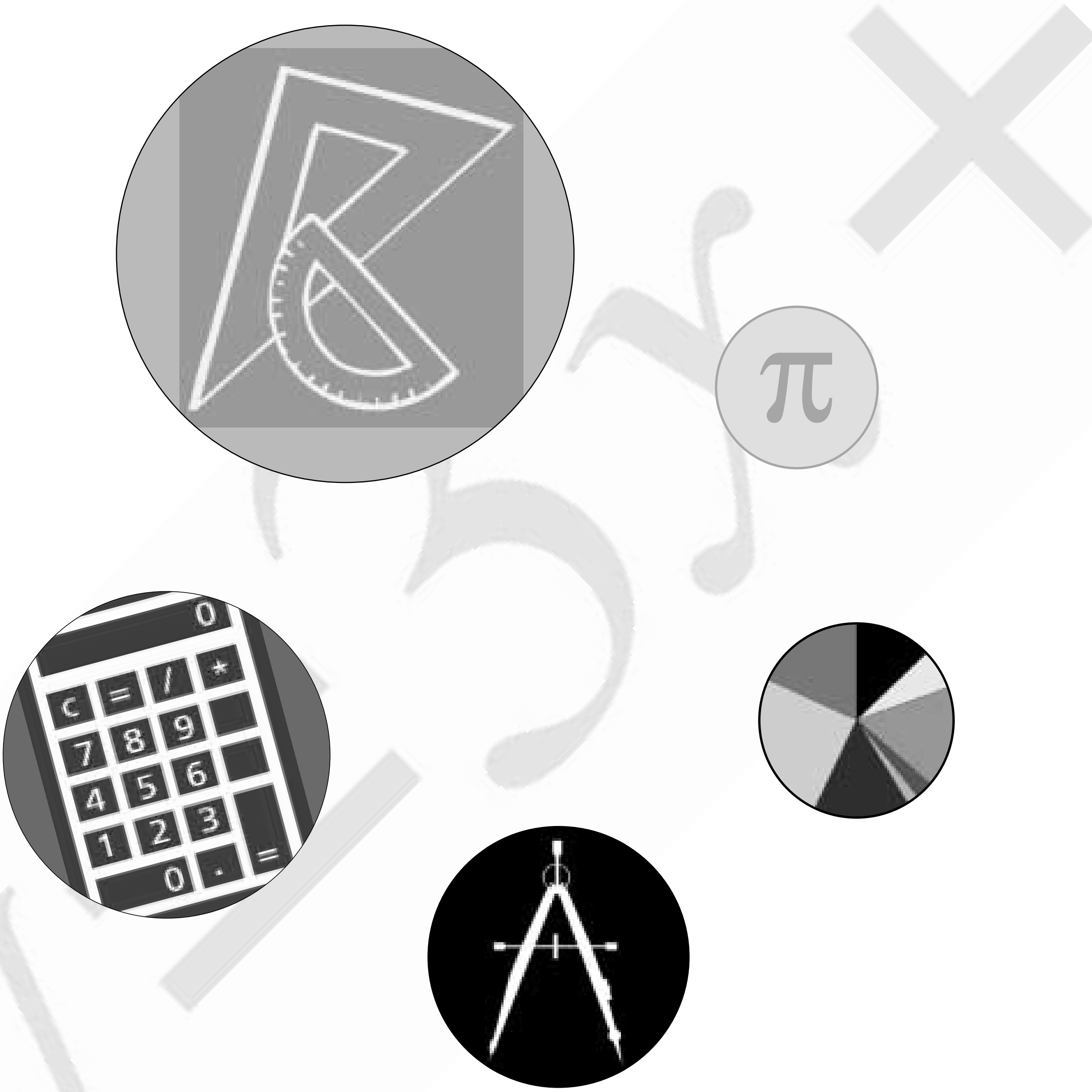


INTRODUCTION

These Guidelines are intended to provide a resource for teachers of Junior Certificate mathematics. In particular they are designed to support the revised Junior Certificate syllabuses (introduced in September 2000) during the first cycle of implementation: that is, until the end of the academic year 2002/2003.

The current *draft* version of the *Guidelines* can be updated after 2003 to reflect the experience and wisdom of teachers who have taught the new syllabuses during their introductory period.

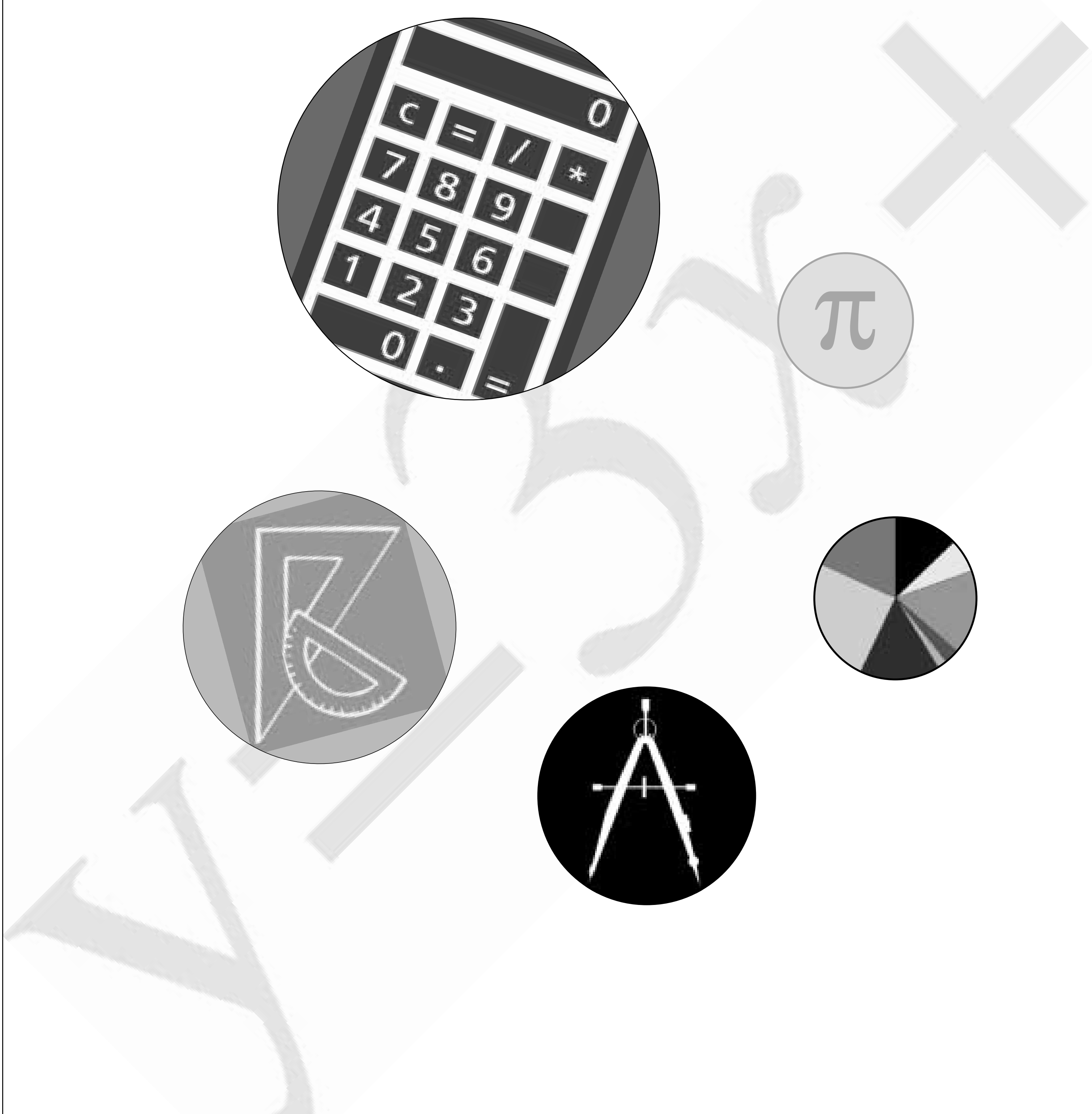
The document aims to address a variety of questions that may be asked about the syllabuses. Questions can be grouped under the headings "why", "what" and "how". The opening sections of the *Guidelines* focus especially on providing answers to questions of "why" type. Section 1 describes the background to and reasons for the revision of the mathematics syllabuses; Section 2, dealing with the specification of aims and objectives, emphasises the rationale and principles underlying their design; and Section 3, on syllabus content, outlines reasons for including specific topics. These sections also address issues of "what" type. For example, Section 3 highlights the changes that were made from the preceding syllabuses and the differences that may be expected in student knowledge and attitude at entry to the junior cycle following the introduction of the revised Primary School Curriculum. In Section 4, "how" questions are considered: how the work might be organised in the school or classroom, and how specific topics might be taught. This section describes some of the insights and practice of experienced teachers who have "made it work" in Irish classrooms. Against this exposition of where the learning of mathematics should be going and how it might get there, Section 5, on assessment, indicates ways in which information can be obtained on whether or not the students have arrived at the intended goals. A number of appendices provide reference material.

Thus, the different parts of the *Guidelines* are likely to be used in different ways. The initial parts offer a general orientation and invite reflection on the purposes of mathematics education in the junior cycle. The central sections provide a resource for the day-to-day work of mathematics teaching: a compendium of ideas, to which teachers may turn when they wish to improve the quality or vary the style of learning in their classrooms. The section dealing with assessment is likely to be of special interest to those preparing students for state examinations, but its brief is wider, with the focus being on formative as well as summative assessment. Altogether, it is hoped that the emphasis on *meaningful and enjoyable learning* will enrich the quality and enhance the effectiveness of our students' mathematics education.

The *Guidelines* are designed to support the teaching of mathematics in the junior cycle in such a way as to meet a wide range of learning needs. However, students with a mild general learning disability often face particular challenges in the area of mathematics. Additional guidelines for teachers of students with a mild general learning disability have been drawn up, and provide a further resource for all teachers of mathematics.

Many people – too many to mention individually – have contributed to the *Guidelines*. A special tribute must be paid to the teachers who provided the "lesson ideas" which form a key part of the document and also to those who made inputs to other sections. The engagement of so many people from outside the course committee, as well as of committee members, is a welcome feature of the process leading to the production of the draft *Guidelines* document. It represents a significant advance in the sharing of mathematics teaching methodologies at national level. As indicated above, the intention is that even more members of the mathematics education community will contribute their ideas over the period of operation of the draft *Guidelines*. The final version can then provide a resource that reflects as much good practice as possible in mathematics teaching in Ireland.

Context of the changes



1.1 HISTORY OF THE SYLLABUSES

DEVELOPMENT OF THE JUNIOR CYCLE SYLLABUSES

The development of the current syllabuses can be traced back to the sixties: that period of great change in mathematics courses world-wide. The changes were driven by a philosophy of mathematics that had transformed the subject at university level and was then starting to penetrate school systems. It was characterised by emphasis on *structure* and *rigour*. Starting from sets, the whole edifice of mathematics could be built up logically, via relations and in particular functions; the structure laws (such as those we know as the commutative, associative and distributive properties) were golden threads tying the different parts of mathematics together. As a summary of the mathematics of the day, it was splendidly conceived and realised by the famous Bourbaki group in France; but it was devised as a state-of-the art summary of the discipline of mathematics, not as an introduction to the subject for young learners. Nonetheless, many countries embraced the philosophy – or at least its outcome, revised subject-matter and rigorous presentation – with great enthusiasm. In Ireland, teachers attended seminars given by mathematicians; the early years of the Irish Mathematics Teachers' Association were enlivened by discussions of the new material.

The first changes in the mathematics syllabuses in this period took place in the senior cycle; new Leaving Certificate syllabuses were introduced in 1964, for first examination in 1966. Meanwhile, thoroughly "modern" syllabuses were being prepared for the junior cycle. They were introduced in 1966, for first examination in 1969, and were provided at two levels: "Higher" and "Lower". (Prior to 1966, two levels had also been offered, but the less demanding of the two had been available only to girls.) These revisions brought in some topics which we now take for granted, such as sets and statistics, as well as a few, such as number bases, which have not stood the test of time. The new syllabuses also addressed a problem in their predecessors: students were finding difficulty with the (comparatively) traditional presentation of formal geometry – difficulty, it can be said, shared by students in many other countries; so Papy's system, based on couples and transformations of the plane, was introduced alongside the existing version. For the examinations, papers clearly delineated by topic – arithmetic, algebra and geometry – were replaced by two papers in which a more integrated approach was taken, in keeping with the "modern" vision of mathematics.

The syllabuses introduced in 1966 ran for seven years. In 1973, revised versions were implemented in order to deal with some aspects that were causing difficulty. Two main alterations were made. First, the hybrid system of geometry was replaced by one entirely in the style of Papy. Secondly, the examination papers were redesigned so that the first section on each consisted of several compulsory multiple-choice items, effectively spanning the entire content relevant to the paper in question. The revised syllabuses were even more strongly Bourbakiste in character than their predecessors, with the ideas of set and function being intended to unify very many aspects of the syllabuses. However, it was hoped that emphasis would be given to arithmetic calculation and algebraic manipulation, which were felt to have suffered some neglect in the first rush of enthusiasm for the modern topics.

The size of the cohort taking the Intermediate Certificate increased in the 1970s. The rather abstract and formal nature of "modern" mathematics was not suitable for all the cohort, and further revisions were needed. In the early eighties, therefore, it was decided to introduce a third syllabus, geared to the needs of the less able. Also, some amendments were made to the former Higher and Lower level syllabuses. The package of three syllabuses, then called Syllabuses A, B and C, was introduced in 1987, for first examination in 1990. In the examination papers for Syllabuses A and B, question 1 took on the role of examining all topics in the relevant half of the syllabus, but the multiple-choice format was dropped in favour of the short-answer one that was already in use at Leaving Certificate level. A very limited choice was offered for the remaining questions (of traditional "long answer" format). For Syllabus C, a single paper with twenty short-answer questions was introduced.

The unified Junior Certificate programme was introduced in 1989. As the Intermediate Certificate mathematics syllabuses had been revised so recently, they were adopted as Junior Certificate syllabuses without further consideration (except that Syllabuses A, B and C were duly renamed the Higher, Ordinary and Foundation level syllabuses). Consequently, there was no opportunity to review the syllabuses thoroughly or to give due consideration to an appropriate philosophy and style for junior cycle mathematics in the 1990s and the new millennium.

The focus so far has been on syllabus *content*, and has indicated that the revolution in mathematics education in the 1960s has been followed by gradual evolution. Accompanying *methodology* received comparatively little attention in the ongoing debates. Choice of teaching method is not prescribed at national level; it is the domain of the teacher, the professional in the classroom. However, pointers could be given as to what was deemed appropriate, and the Preambles to the syllabuses introduced in 1973 and 1987 referred to the importance of understanding, the need for practical experience and the use of appropriate contexts. Now, with the increase in student retention and in view of the challenges posed by the information age, greater emphasis on methodology has become a matter of priority.

RECENT CHANGES AT OTHER LEVELS IN THE SYSTEM

To set the scene properly for the current revision of the Junior Certificate syllabuses, it is necessary to look also at what precedes and follows them in the students' education: the Primary School Curriculum and Leaving Certificate syllabuses.

The *Leaving Certificate syllabuses* were revised in the early 1990s. The revised syllabuses had to follow suitably from the then current Junior Certificate syllabuses, to fit into the existing senior cycle framework, and also to meet the needs of the world beyond school. This imposed some limitations on the scope of the revision. However, content was thoroughly critiqued for current relevance and suitability, some topics being discarded and a limited number of new ones being introduced. For the new Foundation level syllabus – brought in as the Ordinary Alternative syllabus in 1990, and amended slightly and designated as being at Foundation level in 1995 – some recommendations were made with respect to methodology. They emphasised the particular need for concrete approaches to concepts and for careful sequencing of techniques so that students could find meaning and experience success in their work. Much of the work was built round the use of calculators, which were treated as learning tools rather than just computational aids.

The introduction of calculators is also a feature of the revised *Primary School Curriculum* which was published in 1999. The revision was the first to be undertaken since the introduction of the radically restructured Primary School Curriculum in 1971.
The revised curriculum is being implemented on a phased basis, with the mathematics element scheduled for introduction in 2002. As is the case for the second level syllabuses, the revolution of the earlier period has been followed by evolution. The 1971 curriculum emphasised discovery learning, and this has led to considerable use of concrete materials and activity methods in junior classes. The revised Primary School Curriculum focuses to a greater extent on problem-solving and on the need for students to encounter concepts in contexts to which they can relate. Students emerging from the revised curriculum should be more likely than their predecessors to look for meaning in their mathematics and less likely to see the subject almost totally in terms of the rapid performance of techniques. They may be more used to active learning, in which they have to construct meaning and understanding for themselves, rather than passively receiving information from their teachers. The detailed changes in content and emphasis likely to affect second level mathematics are outlined in Section 3.4.

1.2 DEVELOPMENT OF THE REVISED SYLLABUSES

EVALUATIONS OF THE 1987 SYLLABUSES

Under the jurisdiction of the NCCA, the mathematics (junior cycle) course committee was first convened in November 1990, and was asked to analyse the impact of the new junior cycle mathematics syllabus. A somewhat similar brief was given in 1992, coinciding with a wider review: that of the first examination of syllabuses introduced at the inception of the Junior Certificate in 1989. The committee produced a report in response to each request. Among the difficulties identified in one or both of these reports were

- the length of the Higher level syllabus (which had actually been *shortened* in 1987, but the restricted choice in the examination meant that greater coverage was required than before)
- aspects of the geometry syllabus, especially at Higher level

- the proscription of calculators in the examinations, and consequently their restricted use as learning tools and computational aids in the classroom
- design of the Higher and Ordinary level examination papers (the restricted choice being endorsed, but the absence of aims and objectives giving problems in specifying criteria for question design and for formulation of marking schemes).

Both reports included favourable comments on the appropriateness of the Foundation level syllabus – newly introduced at that stage – for most of the students who were taking it.

BRIEF FOR THE REVISIONS

In Autumn 1994, the course committee was asked to critique the Junior Certificate mathematics syllabuses, this time with a view to introducing some amendments if required. Because of the amount of change that had taken, and was taking, place in the junior cycle in other subject areas, *it was specified that the outcomes of the review would build on current syllabus provision and examination approaches rather than leading to a root and branch change of either. Thus, once more, the syllabuses were to be revised rather than fundamentally redesigned.* The review was to take into account

- the work being done by the NCCA with respect to the curriculum for the upper end of the primary school
- the earlier reviews carried out by the committee
- changing patterns of examination papers over recent years
- analysis of examination results since 1990.

The following tasks were set out for the committee:

- To identify the major issues of concern regarding the existing syllabuses in their design, implementation and assessment;
- To address the issues surrounding Foundation level mathematics;
- To draft an appropriate statement of aims and objectives for each of the three syllabuses in line with Junior Certificate practice;
- To prepare Guidelines to assist in improving the teaching of mathematics.

EXECUTION OF THE TASKS

The committee had already identified the main issues, as described above. Consultation with the constituencies, for example at meetings of the Irish Mathematics Teachers' Association, tended to confirm that these were indeed the areas of chief concern to mathematics teachers. Chief Examiners' reports and international studies involving Ireland provided further insights. Altogether, the information pointed to strengths of Irish mathematics education – such as its sense of purpose and focus and the very good performance of the best students – but also to weaknesses, for example with respect to students' basic skills and understanding in some key areas of the curriculum, their communication skills and their ability to apply knowledge in realistic contexts. Consequent changes eventually made to the syllabuses and proposed for the examinations are outlined in Sections 3.3 and 5.4 of this document.

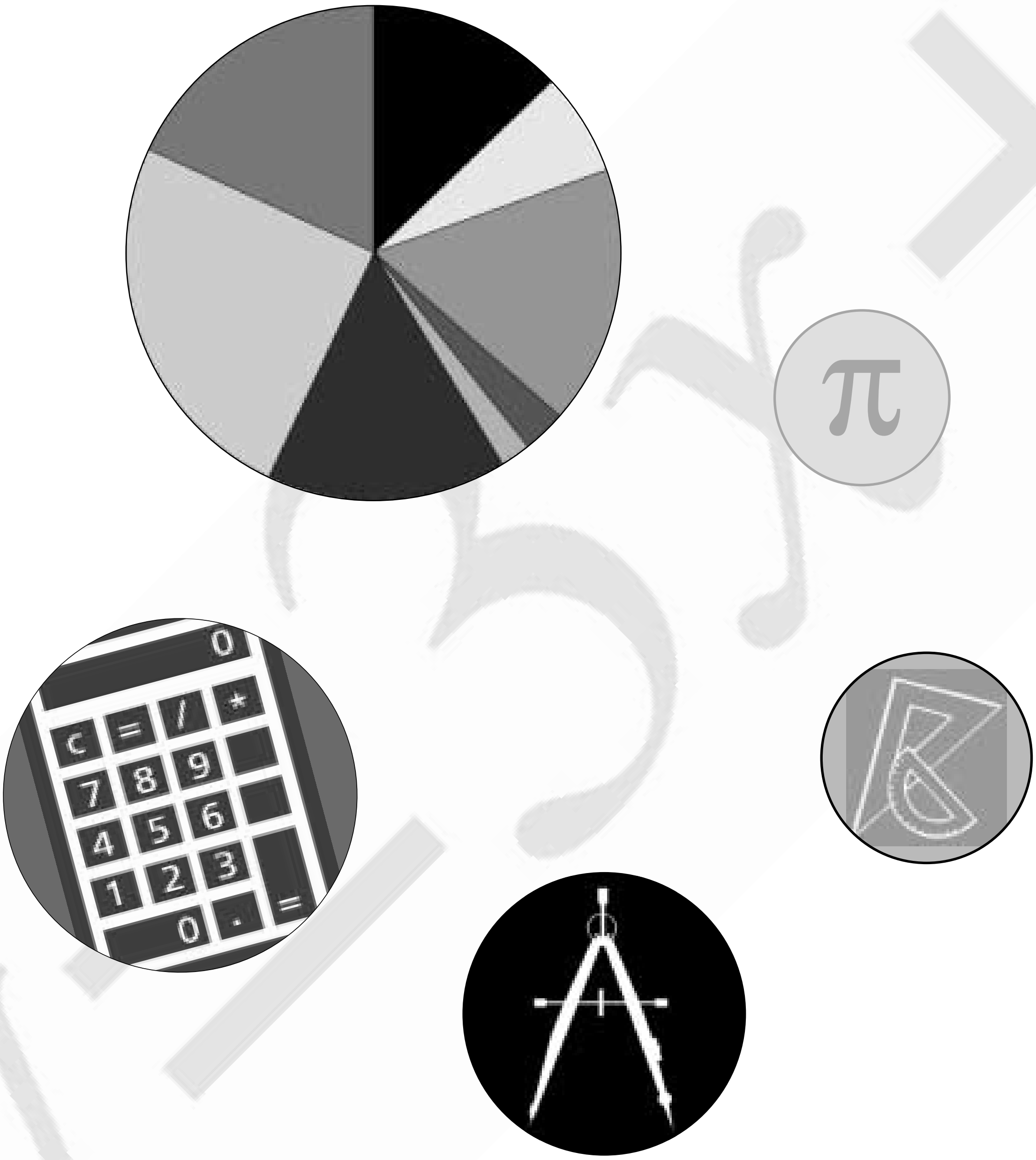
Further consideration of the Foundation level syllabus led to endorsement of its main thrust and its appropriateness for many students at the weaker end of the mathematical ability spectrum. Against this background of general approval, however, the committee recognised some difficulties, both with the syllabus content and with the format of the Junior Certificate examination. Changes were needed to enrich the content and improve the standing of the syllabus and to give students more opportunity to show what they had learnt. The problem of students following the Foundation level syllabus, or taking the examination, when they are capable of working at the Ordinary level, also needed to be addressed. Again, consequent changes are described elsewhere in these *Guidelines* (see Sections 3.3 and 5.4).

FROM INTENTION TO IMPLEMENTATION

The course committee duly presented the final draft of the syllabus to the NCCA Council, and this was approved by Council in May 1998. Later in the same year, the Minister for Education and Science announced his decision to implement the syllabus. It was introduced into schools in Autumn 2000 for first examination in 2003.

Introduction of the syllabus is being accompanied by in-career development for teachers of mathematics. This has been planned so as to focus, not only on the changes in syllabus content, but also on the types of *teaching methodology* that might facilitate achievement of the aims and objectives of the revised syllabus. These *Guidelines* are intended to complement the in-career development sessions. They can also support further study involving teachers or groups of teachers throughout the country.

Aims, objectives and principles of syllabus design



2.1 INTRODUCTION

One of the tasks given to the Course Committee was to write suitable aims and objectives for Junior Certificate mathematics. Accordingly, *aims* and *general objectives* are specified in the Introduction to the syllabus document. They apply to all three syllabuses. To augment these broad aims and general objectives appropriately, each syllabus is introduced by a further specification of its purpose, by means of

- a *rationale*, describing the target group of students, the general scope and style of the syllabus, and aspects deserving particular emphasis in order to tailor the syllabus to the students' needs
- a statement of *level-specific aims*, highlighting aspects of the aims that are of particular relevance for the target group
- a listing of *assessment objectives*: a subset of the general objectives, to be interpreted in the light of the level-specific aims and hence suitably for the ability levels, developmental stages and learning styles of the different groups of students.

Sections 2.2 to 2.4 of the *Guidelines* discuss these features and set them in context. Against this background, Section 2.5 outlines certain principles that guided and constrained design of the syllabuses to meet the aims.

2.2 AIMS FOR JUNIOR CERTIFICATE MATHEMATICS

The aims formulated for Junior Certificate mathematics are derived from those specified for the current Leaving Certificate syllabuses, with appropriate alterations to suit the junior rather than the senior cycle of second level education. The Leaving Certificate aims were based on those specified in the booklet *Mathematics Education: Primary and Junior Cycle Post-Primary* produced by the Curriculum and Examinations Board in 1985. In the absence of a specific formulation for the Junior Certificate, these were taken as the best approximation to contemporary thinking about mathematics education in Ireland.

The syllabus document presents a common set of aims for the three syllabuses (Higher, Ordinary and Foundation level). They can be summarised and explained as follows.

It is intended that mathematics education should:

A. Contribute to the personal development of the students.

   This aim is chiefly concerned with the students' feelings of worth as a result of finding meaning and interest, as well as achieving success, in mathematics.

B. Help to provide them with the mathematical knowledge, skills and understanding needed for continuing their education, and eventually for life and work.

   This aim focuses on what the students will be able to do with their mathematics in the future: hence, on their ability to recognise the power of mathematics and to apply it appropriately.

Section 3.5 of this document describes one vision (not the only possible one) of how these aims might be addressed in the different content areas.

2.3 GENERAL OBJECTIVES FOR JUNIOR CERTIFICATE MATHEMATICS

The aspirational *aims* need to be translated into more specific *objectives* which, typically, specify what students should be able to do at the end of the junior cycle. As with the aims, the general objectives are modelled on those for the current Leaving Certificate syllabuses, notably in this case the most recently formulated set produced for Foundation level.

The objectives listed in the syllabus document can be summarised and explained as follows.

A. Students should be able to *recall* basic facts.

That is, they should have fundamental information readily available for use. Such information is not necessarily an end in itself; rather, it can support (and enhance) understanding and aid application.

B. They should be equipped with the *competencies* needed for mathematical activities.

Hence, they should be able to perform the basic skills and carry out the routine algorithms that are involved in "doing sums" (or other exercises), and be able to use appropriate equipment (such as calculators and geometrical instruments) – and they should also know when to do so. This kind of "knowing how" is called *instrumental understanding*: understanding that leads to getting something done.

C. They should have an overall picture of mathematics as *a system that makes sense.*

This involves understanding individual concepts and conceptual structures, and also seeing the subject as a logical discipline and an integrated whole. In general, this objective is concerned with "knowing why", or so-called *relational understanding*.

D. They should be able to *apply* their knowledge.

Thus, they should be able to use mathematics (and perhaps also to recognise uses beyond their own scope to employ) – hence seeing that it is a powerful tool with many areas of applicability.

E. The students should be able to *analyse* information, including information presented in unfamiliar contexts.

In particular, this provides the basis for exploring and solving extended or non-standard problems.

F. They should be able to *create* mathematics for themselves.

Naturally, we do not expect the students to discover or invent significant new results; but they may make informed guesses and then critique and debate these guesses. This may help them to feel personally involved in, and even to attain a measure of ownership of, some of the mathematics they encounter.

G. They should have developed the necessary *psychomotor* skills to attain the above objectives.

Thus, for example, the students should be enabled to present their mathematics in an orderly way, including constructions and other diagrams where relevant, and to operate a calculator or calculator software.

H. They should be able to *communicate* mathematics, both verbally and in writing.

Thus, they should be able to describe their mathematical procedures and insights and explain their arguments in their own words; and they should be able to present their working and reasoning in written form.

I. They should *appreciate* mathematics.

For some students, appreciation may come first only from carrying out familiar procedures efficiently and "getting things right". However, this can provide the confidence that leads to enjoyable recognition of mathematics in the environment and to its successful application to areas of common or everyday experience. The challenge of problems, puzzles and games provides another source of enjoyment. Aesthetic appreciation may arise, for instance, from the study of mathematical patterns (those occurring in nature as well as those produced by human endeavour), and the best students may be helped towards identifying the more abstract beauty of form and structure.

J. They should be *aware* of the history of mathematics.

The history of mathematics can provide a human face for the subject, as regards both the personalities involved and the models provided for seeing mathematics as a lively and evolving subject.

It is important to note that the objectives, like the aims, are common to all the Junior Certificate syllabuses (Higher, Ordinary, and Foundation level). However, they are intended to be interpreted appropriately at the different levels, and indeed for different students, bearing in mind their abilities, stages of development, and learning styles. This is particularly relevant for assessment purposes, as discussed in Section 5. It leads to the formulation of *level-specific aims* and their application to *assessment objectives,* discussed in Section 2.4.

2.4 RATIONALE, LEVEL-SPECIFIC AIMS AND ASSESSMENT OBJECTIVES

The aims and objectives together provide a framework for all the syllabuses. However, the fact that there are three different syllabuses reflects the varying provision that has to be made for those with different needs. Each syllabus has its own rationale, spelled out in the syllabus document. Key phrases in the three rationales are juxtaposed in the following table in order to highlight the intended thrust of each syllabus.

RATIONALE FOR THE HIGHER LEVEL	RATIONALE FOR THE ORDINARY LEVEL	RATIONALE FOR THE FOUNDATION LEVEL
[This] is geared to the needs of students of above average mathematical ability…. However, not all students … are … future users of academic mathematics.	[This] is geared to the needs of students of average mathematical ability.	[This] is geared to the needs of students who are unready for or unsuited by the mathematics of the Ordinary [level syllabus].
A balance must be struck, therefore, between challenging the most able students and encouraging those who are developing a little more slowly.	[It] … must start where these students are, offering mathematics that is meaningful and accessible to them at their present stage of development. It should also provide for the gradual introduction of more abstract ideas.	[It] must therefore help the students to construct a clearer knowledge of, and to develop improved skills in, basic mathematics, and to develop an awareness of its usefulness.
For the target group, particular emphasis can be placed on the development of powers of abstraction and generalisation and on an introduction to the idea of proof.	For the target group, particular emphasis can be placed on the development of mathematics as a body of knowledge and skills that makes sense and that can be used in many different ways – hence, as an efficient system for the solution of problems and provision of answers.	For the target group, particular emphasis can be placed on promoting students' confidence in themselves (confidence that they can do mathematics) and in the subject (confidence that mathematics makes sense).

In the light of these rationales, *level-specific aims* emphasise the various skills in ways that, hopefully, are appropriate to the levels of development of the target groups. The table opposite presents the aims for the three levels, as set out in the syllabus document.

SPECIFIC AIMS FOR HIGHER LEVEL	SPECIFIC AIMS FOR ORDINARY LEVEL	SPECIFIC AIMS FOR FOUNDATION LEVEL
[This] is intended to provide students with	[This] is intended to provide students with	[This] is intended to provide students with
• a firm understanding of mathematical concepts and relationships	• an understanding of mathematical concepts and of their relationships	• an understanding of basic mathematical concepts and relationships
• confidence and competence in basic skills	• confidence and competence in basic skills	• confidence and competence in basic skills
• the ability to formulate and solve problems	• the ability to solve problems	• the ability to solve simple problems
• an introduction to the idea of proof and to the role of logical argument in building up a mathematical system	• an introduction to the idea of logical argument	• experience of following clear arguments and of citing evidence to support their own ideas
• a developing appreciation of the power and beauty of mathematics and of the manner in which it provides a useful and efficient system for the formulation and solution of problems.	• appreciation both of the intrinsic interest of mathematics and of its usefulness and efficiency for formulating and solving problems.	• appreciation of mathematics both as an enjoyable activity through which they experience success and as a useful body of knowledge and skills.

Against this background, objectives can be specified for assessment leading to certification as part of the Junior Certificate. Mathematics to date in the Junior Certificate has been assessed solely by a terminal examination; consideration of alternative forms of assessment was outside the scope of the current revision. This has a considerable effect on what can be assessed for certification purposes – a point taken up more fully in Section 5 of this document.

The assessment objectives are objectives A, B, C, D (dealing with knowledge, understanding and application), G (dealing with psychomotor skills) and H (dealing with communication). These objectives, while the same for all three syllabuses, are to be interpreted in the context of the level-specific aims as described above. Further illustration is provided in Section 5.

2.5 PRINCIPLES OF SYLLABUS DESIGN

The aims and objectives are important determinants of syllabus content; but so also is the context in which the syllabuses are implemented. The most attractive syllabuses are doomed to failure if they cannot be translated into action in the classroom. With this in mind, several *principles* were specified in order to guide the syllabus design. They are particularly important for understanding the inclusion, exclusion, form of presentation, or intended sequencing of certain topics. The principles are displayed (in "boxes") overleaf.

A. The mathematics syllabuses should provide continuation from and development of the primary school curriculum, and should lead to appropriate syllabuses in the senior cycle.

Hence, the syllabuses should take account of the varied backgrounds, likely learning styles, potential for development, and future needs of the students entering second level education. Moreover, for the cohort of students proceeding from each junior cycle syllabus into the senior cycle, there should be clear avenues of progression.

B. The syllabuses should be implementable in the present circumstances and flexible as regards future development.

They should therefore be *teachable, learnable* and *adaptable*.

The points regarding teachability, learnability and adaptability can be considered in turn.

(a) The syllabuses should be *teachable*, in that it should be possible to implement them with the resources available.

- The syllabuses should be teachable in the time normally allocated to a subject in the Junior Certificate programme.
- Requirements as regards equipment should not go beyond that normally found in, or easily acquired by, Irish schools.
- The aims and style of the syllabuses should be ones that teachers support and can address with confidence, and the material should in general be familiar.

(b) The syllabuses should be *learnable*, by virtue of being appropriate to the different cohorts of students for whom they are designed.

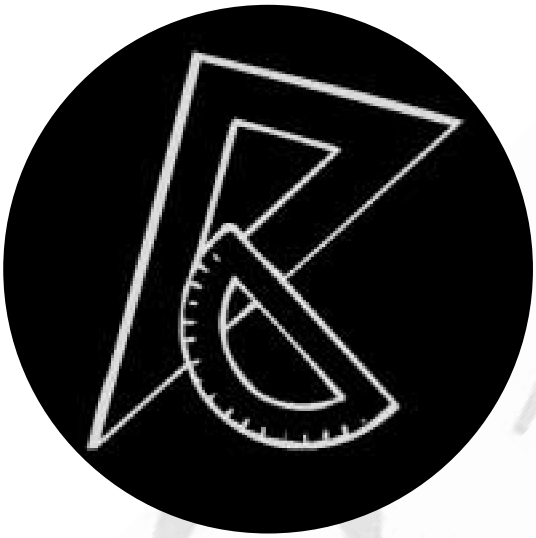
- Each syllabus should start where the students in its target group are at the time, should move appropriately from the concrete to or towards the abstract, and should proceed to suitable levels of difficulty.
- The approaches used should accommodate the widest possible range of abilities and learning styles.
- They should cater for the interests and needs of all groups in the population.
- The materials and methods should be such that students are motivated to learn.

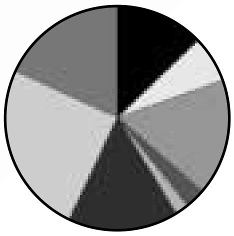
(c) The syllabuses should be *adaptable* – designed so that they can serve present needs and also can evolve in future.

C. The mathematics they contain should be sound, important and interesting.

In order to cater for the differing interests of students and teachers, a broad range of appropriate aspects of mathematics should be included. Where possible the mathematics should be *applicable*, and the applications should be such that they can be made clear to the students (now, rather than in some undefined future) and can be addressed, at least to some extent, within the course.

Syllabus structure and content





3.1 INTRODUCTION

The way in which the mathematical content in the syllabuses is organised and presented in the syllabus document is described in sections 3.2 and 3.3. Section 3.3 also discusses the main alterations, both in content and in emphasis, with respect to the preceding versions. The forthcoming changes in the primary curriculum, which will have "knock-on" effects at second level, are outlined in Section 3.4. Finally, in Section 3.5, the content is related to the aims of the syllabuses.

3.2 STRUCTURE

For the Higher and Ordinary level syllabuses, the mathematical material forming the content is divided into eight sections, as follows:

- Sets
- Number systems
- Applied arithmetic and measure
- Algebra
- Statistics
- Geometry
- Trigonometry
- Functions and graphs

The corresponding material for the Foundation level syllabus is divided into seven sections; there are minor differences in the sequence and headings, resulting in the following list:

- Sets
- Number systems
- Applied arithmetic and measure
- Statistics and data handling
- Algebra
- Relations, functions and graphs
- Geometry

The listing by content area is intended to give mathematical coherence to the syllabuses, and to help teachers locate specific topics (or check that topics are not listed). The content areas are reasonably distinct, indicating topics with different historical roots and different main areas of application. However, they are inter-related and interdependent, and it is not intended that topics would be dealt with in total isolation from each other. Also, while the seven or eight areas, and the contents within each area, are presented in a logical sequence – combining, as far as possible, a sensible mathematical order with a developmental one for learners – it is envisaged that many content areas listed later in the syllabus would be introduced before or alongside those listed earlier. (For example, geometry appears near the end of the list, but the course committee specifically recommends that introductory geometrical work is started in First Year, allowing plenty of time for the ideas to be developed in a concrete way, and thoroughly understood, before the more abstract elements are introduced.) However, the different order of listing for the Foundation level syllabus does reflect a suggestion that the introduction of some topics (notably formal algebra) might be delayed. Some of these points are taken up in Section 4.

Appropriate pacing of the syllabus content over the three years of the junior cycle is a challenge. Decisions have to be made at class or school level. Some of the factors affecting the decisions are addressed in these *Guidelines* in Section 4, under the heading of planning and organisation.

3.3 SYLLABUS CONTENT

The contents of the Higher, Ordinary and Foundation level syllabuses are set out in the corresponding sections of the syllabus document. In each case, the content is presented in the two-column format used for the Leaving Certificate syllabuses introduced in the 1990s, with the left-hand column listing the topics and the right-hand column adding notes (for instance, providing illustrative examples, or highlighting specific aspects of the topics which are included or excluded). Further illustration of the depth of treatment of topics is given in Section 5 (in dealing with assessment) and in the *proposed sample assessment materials* (available separately).

CHANGES IN CONTENT

As indicated in Section 1, the revisions deal only with specific problems in the previous syllabuses, and do not reflect a root-and-branch review of the mathematics education appropriate for students in the junior cycle. The *main* changes in content, addressing the problems identified in Section 1, are described below. A *summary of all the changes* is provided in Appendix 1.

- Calculators and calculator-related techniques

  As pointed out in the introduction to each syllabus, calculators are assumed to be readily available for *appropriate* use, both as teaching/learning tools and as computational aids; they will also be allowed in examinations.

  The concept of "appropriate" use is crucial here. Calculators are part of the modern world, and students need to be able to use them efficiently where and when required. Equally, students need to retain and develop their feel for number, while the execution of mental calculations, for instance to make estimates, becomes even more important than it was heretofore. *Estimation*, which was not mentioned in the 1987 syllabus (though it was covered in part by the phrase "the practice of approximating before evaluating"), now appears explicitly and will be tested in examinations.

  The importance of the changes in this area is reflected in two developments. First, a set of guidelines on calculators is being produced. It addresses issues such as the purchase of suitable machines as well as the rationale for their use. Secondly, in 1999 the Department of Education and Science commissioned a research project to monitor numeracy-related skills (with and without calculators) over the period of introduction of the revised syllabuses. If basic numeracy and mental arithmetic skills are found to disimprove, remedial action may have to be taken. It is worth noting that research has not so far isolated any consistent association between calculator use in an education system and performance by students from that system in international tests of achievement.

  Mathematical tables are not mentioned in the content sections of the syllabus, except for a brief reference indicating that they are assumed to be available, likewise for appropriate use. Teachers and students can still avail of them as learning tools and for reference if they so wish. Tables will continue to be available in examinations, but questions will not specifically require students to use them.

- Geometry

  The approach to *synthetic geometry* was one of the major areas which had to be confronted in revising the syllabuses. Evidence from examination scripts suggested that in many cases the presentation in the 1987 syllabus was not being followed in the classroom. In particular, in the Higher level syllabus, the sequence of proofs and intended proof methods were being adapted. Teachers were responding to students' difficulties in coping with the approach that attempted to integrate transformational concepts with those more traditionally associated with synthetic geometry, as described in Section 1.1 of these *Guidelines*.

  For years, and all over the world, there have been difficulties in deciding how – indeed, whether – to present synthetic geometry and concepts of logical proof to students of junior cycle standing. Their historical importance, and their role as guardians of one of the defining aspects of mathematics as a discipline, have led to a wish to retain them in the Irish mathematics syllabuses; but the demands made on students who have not yet reached the Piagetian stage of formal operations are immense. "Too much, too soon" not only contravenes the principle of learnability (section 2.5), but leads to rote learning and hence failure to attain the objectives which the geometry sections of the syllabuses are meant to address. The constraints of a minor revision precluded the question of "whether" from being asked on this occasion. The question of "how" raises issues to do with the principles of soundness versus learnability. The resulting formulation set out in the syllabus does not claim to be a full description of a geometrical system. Rather, it is intended to provide a defensible *teaching sequence* that will allow students to learn geometry meaningfully and to come to realise the power of proof. Some of the issues that this raises are discussed in Appendix 2.

  The revised version can be summarised as follows.

  - The approach omits the transformational elements, returning to a more traditional approach based on congruency.

  - In the interests of consistency and transfer between levels, the underlying ideas are basically the same across all three syllabuses, though naturally they are developed to very different levels in the different syllabuses.

- The system has been carefully formulated to display the power of logical argument at a level which – hopefully – students can follow and appreciate. It is therefore strongly recommended that, in the classroom, material is introduced in the sequence in which it is listed in the syllabus document. *For the Higher level syllabus, the concepts of logical argument and rigorous proof are particularly important. Thus, in examinations, attempted proofs that presuppose "later" material in order to establish "earlier" results will be penalised. Moreover,* ***proofs using transformations will not be accepted.***

- To shorten the Higher level syllabus, only some of the theorems have been designated as ones for which students may be asked to supply proofs in the examinations. *The other theorems should still be proved as part of the learning process;* students should be able to follow the logical development, and see models of far more proofs than they are expected to reproduce at speed under examination conditions. The required saving of time is expected to occur because students do not have to put in the extra effort needed to develop fluency in writing out particular proofs.

- Students taking the Ordinary and *(a fortiori)* Foundation level syllabuses are not required to prove theorems, but – in accordance with the level-specific aims (Section 2.4) – should experience the logical reasoning involved in ways in which they can understand it. The general thrust of the synthetic geometry section of the syllabuses for these students is not changed from the 1987 versions.

- It may be noted that the formulation of the Foundation level syllabus in 1987 emphasised the *learning process* rather than the product or outcomes. In the current version, the teaching/learning suggestions are presented in these *Guidelines* (chiefly in Section 4), not in the syllabus document. It is important to emphasise that the changed formulation in the syllabus is not meant to point to a more formal presentation than previously suggested for Foundation level students.

Section 4.9 of this document contains a variety of suggestions as to how the teaching of synthetic geometry to junior cycle students might be addressed.

Transformation geometry still figures in the syllabuses, but is treated separately from the formal development of synthetic geometry. The approach is intended to be intuitive, helping students to develop their visual and spatial ability. There are opportunities here to build on the work on symmetry in the primary curriculum and to develop aesthetic appreciation of mathematical patterns.

- Other changes to the Higher level syllabus
  - Logarithms are removed. Their practical role as aids to calculation is outdated; the theory of logarithms is sufficiently abstract to belong more comfortably to the senior cycle.
  - Many topics are "pruned" in order to shorten the syllabus.

- Other changes to the Ordinary level syllabus
  - The more conceptually difficult areas of algebra and coordinate geometry are simplified.
  - A number of other topics are "pruned".

- Other changes to the Foundation level syllabus
  - There is less emphasis on fractions but rather more on decimals. (The change was introduced partly because of the availability of calculators – though, increasingly, calculators have buttons and routines which allow fractions to be handled in a comparatively easy way.)
  - The coverage of statistics and data handling is increased. These topics can easily be related to students' everyday lives, and so can help students to recognise the relevance of mathematics. They lend themselves also to active learning methods (such as those presented in Section 4) and the use of spatial as well as computational abilities. Altogether, therefore, the topics provide great scope for enhancing students' enjoyment and appreciation of mathematics. They also give opportunities for developing suitably concrete approaches to some of the more abstract material, notably algebra and functions (see Section 4.8).
  - The algebra section is slightly expanded. The formal algebraic content of the 1987 syllabus was so slight that students may not have had scope to develop their understanding; alternatively, teachers may have chosen to omit the topic. The rationale for the present adjustment might be described as "use it or lose it". The hope is that the students will be able to use it, and that – suitably addressed – it can help

them in making some small steps towards the more abstract mathematics which they may need to encounter later in the course of their education.

Overall, therefore, it is hoped that the balance between the syllabuses is improved. In particular, the Ordinary level syllabus may be better positioned between a more accessible Higher level and a slightly expanded Foundation level.

CHANGES IN EMPHASIS

The brief for revision of the syllabuses, as described in Section 1.2, precluded a root-and-branch reconsideration of their style and content. However, it did allow for some changes in *emphasis*: or rather, in certain cases, for some of the intended emphases to be made more explicit and more clearly related to rationale, content, assessment, and – via the *Guidelines* – methodology. The changes in, or clarification of, emphasis refer in particular to the following areas.

- Understanding

  General objectives B and C of the syllabus refer respectively to instrumental understanding (knowing "what" to do or "how" to do it, and hence being able to execute *procedures*) and relational understanding (knowing "why", understanding the *concepts* of mathematics and the way in which they connect with each other to form so-called "conceptual structures"). When people talk of teaching mathematics for – or learning it with – understanding, they usually mean *relational understanding*. The language used in the Irish syllabuses to categorise understanding is that of Skemp; the objectives could equally well have been formulated in terms of "procedures" and "concepts".

  Research points to the importance of *both* kinds of understanding, together with knowledge of facts (general objective A), as components of mathematical proficiency, with relational understanding being crucial for retaining and applying knowledge. The Third International Mathematics and Science Study, TIMSS, indicated that Irish teachers regard knowledge of facts and procedures as particularly important – unusually so in international terms; but it would appear that less heed is paid to conceptual/relational understanding. This is therefore given special emphasis in the revised syllabuses. Such understanding can be fostered by *active learning*, as described and illustrated in Section 4. Ways in which relational understanding can be assessed are considered in Section 5.

- Communication

  General objective H of the syllabus indicates that students should be able to *communicate* mathematics, both verbally and in written form, by describing and explaining the mathematical procedures they undertake and by explaining their findings and justifying their conclusions. This highlights the importance of students expressing mathematics in their own words. It is one way of promoting understanding; it may also help students to take *ownership* of the findings they defend, and so to be more interested in their mathematics and more motivated to learn.

  The importance of discussion as a tool for ongoing assessment of students' understanding is highlighted in Section 5.2. In the context of examinations, the ability to show different stages in a procedure, explain results, give reasons for conclusions, and so forth, can be tested; some examples are given in Section 5.6.

- Appreciation and enjoyment

  General objective I of the syllabus refers to *appreciating* mathematics. As pointed out earlier, appreciation may develop for a number of reasons, from being able to do the work successfully to responding to the abstract beauty of the subject. It is more likely to develop, however, when the mathematics lessons themselves are pleasant occasions.

  In drawing up the revised syllabus and preparing the *Guidelines*, care has been taken to include opportunities for making the teaching and learning of mathematics more enjoyable. Enjoyment is good in its own right; also, it can develop students' motivation and hence enhance learning. For many students in the junior cycle, enjoyment (as well as understanding) can be promoted by the *active learning* referred to above and by placing the work in appropriate meaningful contexts. Section 4 contains many examples of enjoyable classroom activities which promote both learning and appreciation of mathematics. Teachers are likely to have their own battery of such activities which work for them and their classes. It is hoped that these can be shared amongst their colleagues and perhaps submitted for inclusion in the final version of the *Guidelines*.

  Of course, different people enjoy different kinds of mathematical activity. Appreciation and enjoyment do not come solely from "games"; more traditional classrooms also can be lively places in which teachers and students collaborate in the teaching and learning of mathematics and develop their appreciation of the subject. *Teachers will choose approaches with which they themselves feel comfortable and which meet the learning needs of the students whom they teach.*

The changed or clarified emphasis in the syllabuses will be supported, where possible, by corresponding adjustments to the formulation and marking of Junior Certificate examination questions. *While the wording of questions may be the same, the expected solutions may be different.* Examples are given in Section 5.

3.4 CHANGES IN THE PRIMARY CURRICULUM

The changes in content and emphasis within the revised Junior Certificate mathematics syllabuses are intended, *inter alia*, to follow on from and build on the changes in the primary curriculum. The forthcoming alterations (scheduled to be introduced in 2002, but perhaps starting earlier in some classrooms, as teachers may anticipate the formal introduction of the changes) will affect the knowledge and attitudes that students bring to their second level education. Second level teachers need to be prepared for this. A summary of the chief alterations is given below; teachers are referred to the revised Primary School Curriculum for further details.

CHANGES IN EMPHASIS

In the revised curriculum, the main changes of *emphasis* are as follows.

- There is more emphasis on
  - setting the work in real-life contexts
  - learning through hands-on activities (using concrete materials/manipulatives, and so forth)
  - understanding (in particular, gaining appropriate *relational understanding* as well as *instrumental understanding*)
  - appropriate use of mathematical language
  - recording
  - problem-solving.
- There is less emphasis on
  - learning routine procedures with no context provided
  - doing complicated calculations.

CHANGES IN CONTENT

The changes in emphasis are reflected in changes to the *content*, the main ones being as follows.

- New areas include
  - introduction of the calculator from Fourth Class (augmenting, not replacing, paper-and-pencil techniques)
  - (hence) extended treatment of estimation;
  - increased coverage of data handling
  - introduction of basic probability ("chance").
- New terminology includes
  - the use of the "positive" and "negative" signs for denoting a number (as in $^{+}3$ [positive three], $^{-}6$ [negative six] as well as the "addition" and "subtraction" signs for denoting an operation (as in $7 + 3$, $24 – 9$)
  - explicit use of the multiplication sign in formulae (as in $2 \times r$, $l \times w$).
- The treatment of subtraction emphasises the "renaming" or "decomposition" method (as opposed to the "equal additions" method – the one which uses the terminology "paying back") even more strongly than does the 1971 curriculum. Use of the word "borrowing" is discouraged.
- The following topics are among those excluded from the revised curriculum:
  - unrestricted calculations (thus, division is restricted to at most four-digit numbers being divided by at most two-digit numbers, and – for fractions – to division of whole numbers by unit fractions)

- subtraction of negative integers
- formal treatment of LCM and HCF
- use of formulae which the children have not developed
- two-step equations and "rules" for manipulating equations (that is, emphasis is on intuitive solution)
- sets (except their intuitive use in developing the concept of number)
- π and advanced properties of circles.

(Some of these topics were not formally included in the 1971 curriculum, but appeared in textbooks and were taught in many classrooms.)

NOTE

The reductions in content have removed some areas of *overlap* between the 1971 Primary School Curriculum and the Junior Certificate syllabuses. Some overlap remains, however. This is natural; students entering second level schooling need to revise the concepts and techniques that they have learnt at primary level, and also need to situate these in the context of their work in the junior cycle.

3.5 LINKING CONTENT AREAS WITH AIMS

Finally, in this section, the content of the syllabuses is related to the aims and objectives. In fact most aims and objectives can be addressed in most areas of the syllabuses. However, some topics are more suited to the attainment of certain goals or the development of certain skills than are others. The discussion below highlights some of the main possibilities, and points to the goals that might appropriately be emphasised when various topics are taught and learnt. Phrases italicised are quoted or paraphrased from the *aims* as set out in the syllabus document. Section 5 of these *Guidelines* indicates a variety of ways in which achievement of the relevant *objectives* might be encouraged, tested or demonstrated.

SETS

Sets provide a conceptual foundation for mathematics and a language by means of which mathematical ideas can be discussed. While this is perhaps the main reason for which set theory was introduced into school mathematics, its importance at junior cycle level can be described rather differently.

- Set problems, obviously, call for skills of *problem-solving;* in particular, they provide occasions for logical argument. By using data gathered from the class, they even offer opportunities for simple introduction to *mathematical modelling* in contexts to which the students can relate.
- Moreover, set theory emphasises aspects of mathematics that are not purely computational. Sets are about classification, hence about tidiness and organisation. This can lead to *appreciation* of mathematics on aesthetic grounds and can help to *provide a basis for further education in the subject.*
- An additional point is that this topic is not part of the Primary School Curriculum, and so represents a new start, untainted by previous failure. For some students, therefore, there are particularly important opportunities for *personal development.*

NUMBER SYSTEMS

While mathematics is not entirely quantitative, numeracy is one of its most important aspects. Students have been building up their concepts of numbers from a very early stage in their lives. However, moving from familiarity with natural numbers (and simple operations on them) to genuine understanding of the various forms in which numbers are presented and of the uses to which they are put in the world is a considerable challenge.

- Weakness in this area destroys students' *confidence and competence* by depriving them of the *knowledge, skills and understanding needed for continuing their education and for life and work.* It therefore handicaps their *personal fulfilment* and hence *personal development*.

The aspect of "understanding" is particularly important – or, perhaps, has had its importance highlighted – with advances in technology.

- Students need to become familiar with the intelligent and appropriate use of calculators, while avoiding dependence on the calculators for simple calculations.
- Complementing this, they need to develop skills in estimation and approximation, so that numbers can be used meaningfully.

APPLIED ARITHMETIC AND MEASURE

This topic is perhaps one of the easiest to justify in terms of *providing mathematics needed for life, work and leisure*.

- Students are likely to use the skills developed here in "everyday" applications, for example in looking after their personal finances and in structuring the immediate environment in which they will live. For many, therefore, this may be a key section in *enabling students to develop a positive attitude towards mathematics as a valuable subject of study*.
- There are many opportunities for *problem-solving*, hopefully in contexts that the students recognise as relevant.
- The availability of calculators may remove some of the drudgery that can be associated with realistic problems, helping the students to focus on the concepts and applications that bring the topics to life.

ALGEBRA

Algebra was developed because it was needed – because arguments in natural language were too clumsy or imprecise. It has become one of the most fundamental tools for mathematics.

- As with number, therefore, *confidence* and *competence* are very important. Lack of these undermine *the personal development of the students* by depriving them of the *knowledge, skills and understanding needed for continuing their education and for life and work.*
- Without skills in algebra, students lack the technical preparation for *study of other subjects in school*, and in particular their *foundation for appropriate studies later on* – including *further education in mathematics itself*.

It is thus particularly important that students develop appropriate *understanding* of the basics of algebra so that algebraic techniques are carried out meaningfully and not just as an exercise in symbol-pushing.

- Especially for weaker students, this can be very challenging because algebra involves *abstractions and generalisations.*
- However, these characteristics are among the strengths and beauties of the topic. Appropriately used, algebra can enhance the students' *powers of communication*, facilitate simple *modelling* and *problem-solving*, and hence illustrate the power of mathematics as a *valuable subject of study.*

STATISTICS

One of the ways in which the world is interpreted for us mathematically is by the use of statistics. Their prevalence, in particular on television and in the newspapers, makes them part of the environment in which children grow up, and provides students with opportunities for *recognition and enjoyment of mathematics in the world around them.*

- Many of the examples refer to the students' typical areas of interest; examples include sporting averages and trends in purchases of (say) CDs.
- Students can provide data for further examples from their own backgrounds and experiences.
- Presenting these data graphically can *extend students' powers of communication and their ability to share ideas with other people*, and may also provide an *aesthetic* element.
- The fact that statistics can help to *develop a positive attitude towards mathematics as an interesting and valuable subject of study* – even for weaker students who find it hard to appreciate the more abstract aspects of the subject – explains the extra prominence given to aspects of data handling in the Foundation level syllabus, as mentioned earlier. They may be particularly important in promoting *confidence and competence* in both numerical and spatial domains.

GEOMETRY

The study of geometry builds on the primary school study of shape and space, and hence relates to *mathematics in the world around us*. In the junior cycle, different approaches to geometry address different educational goals.

Synthetic geometry is traditionally intended to promote *students' ability to recognise and present logical arguments*.

- More able students address one of the greatest of mathematical concepts, that of proof, and hopefully come to appreciate the *abstractions and generalisations* involved.
- Other students may not consider formal proof, but should be able to draw appropriate conclusions from given geometrical data.

- Explaining and defending their findings, in either case, should help students to *further their powers of communication*.
- Tackling "cuts" and other exercises based on the geometrical system presented in the syllabus allows students to *develop their problem-solving skills*.
- Moreover, in studying synthetic geometry, students are encountering one of the great monuments to intellectual endeavour: a very special part of Western culture.

Transformation geometry builds on the study of symmetry at primary level. As the approach to transformation geometry in the revised Junior Certificate syllabus is intuitive, it is included in particular for its *aesthetic value*.

- With the possibility of using transformations in artistic designs, it allows students to encounter *creative aspects of mathematics* and to develop or exercise their own creative skills.
- It can also develop their spatial ability, hopefully promoting *confidence and competence* in this area.
- Instances of various types of symmetry in the natural and constructed environment give scope for students' *recognition and enjoyment of mathematics in the world around them*.

Coordinate geometry links geometrical and algebraic ideas. On the one hand, algebraic techniques can be used to establish geometric results; on the other, algebraic entities are given pictorial representations.

- Its connections with functions and trigonometry, as well as algebra and geometry, make it a powerful tool for the integration of mathematics into a unified structure.
- It illustrates the power of mathematics, and so helps to establish it with students as a *valuable subject of study*.
- It provides an important *foundation for appropriate studies later on*.
- The graphical aspect can add a visually *aesthetic* dimension to algebra.

TRIGONOMETRY

Trigonometry is a subject that has its roots in antiquity but is still of great practical use to-day. While its basic concepts are abstract, they can be addressed through practical activities.

- Situations to which it can be applied – for example, house construction, navigation, and various ball games – include many that are relevant to the students' *life, work and leisure*.
- It can therefore promote the students' *recognition and enjoyment of mathematics in the world around them*.
- With the availability of calculators, students may more easily develop *competence and confidence* through their work in this area.

FUNCTIONS AND GRAPHS

The concept of a function is crucial in mathematics, and students need a good grasp of it in order to prepare a *firm foundation for appropriate studies later on* and *in particular, a basis for further education in mathematics itself.*

- The representation of functions by graphs adds a pictorial element that students may find *aesthetic* as well as enhancing their understanding and their *ability to handle generalisations*.
- This topic pulls together much of the groundwork done elsewhere, using the tools introduced and skills developed in earlier sections and providing opportunities for *problem-solving* and simple *modelling*.

For Foundation students alone, simple work on the set-theoretic treatment of relations has been retained. In contexts that can be addressed by those whose numerical skills are poor, it provides exercises in simple logical thinking.

NOTE

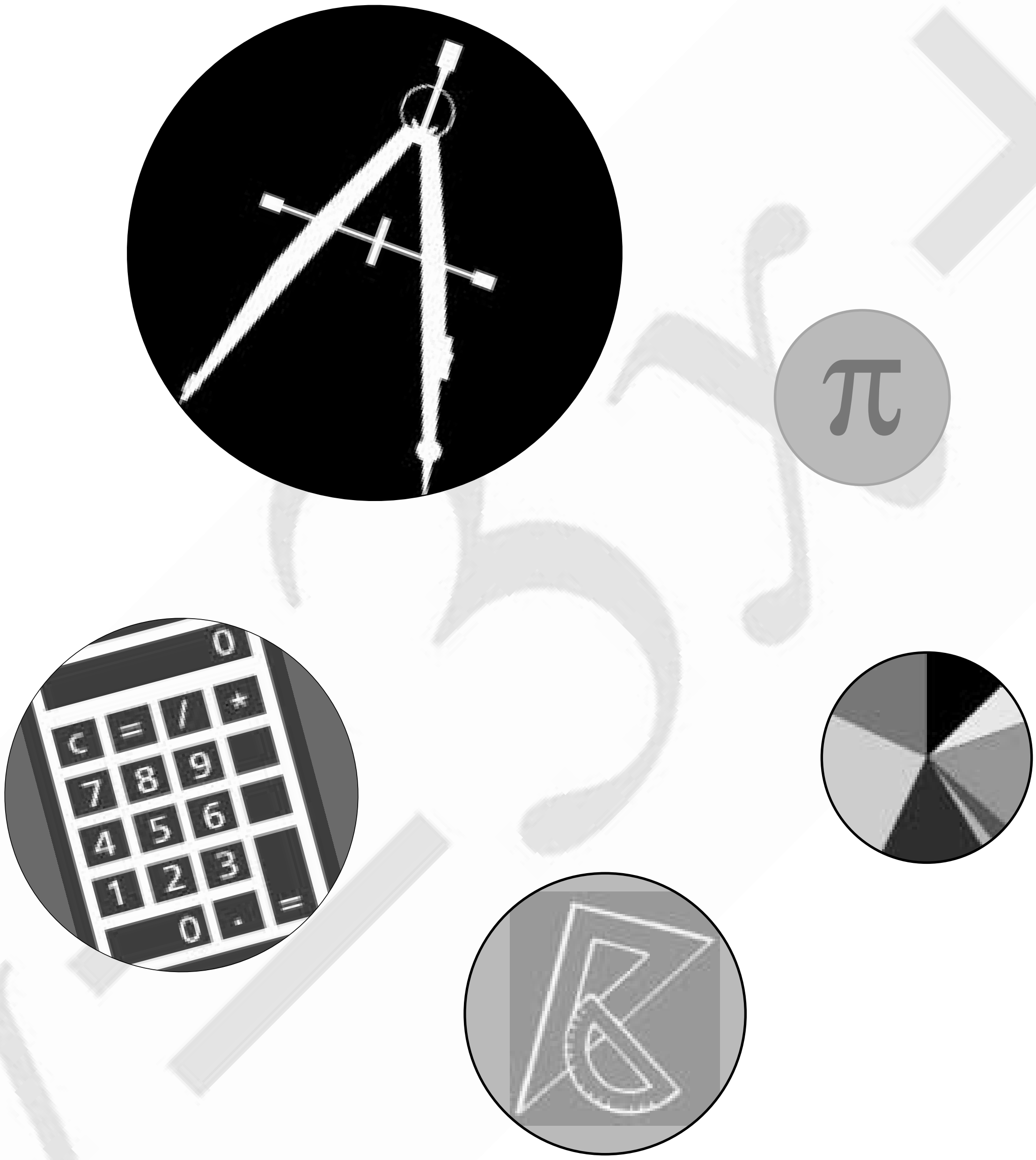
The foregoing argument presents just one vision of the rationale for including the various topics in the syllabus and for the ways in which the aims of the mathematics syllabus can be achieved. All teachers will have their own ideas about what can inspire and inform different topic areas. Their own personal visions of mathematics, and their particular areas of interest and expertise, may lead them to implement the aims very differently from the way that is suggested here. Visions can profitably be debated at teachers' meetings, with new insights being given and received as a result.

The tentative answers given here with regard to *why certain topics are included in the syllabus* are, of course, offered to teachers rather than junior cycle students. In some cases, students also may find the arguments

relevant. In other cases, however, the formulation is too abstract or the benefit too distant to be of interest.
This, naturally, can cause problems. Clearly it would not be appropriate to reduce the syllabus to material that has immediately obvious applications in the students' everyday lives. This would leave them unprepared for further study, and would deprive them of sharing parts of our culture; in any case, not all students are motivated by supposedly everyday topics.

Teachers are therefore faced with a challenging task in helping students find interest and meaning in all parts of the work. Many suggestions with proven track records in Irish schools are offered in Section 4. As indicated earlier, it is hoped that teachers will offer more ideas for an updated version of the *Guidelines*.

Approaches to planning and methodology



4.1 INTRODUCTION

This section describes what arguably is the key aspect of mathematics education: the teaching and learning that takes place in schools or as a result of in-school activities. Section 4.2 addresses *planning*. It raises some of the questions that must be answered – within an individual school – before class groups are formed and timetables drawn up; it also considers the problem of time allocation to different topics in a given class. From Section 4.3 onwards, the focus moves to actual classroom activities.

Section 4.3 describes the *active learning* that is a recurrent theme of these *Guidelines*. The sections that follow contain the collection of lesson ideas contributed by teachers. Most of the ideas are grouped by topic area, so Sections 4.4 to 4.11 deal with the areas into which the content sections of the syllabuses are divided. Some cross-curricular themes are presented in the remaining sections. The suggestions in Section 4 are complemented by a list of *resources for teaching and learning* that appears as Appendix 3.

4.2 PLANNING AND ORGANISATION

The way in which a school organises itself for teaching and learning depends on the vision of education encapsulated, for example, in the school's mission statement, on the collective gifts of the staff, and on the perceived needs of the students. Issues such as time allocated to different subjects and grouping of students within subject areas are addressed against the background of such views. These *Guidelines* cannot presume to dictate, or to offer detailed suggestions; rather, they can pose some key questions that may inform school planning.

TIMING

One of the eight areas of experience which constitute the curriculum at junior cycle level is "mathematical studies and applications". The current NCCA recommendation is that this area should be allocated at least ten per cent of curriculum time. As pointed out in Section 1.2, the length of the former Higher level syllabus, in particular, was one of the problems to be addressed by the revision. In fact both the Higher and the Ordinary level syllabuses have been shortened, and this should facilitate appropriate learning in the time available. Given the concern about recent poor performance in the Junior Certificate examinations, it is important that sufficient time is allocated for students to develop the required concepts and practise the associated skills.

Once time allocations are made, teachers are faced with the problem of *pacing* their teaching suitably. Decisions are a matter for the team of teachers of mathematics in the school (as part of school planning) and for the individual teacher. While no prescriptions can be made in this area, two *recommendations* might be taken into account.

- A slow start, in which an effort is made to establish concepts as well as to develop skills, can provide a firmer basis for later work, and time "lost" at that stage can be regained subsequently.
- The course committee specifically recommends that some work in synthetic geometry is undertaken in each of the three years of the course. However, this does *not* mean that the formal aspects should be addressed in First Year, when "hands-on" practical and discovery-oriented approaches may be more appropriate (see Section 4.3 below).

To facilitate planning and record-keeping, spreadsheets listing the topics on the syllabus for each level are provided in Appendix 4.

GROUPING

With three distinct syllabuses being provided for Junior Certificate mathematics, decisions have to be made at some stage as to the level at which students should work. Depending on school policy with regard to ability grouping, formal *differentiation* – if it occurs at all – may take place as early as the beginning of First Year or as late as shortly before the examinations. The choice of an appropriate time is difficult. On the one hand, premature differentiation may close off avenues of progression at senior cycle level (and hence beyond) for some students; on the other, the three syllabuses are geared to different types of learning, and delayed differentiation may mean that students are faced with material or teaching approaches not suited to their current needs.

Again, prescriptions cannot be made. However, in addition to the issues already raised, the following specific points should perhaps be taken into account.

- Very late differentiation may allow inadequate time for the students to "fine-tune" their approaches to the relevant examination (a source of difficulty suggested in the Chief Examiners' Reports). This applies *a fortiori*, obviously, when a student takes papers at a level for which s/he has not been prepared.

- When the Foundation level syllabus was first introduced (as Syllabus C for the Intermediate Certificate), it was envisaged that it might be taught by methods more familiar at primary than at second level: that is, in ways suitable for the concrete operational level at which students would still be working. If students remain in Ordinary level classes until they are thoroughly confused, and until they are convinced both that mathematics does not make sense and that they will never be able to do it, they may obtain only limited benefits from eventual transfer to Foundation level.
- However, particular problems are raised by early selection for the Foundation level with respect to progression to the senior cycle and to eligibility for various careers as a result.

Certain teaching approaches may help in allowing students to be kept together, and may give them every opportunity of developing their mathematical abilities before decisions have finally to be made. The approaches include those described in Section 4.3.

4.3 ACTIVE LEARNING METHODOLOGIES

All mathematics teachers have their own personal visions of what should happen in the ideal mathematics classroom and what can be achieved in the circumstances in which they teach. These visions differ, just as teachers themselves differ. An approach that suits one teacher, working with a particular class, may not suit another teacher, or indeed the same teacher with a different class. Both so-called "traditional" and "progressive" methods can be successful.

However, some teachers have found that methods that were successful in the past do not work so well with to-day’s students. Some teachers would like to experiment with a broader range of teaching approaches, and would welcome guidance as to how they might do so. This section of the *Guidelines* addresses the issue, offers a rationale for using *active learning methodologies* and describes a range of such methodologies. The following sections provide concrete examples of the methodologies – reflecting work done by teachers in Irish schools – across a wide range of topic areas. Naturally, these approaches are meant to *augment*, not *replace*, existing ones. In particular, provision of opportunities for frequent practice (so that students’ procedural skills become, and then remain, appropriately fluent) will still be an integral and essential feature in the teaching of mathematics. It is hoped that teachers will study the methodologies outlined below, apply those with which they feel comfortable, and adapt their own methods correspondingly, if necessary.

RATIONALE

As indicated in Section 3.3, the revised mathematics syllabus gives greater emphasis than did its predecessor to the following: *increased depth of understanding, effective communication,* and *appreciation and enjoyment of mathematics.* Specifically, objectives C, E, G, H, and I emphasise

- relational learning
- the making of simple mathematical models and the justifying of conclusions
- the exercise of psychomotor skills
- communicating verbally
- appreciating mathematics and recognising it throughout the curriculum and beyond.

It is unlikely that these skills and learning outcomes will be achieved if mathematics teachers rely solely on a teaching methodology which is modelled on "exposition, examples and exercise." Research indicates that, where a broader range of methodologies is used,

- student confidence is enhanced and performance improved
- skills of estimation, approximation, analysis and evaluation can be practised
- a sense of ownership develops around students’ own learning
- enhanced student understanding of mathematical topics occurs
- communication and dialogue improve
- more positive attitudes to mathematics develop
- a lexicon of mathematical terms can be built up gradually in a natural way for students.

RANGE OF ACTIVE LEARNING METHODOLOGIES

There is a wide range of active learning methodologies that mathematics teachers can employ to achieve increased conceptual understanding. Some of these are listed below.

"Hands-on" activities with concrete materials

Project work

Mathematical games

Problem solving

Quiz activities

Use of information and communication technologies, in particular multimedia

Mathematical simulations

Structured discussion

Surveys, for example using questionnaires

Visits to the Science laboratory

Presentations

Demonstrations

Debates

Visits from a mathematics expert

Fieldwork

The lesson ideas that follow involve a number of these methodologies. In particular, many of them attempt to increase the use of *practical work* and *concrete materials*. Other lesson ideas involve student-centred investigative work, and yet others demand group work, student and teacher dialogue and project work where appropriate. The lesson ideas have been tried and tested by practising mathematics teachers in Irish schools. They are not intended to form an exhaustive list, and the range and sample can be extended in the future. Teachers are invited to contribute to this initiative by documenting lesson ideas which they have found useful and effective for a particular topic. Such ideas may relate to how a teacher might introduce a topic, how part of the topic might be taught, or how the work might be revised. For this purpose, a template is included in Appendix 4.

4.4 SETS

SETS LESSON IDEA 1

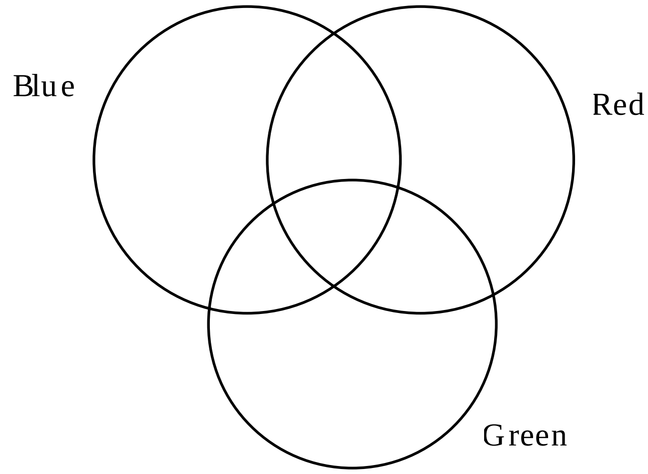
TITLE: VENN DIAGRAMS, SET NOTATION – AND ACTION!

TOPIC: SETS

AIM:
1. That students will engage in an active learning experience of sets.
2. That students will practise changing from English words to set symbols and vice versa.
3. That students will understand the relationship between set symbols, English words and Venn diagrams.

RESOURCES:

A large clear space (gym, courtyard). A set of large cards (at least A4 size) with the numbers 1 to 10 on them. A flip chart and marker or a blackboard. A large Venn diagram with three overlapping circles marked on the floor with chalk/paint/tape/rope. It is preferable if each circle is a different colour, for example blue, green and red.



METHOD:

1. A class of thirty is divided into three groups of ten. One group of ten students represents the ten elements from 1 to 10 (each holding one card from the range 1 to 10). Another group of ten students will give instructions in English (E) regarding where the students representing the elements 1 to 10 will be placed in relation to three Venn diagrams. The final group of ten students will give instructions in mathematics (M) (i.e. using symbols) on a flip chart to replicate the instructions given in English by the previous group of students.

2. Here is an example of how the game might progress. The first E student calls out: *Number 1 is in the blue set but he's not in the red set or the green set*. So, number 1 element goes and stands in the relevant section.

3. The first M student writes on flip chart: $1 \in B \setminus (R \cup G)$ or similar set notation to describe the position of the student representing the element 1.

4. The next E student calls out: *Number 2 is in all of the sets*. The student representing the number 2 element goes and stands in the relevant section.

5. The next M student writes on flip chart: $2 \in B \cap R \cap G$

6. The game continues until all of the elements 1 to 10 have been placed.

7. Some possible alternatives to this include:

   M students give instructions first and then E students describe where the element is standing

   or

   The elements go and stand wherever they like and M and E students then have to describe in set notation and English words respectively where each of the elements from 1 to 10 is standing.

8. After each game is completed, students could switch to a different group and then repeat the exercise.

CLASSROOM MANAGEMENT IMPLICATIONS:

This active learning methodology depends on having access to a large open space area. A cut-down version of this can be managed in the normal mathematics classroom with three hula-hoops or three small ropes (or a poster of the diagram) and students placing counters in the various sections on the instructions of their classmates. The development of students' communication skills in mathematics is a key outcome of this learning activity and the group exercise facilitates this process. This activity could also be suitable – although not quite as much fun – as a chalk and talk exercise, with students actively communicating in the language of set theory.

4.5 NUMBER SYSTEMS

NUMBER SYSTEMS LESSON IDEA 1

TITLE: COUNTDOWN – ADAPTED FROM THE TV PROGRAMME OF THE SAME NAME!

TOPIC: BASIC ARITHMETIC WITH NATURAL NUMBERS

AIM: Practice in +, –, ×, ÷, and brackets. (This provides good numeracy practice without a calculator but could be used also to reinforce the importance of order of operations with a calculator.)

RESOURCES:

Make out fourteen cards as shown (or simply write the numbers onto the board). The top row contains four large numbers and the other three rows contain the numbers from 1 to 10.

100 75 25 50

8 4 1

10 7 5 3

2 6 9

METHOD:

1. If one is using cards, mix them up and turn them face down (keeping the four large numbers in the top row).
2. Pick six numbers at random from the cards (one from the top and five from anywhere else).
3. Pick a three-digit target number at random.
4. By adding, subtracting, multiplying, dividing some or all of the six chosen numbers, try to reach the target number, or to get as close as possible. Brackets can be used.

EXAMPLE:

Numbers chosen: 100, 5, 7, 9, 4, 1

Target number: 654

Solution: $100 \times 7 - 5 \times (9 + 1) + 4 = 654$

If students are weak at arithmetic, the "random numbers" could be pre-arranged!

EXAMPLE:

Numbers chosen: 100, 5, 2, 3, 7, 1

Target number: 512

Solution: $(100 \times 5) + 2 + 3 + 7 = 512$

CLASSROOM MANAGEMENT IMPLICATIONS:

The students can work in teams or individually, and a time limit can be imposed (e.g. 60 seconds), as appropriate to the class.

NOTE:

This activity can be incorporated into the teaching of natural numbers, or it can be used as light relief after completing a difficult topic.

NUMBER SYSTEMS LESSON IDEA 2

TITLE: NUMBER PUZZLE

TOPIC: BASIC ARITHMETIC WITH NATURAL NUMBERS

AIM: Practice in adding natural numbers in an entertaining way.

RESOURCES:

The puzzle opposite can be photocopied or reproduced on overhead or blackboard.

METHOD:

						42
		4	5	2	7	29
2	7		7			36
	1	6	7	8		31
3		6	4	6		26
4		2	1	5	7	25
	7			5	7	38
25	34	32	27	34	33	32

Try to fill in the missing numbers. The missing numbers are integers between 0 and 9. Numbers in each row add up to the totals on the right of the row. Numbers in each column add up to the totals at the bottom of the columns. The diagonal lines (to top right and bottom right) also add up to the totals given.

Solution:

						42
3	8	4	5	2	7	29
2	7	6	7	8	6	36
5	1	6	7	8	4	31
3	5	6	4	6	2	26
4	6	2	1	5	7	25
8	7	8	3	5	7	38
25	34	32	27	34	33	32

CLASSROOM MANAGEMENT IMPLICATIONS:

This activity can be done in pairs or in small teams of three or four in a quiz-like fashion. An interesting follow-up exercise could be to ask students to produce such a puzzle themselves. Some discussion may lead to the conclusion that the best way of producing such a puzzle is to prepare a full solution first and then to leave out certain numbers.

NUMBER SYSTEMS LESSON IDEA 3

TITLE: BUZZ GAME

TOPIC: NUMBERS

AIM: To revise and reinforce number patterns.

RESOURCES:

None required.

METHOD:

(Many variations)

1. The class is divided into small groups (say, less than 7 to a group). In each group, students in sequence rattle off the natural numbers. When a multiple of, say, 7 is met, the word "buzz" is used instead of the number and the order of students is reversed.

   For example: 1, 2, 3, 4, 5, 6, buzz (reverse), 8, 9, 10, 11, 12, 13, buzz (reverse), …

2. The person who errs is “knocked out” and the next person starts a new sequence.

CLASSROOM MANAGEMENT IMPLICATIONS

None

NOTE

If it is not convenient to form small groups, the "reverse" move may be omitted. The game can also be used with multiples, divisors of numbers such as 48, primes, sequences, etc. and even combinations of these (with other "buzz" words). This is a useful mainly with First Years.

NUMBER SYSTEMS LESSON IDEA 4

TITLE: THE PLUS AND MINUS GAME

TOPIC: INTEGERS

AIM: To help students in the addition and subtraction of integers.

RESOURCES:

None.

METHOD:

1. This activity helps to present the addition and subtraction of integers without having another *set of rules* to learn.

2. Consider the following example:

   Evaluate: -3 + 4 – 1 + 5 + 2

   The student creates a scoreboard and fills in the scores for the "+" team and the " - " team as follows:

+	-
4	3
5	1
2	

3. The scores are then added for the two teams and the answer to the question arrived at by asking who wins and by how much.

+	-
4	3
5	1
2	
11	4

In this case the "+" team wins by 7 so the answer to the problem is +7.

If the game is a draw, then no one wins and the answer is zero.

CLASSROOM MANAGEMENT IMPLICATIONS:

None

NOTE:

1. This method can be carried on to simplifying algebra, where a separate "plus and minus game" is played for each group of like terms.

2. One of the most frustrating things in dealing with integers is the ease with which the rules for adding and subtracting integers can be confused with those for multiplying and dividing integers. Many students look at -3 – 2 and, incorrectly, get +5 thinking "like signs give plus". It is important that students learn a full rule rather than a partial one: "when multiplying or dividing two integers, like signs give plus". Studying patterns like the following, may serve to situate the rule on firm ground:

$$
\begin{aligned}
2 \times 3 &= 6 \\
2 \times 2 &= 4 \\
2 \times 1 &= 2 \\
2 \times 0 &= 0 \\
2 \times -1 &= -2 \\
2 \times -2 &= -4
\end{aligned}
$$

3. Throughout the lesson ideas in these Guidelines, and in the proposed new-style examination papers, a distinction is drawn between the negation sign (-) and the symbol for the operation of subtraction (–); the latter is longer in appearance.

NUMBER SYSTEMS LESSON IDEA 5

TITLE: THE GOLDEN RATIO: AN INTRODUCTION TO RATIO AND PROPORTION FOR FIRST YEAR STUDENTS

TOPIC: RATIO AND PROPORTION

AIM: To help the students to develop an understanding of ratio and proportion.

RESOURCES:

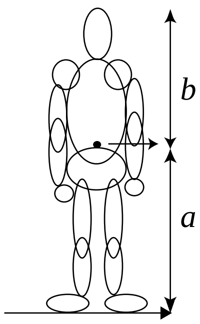
The main resource is something very close to the students' hearts – their own bodies. Also required: metre sticks, calculators, basic geometry sets and space for the students to measure themselves!

INTRODUCTION:

Students will find from this practical work that nearly all of us are built to mathematical formulae! The underlying ratio is the Golden Ratio (or Golden Mean), approximately 1.6:1. The Greeks utilised this geometrical ratio in designing their buildings; it was also known to them as the Divine Ratio.

Students should be directed to research the Golden Ratio in other subject areas – they will find the Art department a great fund of information.

METHOD:



1. Students should measure and note the distance *a* from their navel to their toes and the distance *b* from their navel to the top of their head. They should find that, no matter what their height, the ratio of the two numbers $\frac{a}{b}$ will usually give a result very close to 1.6.

   For some young students the ratio will not be too near to 1.6 (it is not unusual to have a range 1.49 to 1.72 but in general the majority are within 1.55 to 1.67). This is easily explained as they are still growing and the ratio will be achieved in later years.

2. Proportion as a concept arises when the ratios are compared.

3. During this work there are excellent opportunities to debate matters like:

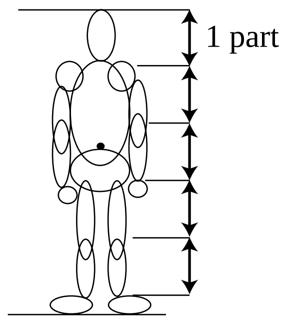
   (i) How to measure accurately using metres and centimetres.

   (ii) How to write the measurements in decimal form (one student recorded a measurement of length as "one metre fifteen inches"!).

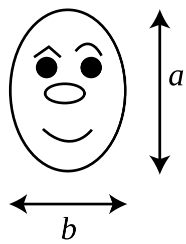
   (iii) How to round up or down the calculator's over-accurate answer.

   (iv) What would be the result if one did the reverse division $\frac{b}{a}$?

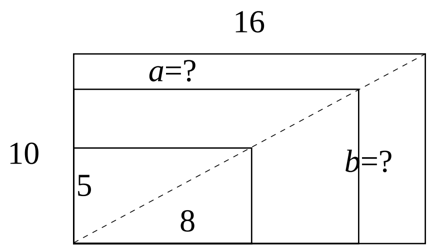
4 A very important concept will arise when students realise the nuance of using units, metres, centimetres and *parts*. The art class might come to the rescue where often, in figure drawing, the "head" is used as the unit of measurement, i.e. the one *part*, and the body is then divided into different numbers of parts no matter what the length of the head is.



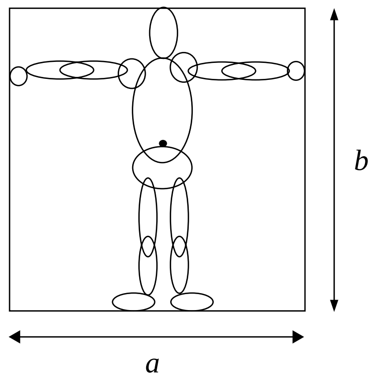
The mystery of this ratio grows when students compare the ratio of length to width of their hand and head; it also conforms to the Golden Mean.

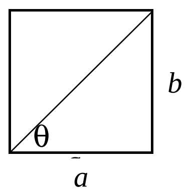


This exercise can be turned into a more abstract geometrical form by drawing a rectangular shape of sides in the ratio 1.6:1, say 8 cm : 5cm, enlarging this to 16 cm : 10 cm, and then examining the result of the ratio of any of the comparable lengths in between these limits.
This drawing can be used to help students begin to understand the trigonometric ratio "tan" as the ratio of the lengths of two line segments.



Finally, a unique one-to-one ratio for a human being is given by the Vitruvian Man. This is Leonardo da Vinci's famous drawing that will be familiar to nearly all of the students from television's World in Action programme.
If they measure their height and then the width of their outstretched arms, they will find the ratio of the two lengths is 1. It gives a whole new meaning to being called "a square"! For the more abstract thinkers the geometrical figure in the square can be related to $\tan 45° = 1$





$$\tan\theta = \frac{b}{a} = 1$$

CLASSROOM MANAGEMENT IMPLICATIONS:

Students can work in pairs for the practical work.

NUMBER SYSTEMS LESSON IDEA 6

TITLE:	I HAVE … YOU HAVE
TOPIC:	NUMBER SYSTEMS: NUMBER OPERATIONS
AIM:	To give practice in mental arithmetic

RESOURCES:

A set of cards of the form:

I have [a number]. Who has [some function of it]?

Examples might be:

1. I have 10; who has half that number?
2. I have 5; who has 40 more?
3. I have 45; who has the result of dividing this by 5?
4. I have 9; who has the square root of my number?

The cards may form a "cycle"; for the above set, the last card might read

- I have 50; who has one-fifth of this?

Alternatively, there can be a first card and a final card, the latter reading (in this case):

- I have 50; that is all.

There should be enough cards for each student in the class to have two or three.

The teacher may have a printed list of all the cards, in sequence.

METHOD:

1. The cards are shuffled and given out to the students. The teacher retains two successive cards in the "cycle" (*or* the first and final cards). The students hold their cards in such a way that they can see the writing on all of them but so that their neighbours cannot.

2. The teacher starts by reading the *second* card of the two successive ones s/he holds (or the first card of the set): saying slowly and clearly (for example) "I have 10 [pause]; who has half that number?"

3. *All students have to do the calculation to see if they hold the next card*. The student who has that card should then read it: "I have 5 [pause]; who has 40 more?" The student then places that card face downwards on his/her desk.

4. The game proceeds. If a student speaks out of turn as a result of doing a calculation incorrectly, hopefully the teacher and various students will notice and (for example) call out "No!" The student who holds the correct card should also be speaking up, reading the card. *The teacher may feel more at ease if s/he has the complete list of cards in sequence, but the game is perhaps more fun for all concerned if the teacher is also relying on his/her ability to calculate mentally.*

5. If all goes well, the game ends when the teacher hears his/her cue: the calculation that gives the first number (10, for the example described here) or the final one. At this point, all the cards should have been used, and so should be face downwards on the desks. If any student still holds a card, a mistake has been made.

6. The cards can be collected, shuffled, and dealt again. For the second round of the game, the cards might be read more quickly.

CLASSROOM MANAGEMENT IMPLICATIONS:

If each student has (say) three cards initially, most students remain "in the game" for a considerable period of time. Those who have played all their cards can stay involved by watching out for mistakes by others, but students are more likely to stay on task if they are in danger of "missing their cue".

NOTE:

1. The first time the game is played, a "trial run" may be needed to ensure that students know what to do (and to encourage them to read their cards slowly and clearly).

2. The exercise may not occupy a whole class. Especially when the game is familiar, it can be a "warm-up activity" or an end-of-lesson reward for good work (on Friday afternoon?) – not necessarily with the same set of cards each time.

3. For some weaker or more nervous students, *arranging the cards in their correct sequence* could provide practice without the fear of going wrong in front of their peers.

4. Sets of cards may be available commercially, but teachers can make their own, pitching them at an appropriate level for a particular class (and perhaps concentrating on particular number operations).

5. When the game is familiar, students might be asked to *make* a set of cards (perhaps working in pairs, perhaps for homework).

6. With very weak students, the game might be played with calculators available – the emphasis being on "what do I do to get 'half'?", "what does '40 more' mean?", and so forth.

NUMBER SYSTEMS LESSON IDEA 7

TITLE: MENTAL MATHEMATICS

TOPIC: VARIOUS

AIM: Revision of material covered

RESOURCES:

It is advisable to have questions previously prepared, but it is not essential. A sample sheet of fifteen questions is included below.

METHOD:

1. The class is split into groups (these could be based on rows, depending on how the classroom is organised).
2. These groups (teams) compete against each other, with 3 points awarded for a correct answer; a wrong answer gets passed onto the next team for a bonus of 2 points or to the next team after that for a bonus of 1 point.
3. The winning team has the carrot of no homework that night! The teacher can also throw in "open" questions answerable by the first person to raise a hand: correct answer 5 points, say; incorrect answer -5 points.
4. The idea should be to use one class period to deliver the questions, and to mark and score the results. It is unlikely that the teacher will get through more than twenty questions. Only essential information and items relevant to each question should be written on the board. Questions may have to be read out twice and the importance of listening emphasised.

SAMPLE WORKSHEET: MENTAL MATHEMATICS

1. What is two thirds of 60?
2. $16 \times 0.2 = ?$
3. Calculate 5.4 – 2.8
4. 5.7 divided by 0.3 = ?
5. Find ¾ of 60
6. If I travel 10 km in 10 minutes, what is the average speed in km/h?
7. 80% of a number is equal to 64. What is the number?
8. Find the mean of 1, 2, 4 and 5.
9. What is one third of ¾ of 8?
10. Calculate 29 + 14.8.
11. What is the last prime number before 50?
12. 20% of a number is 15. What is the number?
13. Two thirds of a number is 16. What is one quarter of this same number?
14. A car uses 5 litres of petrol to travel 80 km. How many litres are required to travel 320 km?
15. 8 people can build a wall in 15 days. How long would it take 5 people to build the same wall?

CLASSROOM MANAGEMENT IMPLICATIONS:

None (limit the excitement!)

NOTE:

1. Questions need to be simple enough to be done in the head yet broad enough to cover the material dealt with either on a topic by topic basis or covering a series of topics. This exercise is good with all groups in the junior cycle.
2. A variation of a quiz can occur when students make up questions that have a given answer. For example, if the answer is 7, what is the question? This simple exercise/game aims to make the student and teacher think in a different way and requires few or no resources. If the questions are numerical, a scoring system can be drawn up assigning for example: One point for using + or –, two points for using × or ÷, and three points for using √.

4.6 APPLIED ARITHMETIC AND MEASURE

APPLIED ARITHMETIC AND MEASURE LESSON IDEA 1

TITLE: MEASUREMENT OF LINES AND CURVES

TOPIC: MEASURE

AIM: Students are encouraged to measure various objects accurately using tape or ruler. This helps them to develop a feel for size and an understanding of units, and increases their power of estimation. (How many students can describe their height, even approximately, in centimetres?)

RESOURCES:

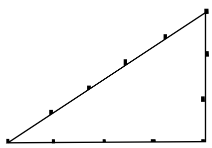
Various objects, most of which will be either in the students' school bags or in their classroom. Ruler and tape measures. Copybook to record results.

METHOD:

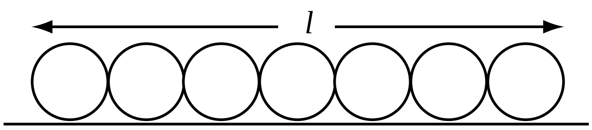
Students are urged to measure the length and breadth of their textbooks, calculators, school desks, and so forth. Problems such as "How could the thickness of a single page of the textbook be estimated?" could be proposed and solved.

Follow-up work could be along the following lines:

1. With the use of photocopying machines it is possible to provide student work-sheets containing lines, both straight and curved, the lengths of which are to be determined. The measurement of the curved line might be an opportunity for class discussion and dialogue.
2. Outdoor area activities: Area calculations of the following could be made as they are in everyday use: basketball/tennis/volleyball courts, football pitches.
3. Car-park space: determine the maximum number of cars that can be comfortably parked in the school car-park.
4. Students are asked to peg out a rectangular area of 6m x 5m. A rope with knots equally spaced and shaped in the form of a 3, 4, 5 triangle will provide an accurate right-angle. The accuracy of this experiment can be tested by measuring the two diagonals.



5. The previous field-work can be related to the pegging out of a site for a new house. If one of the students in the class has a parent or relative involved in house building then this could be an ideal opportunity to invite that person in to show how this work is achieved (most likely now using a theodolite with a rotating telescope). The opportunities for discussion using mathematical language are immense when using the "Guest Speaker" methodology.
6. The diameter (and hence radius) of a small circular object may be found by placing a number of such objects, for example small coins, in a straight line, measuring the total length l and dividing by the number of objects *n*. This gives greater accuracy than would be obtained by measuring one such object on its own.



CLASSROOM MANAGEMENT IMPLICATIONS:

Many of the follow-up suggestions involve activities outdoors. The obvious benefit is that students can begin to relate mathematics to the real world.

APPLIED ARITHMETIC AND MEASURE LESSON IDEA 2

TITLE: DISCOVERING π

TOPIC: INTRODUCTION TO π

AIM: To give students an appreciation of "pi", how it might have come about and its use in formulae.

RESOURCES:

1 wheel (disc)

1 sheet of cardboard

Some paint

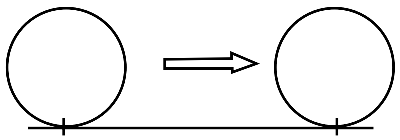
Two books

METHOD:

Place a mark on the wheel.

Roll the wheel on the cardboard and indicate on the cardboard the two positions that correspond to the mark on the wheel (see diagram).

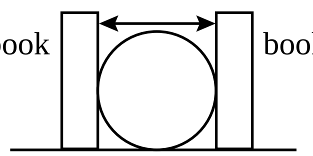
Measure the distance between the two marks.



Find the diameter using two books, as shown below.

Divide the diameter length into the result obtained above.

This should give 3.14 approximately.



Follow-up homework can be along the following lines:

Select a cylindrical object at home, e.g. a can of beans.

Measure the circumference with a piece of string.

Measure the diameter using books or blocks as above.

Calculate the ratio:

$$\frac{\text{length of circumference}}{\text{length of diameter}}$$

Repeat a number of times with different cylinders.

Calculate the mean value, to give an approximation to π.

The following table can be used for recording purposes:

Complete the table:

Object	Circumference	Diameter	Ratio
Can of beans			
Soft drink can			
Jam jar			
Favourite CD			

Mean value =

CLASSROOM MANAGEMENT IMPLICATIONS:

This activity can be done individually by students or in pairs.

NOTE:

The approximate value of "pi" will be close to 3.14 and students will soon begin to appreciate the concept of approximations and why the value of "pi" is still undetermined.

APPLIED ARITHMETIC AND MEASURE LESSON IDEA 3

TITLE: ARRIVING AT πr^2

TOPIC: AREA OF A CIRCLE

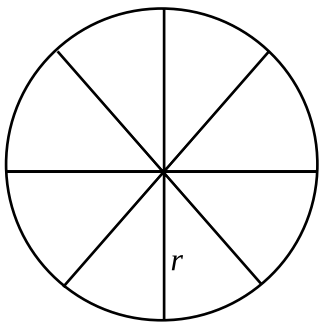
AIM: To demonstrate an idealised way of finding the area of a circle.

RESOURCES:

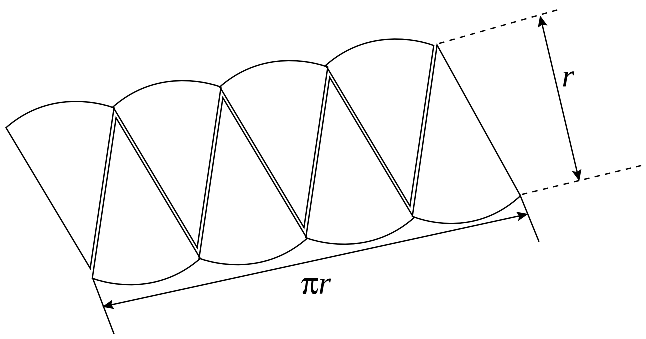
Coloured card, scissors and geometry set.

METHOD:

1. Cut out a coloured disc, radius *r*, and mark out eight sectors as shown.



2. Cut out the eight sectors and slot them together to form a "bumpy parallelogram" as in the diagram.



3. The students should be challenged to give the approximate lengths of the sides. They should use their previous knowledge of the relationship between the diameter (D) and the circumference (C).

$$C = \pi D$$
$$C = \pi.2r$$
$$\tfrac{1}{2}C = \pi r$$

4. The outcome of the work should be knowledge of how the formula "area = πr^2" came into existence, its meaning and an enhanced facility to remember the formulae.

5. The final diagram should be cut out of coloured card and will make a useful wall-display as an aide-memoire for all classes.

CLASSROOM MANAGEMENT IMPLICATIONS:

This activity can be done individually by students or in pairs.

NOTE:

1. Dexterity and accurate measuring will be essential in this activity and the students should show a pride in presenting their finished drawings.

2. Mathematical terms like sector, circumference, approximation, formulae, division and sub-division should be frequently used so that such words become part of the students' lexicon.

APPLIED ARITHMETIC AND MEASURE LESSON IDEA 4

TITLE: FINDING THE VOLUME OF A SPHERE

TOPIC: VOLUME

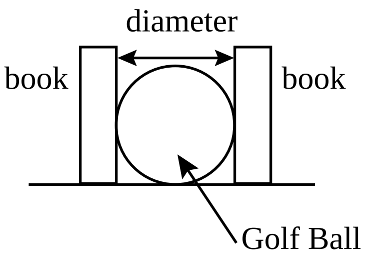
AIM:
1. To find the volume of a sphere using a "hands-on" approach.
2. To verify result using the "formula" approach.
3. To use both cm^3 and ml as units of volume.

RESOURCES:

Golf ball, two books, ruler, graduated cylinder, elastic bands.

METHOD:

Measure the diameter of the golf ball.



Determine the radius *r*.

Calculate the volume using the formula

$$\text{Volume} = \frac{4}{3}\pi r^3$$

Check the result as follows.

Approximately half fill a graduated cylinder with water.

Mark the level of water with an elastic band or marker.

Note the volume V_1.

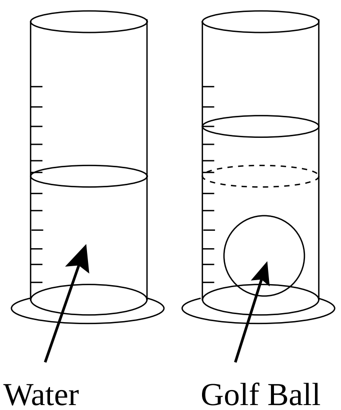
Carefully add the golf ball to the graduated cylinder.

Mark the new height. Note V_2.

Calculate $V_2 - V_1$.

Compare with the original calculation for the golf ball using the formula.

Graduated Cylinder



The following table can be used to record the results.

Results

Diameter of sphere	cm
Radius of sphere	cm
Volume of sphere $\frac{4}{3}\pi r^3$	cm^3
V_1	cm^3
V_2	cm^3
$V_2 - V_1$	cm^3

CLASSROOM MANAGEMENT IMPLICATIONS:

This is an excellent opportunity to make a cross-curricular link with science teachers in the school. A visit to the science laboratory for this methodology can be arranged for more comfort.

This activity can be done individually by students or in small groups. Group work offers more opportunities for communication, dialogue and inquiry.

NOTE:

Homework can be used to follow up this work by asking students to repeat the experiment at home using a measuring jug. (Remember, one millilitre or 1 ml is the same as 1 cm^3.)

APPLIED ARITHMETIC AND MEASURE LESSON IDEA 5

TITLE: MEASURING THE SURFACE AREA OF A CYLINDER

TOPIC: AREA

AIM: To introduce students to the formula for the area of a cylinder.

RESOURCES:

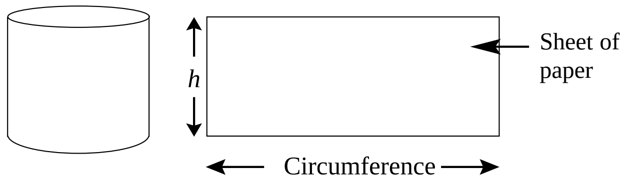
Any cylindrical object and a sheet of paper.

METHOD:

The curved surface area of cylinders can be measured by wrapping a suitably folded sheet of paper around the cylinder. The area of the folded sheet can then be easily worked out using the familiar formula:

Area = $l \times w$

Students will see readily that the length l = circumference = $2\pi r$ and that the width $w = h$



Thus, area = $2\pi r \times h = 2\pi rh$

CLASSROOM MANAGEMENT IMPLICATIONS:

This activity can be done individually by students or in pairs.

APPLIED ARITHMETIC AND MEASURE LESSON IDEA 6

TITLE: GETTING A FEEL FOR VOLUME

TOPIC: VOLUME

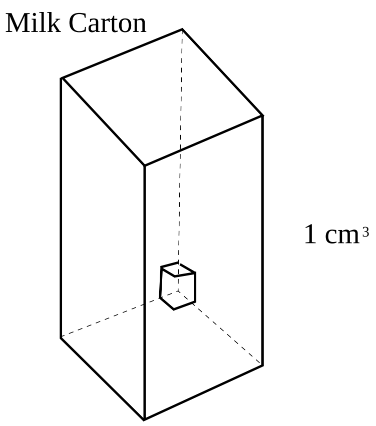
AIM: To introduce students to the volume of common objects and associated units.

RESOURCES:

Milk cartons, cubes, cola cans, tennis balls and golf balls, calculator.

METHOD:

Solids occupy space and students should be encouraged to "experience" or "feel" the volume phenomenon and appreciate that it has associated units which differ from those of length or area.



1. Start by taking a rectangular box-shaped milk carton (1 litre capacity) – it provides the student with a clear picture of what a litre is. [Students might like to know that the volume of the average student is equivalent to approximately fifty of these cartons.]
2. It can be pointed out to students that 1 litre = 1000 cm^3. A small piece of plasticine shaped into a cube of side 1 cm can serve as a visual aid for 1 cm^3 – it is literally a one centimetre cube.
3. Repeat this exercise with other common objects as mentioned above.

CLASSROOM MANAGEMENT IMPLICATIONS:

Follow-up work could involve some or all of the following activities:

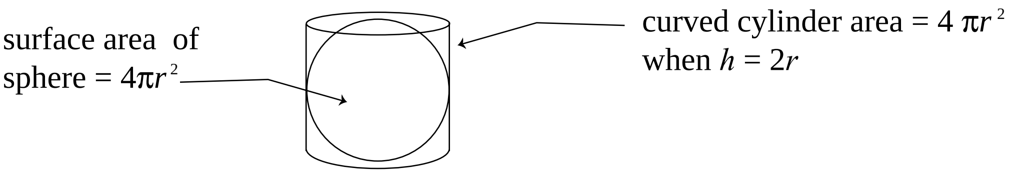
1. A soft-drinks can approximates to a perfect cylinder. Its volumetric content may be read from the attached label or measured directly using a graduated cylinder. Does calculation produce the same result using the formula

$\pi r^2 h$? The calculator will help enormously to take the drudgery out of these calculations and students will also learn the value of rounding off answers.

2. Golf balls or tennis balls are suitable for the study of spheres (for younger classes use a basketball/football/ volleyball). The diameter can be measured using a sandwich method and ruler.

NOTE:

Archimedes is credited with showing that the surface area of a sphere is equal to the curved surface area of the smallest cylinder that contains the sphere.



APPLIED ARITHMETIC AND MEASURE LESSON IDEA 7

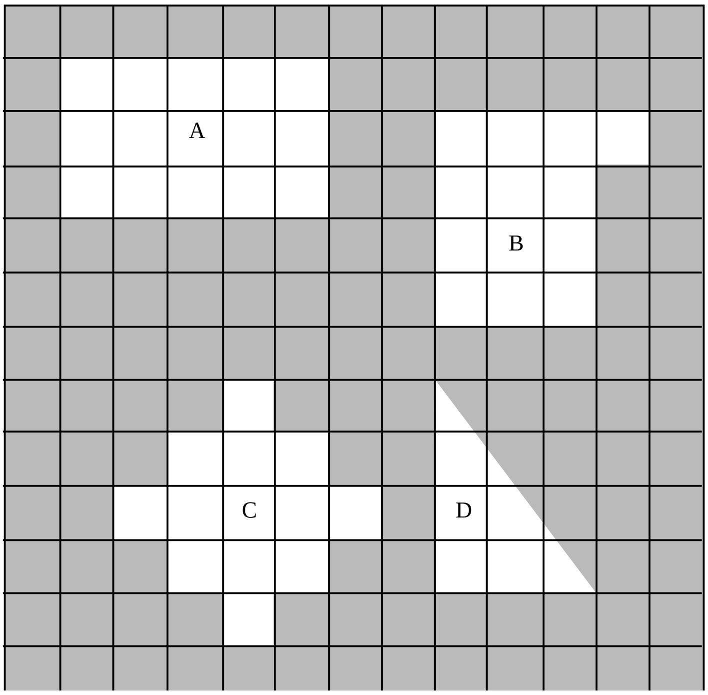
TITLE: PERIMETER AND AREA

TOPIC: PERIMETER AND AREA OF VARIOUS FIGURES

AIM: To give students practice at finding the area and perimeter of various figures.

RESOURCES:

The grid below, more copies of which can be constructed easily from a standard word processing package that incorporates a basic set of drawing tools.



METHOD:

Examine the four shapes labelled A,B,C,D and complete the table below.

Insert the perimeter and area of each shape in the space provided in the table below.

The perimeter of A is given.

	A	B	C	D
Perimeter	16cm	cm	cm	cm
Area	cm^2	cm^2	cm^2	cm^2

The grey shaded area is = ☐ cm^2.

CLASSROOM MANAGEMENT IMPLICATIONS:

None

4.7 ALGEBRA

ALGEBRA LESSON IDEA 1

TITLE: SNOOKER BALLS AND VARIABLES!

TOPIC: ALGEBRA

AIM: 1. To introduce students to the use of variables by means of a snooker analogy.

2. To give the students confidence in using variables – algebra is not just for the good classes!

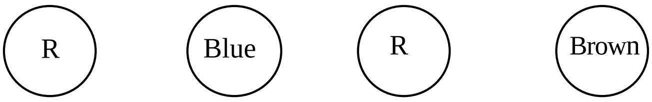
RESOURCES:

A set of snooker balls or different coloured discs or coloured circles drawn on the overhead or blackboard; a large sheet showing the values of the different coloured balls in snooker; prepared worksheets.

METHOD:

1. Opening discussion about snooker.
2. Get the students to give the value of different snooker balls. Use the large sheet showing the values of different coloured balls in snooker as an aid.
3. Write up a pretend break on the board as follows:

Pretend break.

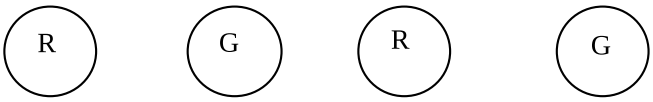


Ask students to work out the value of the break score.

More examples can be done and the scores found.

4. Students can be given a prepared worksheet (number 1) to complete using similar examples which can be corrected immediately.
5. The next step is to ask students to write out the pretend break in words. The teacher should write clearly what each letter stands for:

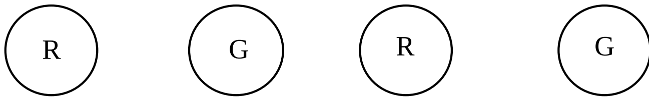
R = Red G = Green U = Blue K = Black N = Brown



So, students are asked: How many of each colour are there?

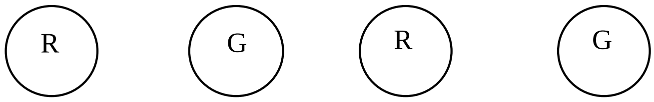
Answer: 2 red and 2 green.
(answer to be given in this way)

6. Students are then given another worksheet (number 2) with similar examples to complete.
7. When the second worksheet is corrected an identical worksheet (number 3) is given out. On this occasion students are asked to write the answer as follows:



Answer: 2R + 2G

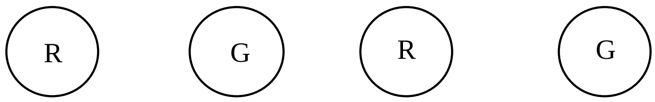
8. The same worksheet (number 4) is again given to the students. Now, students are asked to substitute in the value of each snooker ball and arrive at the total break:



Answer: 1 + 3 + 1 + 3 = 8 Example: 2R + 2G = 8

9. A similar worksheet (number 5) is given to students to complete. On this occasion the variable expression is given under each example and students asked to substitute in the correct value and arrive at the score break.

The teacher shows an example first:



Answer: 2R + 2G = 2(1) + 2(3) = 2 + 6 = 8

The students are now substituting values for variables. They can then progress to using x and y or other letters.

It is possible for the teachers to combine the last two worksheets into one if so desired.

CLASSROOM MANAGEMENT IMPLICATIONS:

Srudents can work individually or in pairs.

NOTE:

Strictly speaking, the letters in the above analogy stand for objects and not variables per se. However, teachers have found the lesson idea effective when dealing with students who find algebra difficult.

WORKSHEET 1: SNOOKER: WORK OUT THAT BREAK!

1	2	3	4	5	6	7
Red	Yellow	Green	Brown	Blue	Pink	Black

R = Red G = Green

Work out the scores for each of the breaks below.

1.

R Blue R G

Break =

2.

R G R Black R Black

Break =

3.

R Black R Black R Pink R

Break =

4.

R Blue R Brown

Break =

WORKSHEET 2: SNOOKER: BREAKS AND WORDS!

R = Red | Y = Yellow | G = Green

U = Blue | K = Black | N = Brown | P = Pink

How many of each colour are in the following breaks?

1.

R G R G

2.

R Y R Y R

3.

R P R P R P R

4.

R U R U

WORKSHEET 3: SNOOKER: BREAKS AND LETTERS!

R = Red	Y = Yellow	G = Green	
U = Blue	K = Black	N = Brown	P = Pink

How many of each letter are in the following breaks?

1.

R G R G

2.

R Y R Y R

3.

R P R P R P R

4.

R U R U

WORKSHEET 4: SNOOKER: LETTERS AND ADDITION!

1	2	3	4	5	6	7
Red	Yellow	Green	Brown	Blue	Pink	Black

R = Red   Y = Yellow   G = Green

U = Blue   K = Black   N = Brown   P = Pink

Write out the scores for each of the following.

1.

R   G   R   G

Example: 2R + 2G =

2.

R   Y   R   Y   R

3.

R   P   R   P   R   P   R

4.

R   U   R   U

WORKSHEET 5: SNOOKER: LETTERS AND SUBSTITUTION!

1	2	3	4	5	6	7
Red	Yellow	Green	Brown	Blue	Pink	Black

R = Red Y = Yellow G = Green

U = Blue K = Black N = Brown P = Pink

Write out the scores for each of the following.

1.

R G R G

Example: 2R + 2G = 2( ) + 2( ) = ________

2.

R Y R Y R

3R + 2Y = = ________

3.

R P R P R P R

4R + 3P = = _________

4.

R U R U

2R + 2U= = _________

ALGEBRA LESSON IDEA 2

TITLE: SIMPLIFYING ALGEBRAIC EXPRESSIONS WITH LIVEMATH

TOPIC: ALGEBRA

AIM:
1. To give students an opportunity of using a computer algebra software package.
2. To show students how LiveMath can be used to simplify algebraic expressions.

RESOURCES:

Computers with LiveMath installed. A basic working knowledge of LiveMath on the part of the teacher.

METHOD:

Follow the sequence of steps below using LiveMath

Declarations

Algebraic expressions are entered as they are written in ordinary mathematics:

☐ $3x - 7y$

To represent the symbol x multiplied by the symbol y, we must separate them by a space:

☐ $x\ y$

To divide two things, we press the "forward slash", *x/y*, and LiveMath arranges the fraction correctly:

☐ $\frac{x}{y}$

Note the difference between the following:

☐ $\frac{x}{y+1}$

☐ $\frac{x}{y}+1$

Note: to leave the denominator, press the "Esc" key.

The first use of LiveMath in algebra is in simplifying algebraic expressions.

We have the choice of using "Simplify" from the "Manipulate" pull-down menu, or using the brush on the Palette:

☐ $5a+3b-4a+3b-5a+6b$

Δ $5a+3b-4a+3b-5a+6b=-4a+12b$ *Simplify*

To highlight the expression, click on the "square"; then choose "Simplify".

When students are comfortable with typing in the expressions correctly and using the simplify feature they can be asked to simplify the following algebraic expressions:

Simplify: $5a + 3ab - 5b + 6ab - 4a + 7b$

$2x^2 - 3x + 5x^2 - 4x - 3x^2 + 7x$

$\frac{x^2-5x+4}{x-4}$

CLASSROOM MANAGEMENT IMPLICATIONS:

Students may initially like to work in pairs and communicate using mathematical terms such as "variables" and "expressions".

ALGEBRA LESSON IDEA 3

TITLE: GUESS MY RULE

TOPIC: ELEMENTARY ALGEBRA

AIM: To develop the concept of a variable.

RESOURCES:

No special ones.

METHOD:

1. The teacher thinks of a simple (verbal) rule such as "double and add 1", and challenges the students to guess what this rule is.

   The teacher asks the students to think of numbers between, say, 0 and 20.

   Then:

   - a student provides a number
   - the teacher gives the result of applying the rule;

   and this cycle is repeated several times.

   At any stage, a student may offer a guess as to what the rule is. If the guess is wrong, then the student cannot guess again (or put forward another number) during this game.

   Eventually – hopefully – the students guess the rule.

   Some time is spent checking, to see that all students understand and can apply the rule. *Note: at this stage, symbols should be avoided and the rule formulated in words (unless the students themselves suggest some symbolic representation).*

2. The game is played again, using a different rule (perhaps formulated by, or with input from, the student who identified the rule correctly in the first game). If the students have not already seen the need to record number pairs systematically, the teacher suggests the strategy and an appropriate format is agreed.

3. The students form small groups and play the game. The one who guesses the rule formulates the next rule. Meanwhile, the teacher circulates in order to arbitrate where necessary and to check that the rules are being applied correctly. The students have to write the rules in their own words and explain them to the teacher when s/he visits the group.

4. For homework, students are asked to try out the game at home, and to report any interesting incidents the next day. (On the next day, the teacher asks for feedback, for instance asking if anyone suggested a "short cut" – but not yet introducing symbolic notation unless the students suggest it.)

CLASSROOM MANAGEMENT IMPLICATIONS:

It is helpful if the students are used to working in groups, and ideally to negotiating and sorting out problems themselves before appealing to the teacher as "referee".

NOTE:

1. The game can be played with various types of rule – say of type $ax + b$, $a(x + b)$, x^2, $x^2 + 1$, $x/2$, and so forth – for some time. Eventually, when the students are tired of writing the rules in English, symbolic notation can be introduced, in stages, say:

   $2 \times$ number + 1

   and, after a while,

   $2 \times N + 1$

   (or using some symbol of the students' choice in place of N, but eventually introducing a letter to stand for "any number").

   The multiplication sign should be retained for some time (as in the revised *Primary School Curriculum).*

2. This introduces the idea of a *variable* – which can take many values – rather than an *unknown* (which has a specific value, for example as in "$x + 7 = 10$"). Starting with a variable may help to avoid problems which can arise with the similar game, (say) "my number plus 7 is 10, what is my number?" which can lead eventually to protests of form "but yesterday we decided x was 6…".

3. The game can be used also to introduce the idea of a *function.*

ALGEBRA LESSON IDEA 4

TITLE: I HAVE AN EXPRESSION… AND YOU HAVE AN EXPRESSION

TOPIC: ALGEBRAIC SKILLS AND CONCEPTS

AIM: To give practice in manipulating algebraic expressions.

Note: this is the same game as that described in Number systems lesson idea 6, but with algebraic expressions taking the place of numbers.

RESOURCES:

A set of cards of the form:

I have [an algebraic expression].
Who has [some function of it]?

Examples might include:

- I have $2x + 3$; who has 5 less than this?
- I have $4y – 3$; who has this expression minus $2y$?
- I have $x^2 + 2x + 1$; who has the square root of this? (the difficulty level being geared to the class).

See Number systems lesson idea 6 for details of the structure of the set of cards.

METHOD:

See Number systems lesson idea 6.

CLASSROOM MANAGEMENT IMPLICATIONS:

See Number systems lesson idea 6.

ALGEBRA LESSON IDEA 5

TITLE: PATTERNS AND FORMULAE

TOPIC: ELEMENTARY ALGEBRA

AIM: To create a need for learning to manipulate algebraic expressions

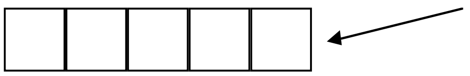
RESOURCES:

No special ones.

METHOD:

1. The teacher draws or displays a diagram consisting of a row of squares such as that shown, and challenges the class: how many matches (or lollipop sticks, or other suitable "rods") are needed to make it?

This side is one "rod" in length



2. The students are asked to draw a similar diagram, but with more or fewer squares (still arranged in a line). How many matches are needed?

3. The students are then challenged to find a relationship between the *number of squares* and the *number of matches* by looking at their own diagrams and perhaps the (probably different) diagrams of their neighbours. This relationship should be expressed as a verbal rule, for example "three for each square, plus one extra for the first square" or "twice the number of squares (for the top and bottom) plus one more than the number of squares (for the uprights)."

   If different versions do not emerge spontaneously, the students can be challenged to find them.

4. The teacher can then ask: can we be sure that the different rules will always give the same answer? To find out, the verbal rules can be written as *algebraic expressions.* Examples may include:

   $3n + 1$
   $2n + (n + 1)$
   $4n - (n - 1)$ {The teacher might provide this one if the students do not do so.}

   The teacher can work with the class in order to formulate the expressions correctly on the blackboard or OHP.

   In order to reconcile the various formulae, the students then have to learn how to simplify the expressions, or have to apply what they have already begun to learn about algebraic manipulation.

5. As a follow-up activity, or in another lesson, the "Guess my rule" game (Algebra lesson idea 3) can be used in this context. For instance:

You say:	2	7	3	1	0
I say:	12	32	16	8	4

   This can be interpreted as either $4(x + 1)$ or $4x + 4$

6. The teacher can provide a set of appropriate exercises, or students can then make up their own examples.

CLASSROOM MANAGEMENT IMPLICATIONS:

None

4.8 STATISTICS

STATISTICS LESSON IDEA 1

TITLE: INVESTIGATING THE PROPERTIES OF PIE CHARTS USING A SPREADSHEET

TOPIC: STATISTICS

AIM: To encourage the students to experiment with graphing pie charts and to see the relationship between changing data and the graphical representation of the data.

RESOURCES:

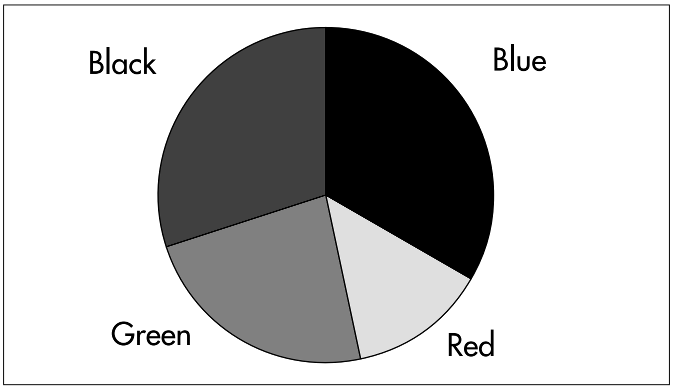
Computers with spreadsheet software installed.

METHOD:

1. Students enter data (similar to those in the table below) into the spreadsheet.

Favourite Colour	Blue	Red	Green	Black
Number of students	10	4	7	9

2. Students are instructed to use the standard spreadsheet graphing tools to create a pie chart along the following lines:



The advantage of the spreadsheet is that changes made to the table will be automatically reflected on the pie chart.

3. Now students can investigate the answers to questions such as the following (some suggested answers are given).

Q. *What happens to the pie chart if I double the number of students who chose blue as their favourite?*

A. The blue section increases in size (but it does not double).

Q. *What happens to the pie chart if I double the number of students who chose each colour?*

A. Nothing happens. (*Why?*) Because the relative size of the numbers stays the same. 10 out of 30 people is the same fraction as 20 out of 60 people.

Q. *What happens to the pie chart if I halve the number of students who chose each colour?*

A. Nothing happens. (*Why?*) Because the relative size of the numbers stays the same.

CLASSROOM MANAGEMENT IMPLICATIONS:

If a reasonable number of computers is not available for use by mathematics students, the above ideas could be demonstrated by the teacher to groups of students gathered around a single computer, or with the aid of a data projector, if available. Students could invent their own data sets or use a data set gathered by means of a simple classroom survey that might be part of a project in a subject area such as CSPE. Students will probably also have their own suggestions for *"What would happen if…"* questions.

STATISTICS LESSON IDEA 2

TITLE: THE MEAN: WHAT DOES IT MEAN?

TOPIC: MEASURES OF CENTRAL TENDENCY

AIM: To develop an understanding of the mean and the mode, and of suitable uses for each of them.

RESOURCES:

No special ones.

It is assumed that the students have already met both the mode and the mean. Alternatively, some of the ideas here could be used in introductory lessons for either concept, and some could be incorporated in later lessons, as appropriate.

METHOD:

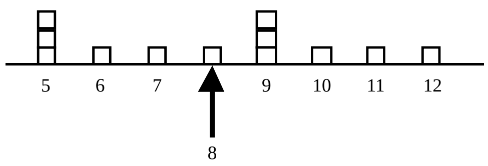
1. Recall: the *mode* is the "most fashionable" (most frequently occurring) number.

2. So what is the *mean?*

   - Take as an example the marks scored by a group of twelve children in a test:

10	5	6	8	9	5
5	11	7	9	9	12

   - What is the mean? 96/12 = 8.
   - *So, suppose the twelve people shared the marks out equally among themselves; they would each get 8.*
   - Also, imagine the marks sitting on a number line, and that the number line forms a seesaw pivoted at some point:



   8 is the *balance-point: the point such that the seesaw balances.*

   - What happens if we change one mark? Suppose the test was out of 20 and the last person got 20, not 12; the seesaw will tip … and the balance-point will be more to the right. Or suppose the person scoring 7 got 11; again the seesaw will tip a bit, and a new balance-point will be needed.
   - In general, changing any mark changes the mean; this is *not* usually the case for the mode.

3. Consider an example: shoe sizes for the class; collect data (or use previously collected data) and find the mode and the mean. *If the mean turns out to be a natural number, add the teacher's shoe size to the collection; hopefully the answer is no longer a natural number!*

   - What can we say about the mean shoe size? Is it the most usual shoe size? Is it a shoe size at all? Where might the mode be more useful … less useful? [Students can discuss in pairs, say, and record their opinions – there are many acceptable answers.]

4. Repeat for some of the following (again, perhaps, using previously collected data, and utilising calculators where appropriate for the calculations): height of students; number of children in the family; age (to the nearest day); distance travelled to school (to nearest half-kilometre). [For height, age and distance, non-integral values are "possible values"; moreover, there may be no modal value, or the mode may be no guide to where the data cluster.]

5. For homework, students write up an explanation of uses for the mode and the mean.

CLASSROOM MANAGEMENT IMPLICATIONS:

None

STATISTICS LESSON IDEA 3

TITLE: FROM "ADD UP AND DIVIDE" TO "FORMULA FOR MEAN OF A FREQUENCY DISTRIBUTION"

TOPIC: STATISTICS: MEAN OF A FREQUENCY DISTRIBUTION

AIM: To establish the *formula for* and a *method for calculating* the mean of a frequency distribution.

RESOURCES:

No special ones.

METHOD:

1. Consider a frequency distribution (perhaps using class data, but numbers which might be marks from a test are provided here by way of an example – choose easier ones for Foundation students):

X	f
0	1
1	1
2	0
3	2
4	1
5	2
6	5
7	5
8	6
9	5
10	2

   Can we find the mean? Guess possible / likely values (could it be 10?).

2. Relate to an already-known method: the numbers are

   0 1 3 3 4 5 5 6 6 6 6 6 7 7 7 7 7 8 8 8 8 8 8 9 9 9 9 9 10 10

   … and there are 30 numbers. So the mean is

   (0 + 1 + 3 + 3 + 4 + 5 + 5 + 6 + 6 + 6 + 6 + 6 + 7 + 7 + 7 + 7 + 7 + 8 + 8 + 8+ 8 + 8 + 8 + 9 + 9 + 9 + 9 + 9 + 10 + 10) ÷ 30, or 199 ÷ 30.

3. This is not a nice sum, and it would be worse if there were 1000 numbers. (Even with a calculator, one might easily make a data-entry error.) Is there a better way?

4. Yes; note it reduces to (indeed, came from):

   (0 + 1 + 3×2 + 4 + 5×2 + 6×5 + 7×5 + 8×6 + 9×5 + 10×2) ÷ 30

   which is probably easier to handle….

5. … and in fact we have an easy way of setting out the calculation by using the frequency table (and so ending with vertical rather than horizontal additions) [bright students might jump from step 1 almost directly to here]:

X	f	fX (or Xf)	
0	1	0	*not 1!!*
1	1	1	
2	0	0	*not 2!!*
3	2	6	
4	1	4	
5	2	10	
6	5	30	
7	5	35	
8	6	48	
9	5	45	
10	2	20	
	$\Sigma f = 30$	$\Sigma fX = 199$	

   So the mean is the sum of the "fX"s divided by the sum of the "f"s:

$$\frac{\Sigma fX}{\Sigma f}$$

   … work it out, and check the result for plausibility! Yes, 6.66… is reasonable.

6. Note that the formula can also be read as a sequence of instructions:

   - Add the "f"s (to get the total number of scores)
   - Multiply each "X" by the corresponding "f"
   - Add the "fX"s
   - Divide

7. Practise other examples (checking each answer for plausibility).

CLASSROOM MANAGEMENT IMPLICATIONS:

None

NOTE ON DATA HANDLING

The remaining lesson ideas in this section focus in particular on the "*data handling*" *section of the Foundation level syllabus* (see syllabus page 31). They suggest ways in which students might move between the following activities:

- *looking at graphs* – perhaps initally at sketch-graphs, in order to study the general shape and salient features rather than the precise details;
- *looking at tables of data;*
- *listening to or creating stories* round the data presented in either form.

The problems which these lesson ideas present are "realistic", in that they are based on contexts with which students can identify. Some are real-life contexts; others involve games which the students enjoy. In all cases they may promote discussion and involve the students in mathematics that in some way is personal to them. This can help in developing a positive attitude to mathematics.

It is suggested in the mathematical education literature that work of this type should *precede* the formal introduction of algebraic notation. For example, lesson ideas 2 and 3 point to ways in which the work can lead naturally to the introduction of variables and/or the idea of a function. Hopefully, students will then see some good reason (in their own terms) for devising the corresponding terminology and conventions.

The approach to "data handling" described here was included specifically in the Foundation level syllabus because students at this level have such great difficulties with basic algebra. The placing of the relevant section of the syllabus before the sections on algebra and functions is intended to suggest a corresponding teaching sequence. In fact students other than those working at Foundation level might also benefit from such sequencing.

STATISTICS AND DATA HANDLING LESSON IDEA 1

TITLE: SKETCHING THE GRAPH, TELLING THE STORY

TOPIC: INTRODUCTION TO DATA HANDLING; PREPARATION FOR TREND GRAPHS AND FUNCTIONS

AIM: To develop a feel for the way in which the shape of a graph "tells a story".

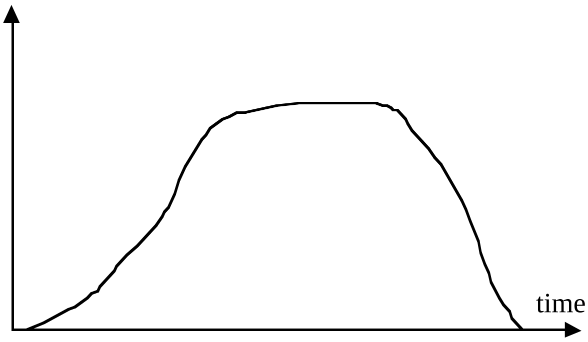
RESOURCES:

No special ones are necessary, but it may be helpful to have prepared a set of cards containing appropriate graphs or stories (related to the students' interests).

METHOD:

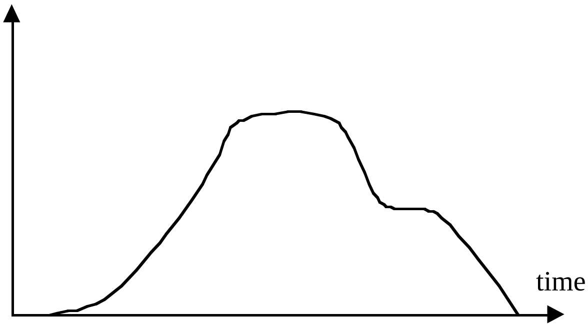
1. The graph below shows attendance at a football match against time. The teacher displays the graph and asks the students to tell a story accounting for the shape of the graph.

number of people



2. The second graph also shows attendance at a football match against time. Once more the students are asked to tell a story accounting for the shape of the graph. They can then compare their stories with their neighbours' versions.

number of people



3. The students now make up another story about attendance at a match, and see if their neighbour can draw a graph to represent it.
4. Stories and graphs can be shared around the class (or the teacher can use the set of cards suggested above, some showing graphs and some telling stories, so as to provide an element of more controlled reinforcement).

CLASSROOM MANAGEMENT IMPLICATIONS:

It is helpful if the students are used to working in groups, and ideally to negotiating and sorting out problems themselves before appealing to the teacher as "referee".

NOTE:

The students can tell their *own stories* and so "buy in" to the activity, debating with friends over the merits of their various versions.

STATISTICS AND DATA HANDLING LESSON IDEA 2

TITLE: SLEEP PATTERNS

TOPIC: DATA HANDLING

AIM: To link verbal rules with tables of data and graphs.

RESOURCES:

No special ones.

METHOD:

1. The teacher suggests the following *rule*: "The number of hours sleep you need per night is given by the rule: Sixteen minus half your age."
2. The students are invited to discuss this; is it a good rule? Do individual students feel that it represents their sleep patterns over the years?
3. The students draw up a table for ages from babyhood to adulthood and corresponding numbers of hours of sleep, using the rule. They consider: how would it work for their parents – or for the teacher(!)? How about their grandparents? (Answers may differ, depending on whether the students think that their grandparents sleep for more hours or less hours than do their parents.)
4. The students are invited to draw up another table which might be better for the older age-groups, and to try and describe the modified rule in words.
5. The students draw or sketch a graph for an extended version of this table.
6. The class can *discuss* different graphs and rules.

CLASSROOM MANAGEMENT IMPLICATIONS:

Students may need to learn to *discuss* in mathematics class, and to put forward and argue for their own ideas. Hopefully, once they get used to the idea, they take ownership of their results and feel personally involved.

STATISTICS AND DATA HANDLING LESSON IDEA 3

TITLE: SHOE SIZES

TOPIC: DATA HANDLING; A BASIS FOR ALGEBRA

AIM: To link verbal rules with tables of data and graphs; to introduce the idea of a variable and/or a function.

RESOURCES:

No special ones

METHOD:

1. The teacher invites the class to try the following relationship:
   - measure the length of your foot (in centimetres)
   - multiply by 1.5
   - add 1

   The result should be your [continental] shoe size. Does it work?
2. The students measure their feet and test the rule.
3. The students are asked to sketch a graph for the rule. What shape is the graph?
4. The students are challenged to make up a rule for UK shoe sizes.
5. The original rule can be used as the basis for introducing variables – slowly (not necessarily in the same lesson, unless preliminary work has been done earlier):
   - the rule is first stated in words
   - then a function machine might be used
   - then an abbreviated form of the rule, such as "length $\times$ 1.5 + 1 = size", might be written (to "save time")
   - this can lead to a simpler formulation "$L \times 1.5 + 1 = S$"
   - this can be written also as "$S = 1.5 \times L + 1$"

   The multiplication sign should be kept in the formula for some time.

CLASSROOM MANAGEMENT IMPLICATIONS:

None

STATISTICS AND DATA HANDLING LESSON IDEA 4

TITLE: PAYING FOR THE BUS JOURNEY

TOPIC: DATA HANDLING; A BASIS FOR ALGEBRA AND FUNCTIONS

AIM: To link tables of data with graphs

RESOURCES:

A bus (or other transport) timetable, showing fare stages and prices.

METHOD:

1. The teacher displays data from a page of a (fictional and old) bus timetable supposedly found in some interesting circumstances recently:

Number of fare stages	1-3	4-6	7-9	10-12
Fare	50p	70p	80p	85p

   Students are invited to consider the table. Does the fare go up or go down as the journey gets longer? Does it go up as much for long journeys as for short ones? Which is "better value," a long journey or a short one? How much might you pay for 14 stages?

2. The students consider: do Bus Éireann (or other relevant local) fares follow this pattern? If the "timetable page" considered initially is one for the same company, how long ago might the timetable on the page have been in operation?

3. The students are asked: can you *sketch* a graph showing price against number of stages (for the given data, or for current data)? How might the graph go for a greater number of fare stages?

CLASSROOM MANAGEMENT IMPLICATIONS:

None

STATISTICS AND DATA HANDLING LESSON IDEA 5

TITLE: SUNSET

TOPIC: DATA HANDLING; A BASIS FOR ALGEBRA AND FUNCTIONS

AIM: To link tables of data with graphs

* It is envisaged that this lesson would follow the one using lesson idea 4. For more able classes, the material might be covered in one period.

RESOURCES:

None necessary, but tables of sunset and sunrise times (perhaps from different parts of the world) would be useful

METHOD:

Day	1	2	3	4	5
Sunset time	19.35	19.33	19.30	19.28	?

Consider the sunset times shown in the table. What time of year is it? [The times are those for sunsets in Ireland from September 17th.] How did you know?

What time do you think the sun would set on day 5? ... on day 6?

Why are the time intervals not always the same between successive days? [Answers can relate to approximation as well as to geographical features.]

Can you sketch a graph showing time against day?

How does the shape of this graph compare with the shape of the previous one?

Can you sketch the graph for a one-year period?

[For students with appropriate geographical knowledge] What differences might be found in other countries [especially any that class members have visited or with which they are familiar from television]?

CLASSROOM MANAGEMENT IMPLICATIONS:

None

4.9 GEOMETRY

GEOMETRY LESSON IDEA 1

TITLE: DIFFERENT TYPES OF TRIANGLES

TOPIC: TRIANGLES

AIM:

1. That students will be able to recognise various types of triangles.
2. That students will be provided with concrete experience dealing with triangles.

RESOURCES:

The worksheet opposite with various types of triangles.

METHOD:

The students are introduced to a series of triangles on a worksheet, an example of which is presented opposite. The triangles comprise a mixture of isosceles, right angled, scalene and equilateral triangles. Their task is to determine the lengths of the sides, the magnitudes of the angles and consequently the type of each triangle presented. The results can be presented in tabular form, as shown. For ease of reference, angles and sides of each triangle may be labelled 1,2,3.

CLASSROOM MANAGEMENT IMPLICATIONS:

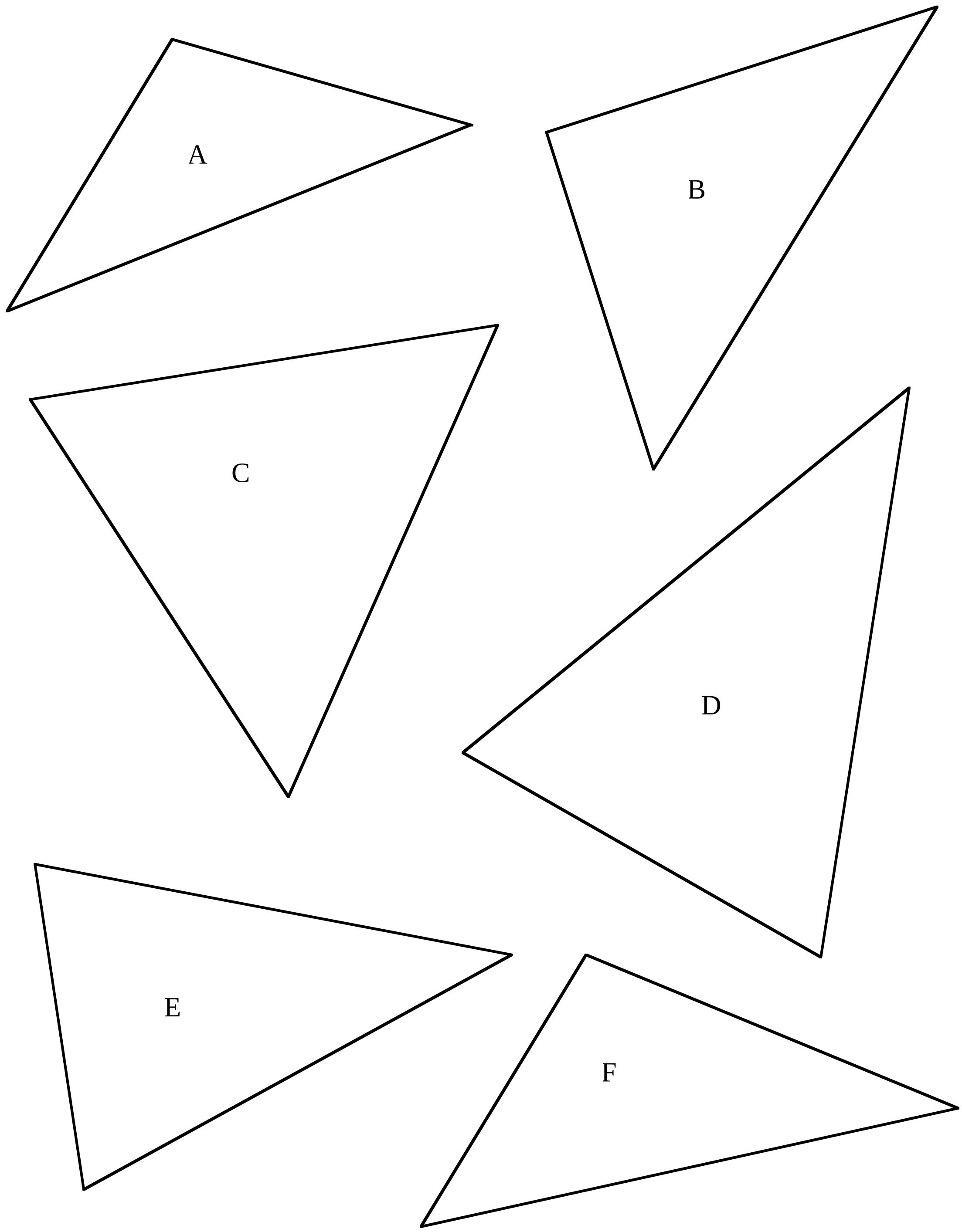
Students can work individually on the above exercise but should then be encouraged to share and discuss their results with their peers as the exercise goes on.

NOTE:

1. The worksheet opposite was constructed using a standard word processing package that incorporated a basic set of drawing tools.
2. Teachers familiar with dynamic geometry software will know that such packages can be used to prepare these worksheets also. An additional advantage is that they can also provide a teacher’s master copy with a printout of the lengths of sides and angles in the given triangles.
3. Familiarity with isosceles (iso-skeles means "equal legs") and equilateral triangles can be further enhanced if students use cut-out triangles from the same sheet. The sides of each triangle can be compared by folding along a suitable axis.
4. The Irish phrases used to describe these different types of triangles can give an insight into their properties.

   Thus, for example, "triantán comhchosach" (equal legs) describes the isosceles triangle, while "triantán comhshleasach" (equal sides) describes the equilateral triangle.

Student Worksheet – Triangles



Triangle	Angle 1	Angle 2	Angle 3	Side 1	Side 2	Side 3	Equilateral	Isosceles	Right Angled	Scalene
A										
B										
C										
D										
E										
F										

GEOMETRY LESSON IDEA 2

TITLE: PROPERTIES OF TRIANGLES, PARALLEL LINES AND PARALLELOGRAMS

TOPIC: TRIANGLES, PARALLEL LINES AND PARALLELOGRAMS

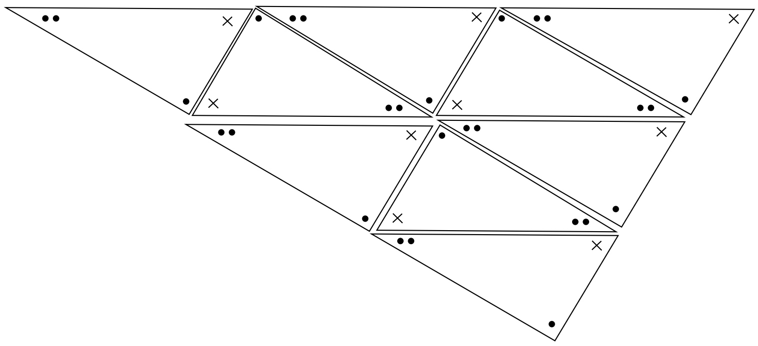
AIM:
1. That students begin their study of geometry with a number of concrete experiences and spatial visualisations as geometry is the study of shape and space.
2. In particular that students will make a number of discoveries concerning triangles, parallel lines and parallelograms.

RESOURCES:

A set of triangular tiles constructed from cardboard

METHOD:

1. Mark off the corresponding angles in each triangle as shown.
2. The following is just a selection of some of the discoveries that students can be encouraged to make:
   - the equality of alternate, corresponding and vertically opposite angles
   - the external angle of a triangle is equal to the sum of the two interior opposite angles
   - opposite sides and opposite angles of a parallelogram are, respectively, equal in measure
   - a diagonal bisects the area of a parallelogram.



CLASSROOM MANAGEMENT IMPLICATIONS:

Students can work individually on the above exercise but should then be encouraged to share and discuss their results with their peers as the exercise goes on.

GEOMETRY LESSON IDEA 3

TITLE: BISECTING ANGLES AND CONSTRUCTING THE INCIRCLE OF A TRIANGLE USING A DYNAMIC GEOMETRY PACKAGE

TOPIC: GEOMETRY CONSTRUCTIONS

AIM:
1. That students will learn how to bisect an angle using dynamic geometry software.
2. That students will observe, by altering the size of the angle, how the angle bisector alters to divide the new angle into two equal parts.
3. That students will use the knowledge gained to bisect the three angles of a triangle and thus find the incentre.
4. That students will use the incentre to construct the incircle of the triangle and observe how the construction is preserved when a vertex of the triangle is dragged.

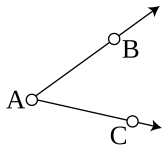
RESOURCES:

Computers with dynamic geometry software installed.

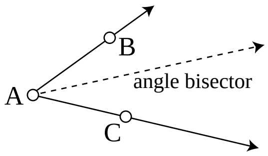
Basic working knowledge of the package. (The method outlined opposite is based on Geometer's Sketchpad.)

METHOD:

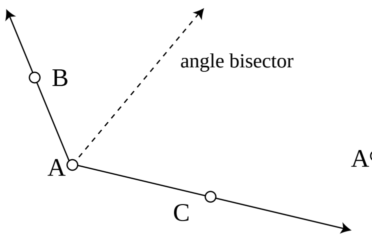
1. On a new sketch with the Half-Line tool construct an angle, ∠BAC.

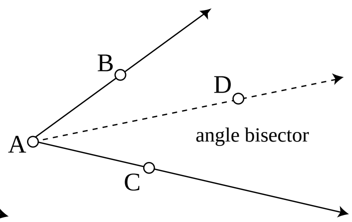


2. Select each of the points B, A, and C making sure to select the common starting point second. With these points selected from the Construct Menu select Angle Bisector. Using the Display Menu show the bisector as a red broken line.



3. Click and drag on the point B and as the size of the angle changes notice how the angle bisector alters to divide the new angle into two equal parts. Construct a point D on the bisector. Record the measures of the angles indicated in the table. Click and drag on B a number of times to complete the table.





| ∠BAD|	| ∠CAD|	| ∠CAD| + | ∠BAD|	| ∠BAC|

4. On a new sketch, with the line segment tool construct a triangle, ΔABC.

   Select each angle in turn, taking care to select the vertex point second in each case, and construct the bisector of each angle.

   Construct the point of intersection D of the bisectors by selecting any two of them and from the Construct Menu choose point of intersection.

   From the point D drop a perpendicular line to the line segment [AC]. Find the point E where this perpendicular line intersects [AC].

   First select the point D, hold down shift and select the point E.

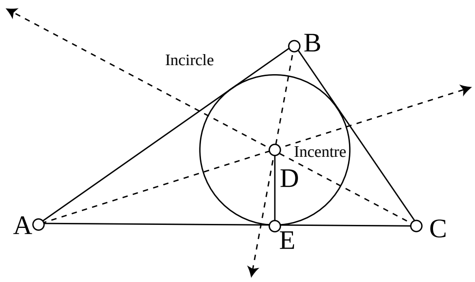
   From the Construct Menu choose Circle by centre and point.

   Display this circle by a thick green line.

   D is called the incentre and the circle is called the incircle.

   Click and drag on A, B and C in turn and notice how the incircle construction is preserved.

   What can you say about the incentre when the triangle is isosceles, equilateral ?



The skills acquired in the above lesson can be reinforced by asking students to carry out the following tasks.

1. On a new sketch,
   - (i) Construct ∠BAC measuring 120º.
   - (ii) Construct the angle bisector.
   - (iii) Construct a point D on the bisector.
   - (iv) Measure ∠BAD and ∠CAD.
   - (v) Check that ∠BAC = ∠BAD + ∠CAD. Show all angle measurements on the screen.
   - (vi) Save your work.
2. On a new sketch,
   - (i) Construct a triangle.
   - (ii) Bisect the angles.
   - (iii) Construct the incentre.
   - (iv) Construct the incircle.
   - (v) Save your work.

CLASSROOM MANAGEMENT IMPLICATIONS:

Students might like to work in pairs on the initial skill-building exercise but they should then be encouraged to try out the exercises on their own and to share their mathematical experiences. There is ample opportunity here for enhancing their communication skills in mathematics.

GEOMETRY LESSON IDEA 4

TITLE: THE ANGLES IN A TRIANGLE SUM TO 180º – A DEMO!

TOPIC: THEOREMS – AN EXPERIMENTAL APPROACH

AIM: That students will demonstrate how the three angles in a triangle sum to 180º.

RESOURCES:

Paper or card, scissors, straight-edge.

METHOD:

1. Use a straight-edge to draw a triangle onto a sheet of paper or card (Fig. 1). (It is interesting to see how many of our students automatically draw an equilateral, isosceles or right-angled triangle – this gives a clue as to which kind and orientation of triangle they are most used to seeing.)

2. Cut it out carefully. Label the three corners A, B, and C (Fig. 2).

3. Now measure each angle with a protractor. The three angles may not add up to exactly 180º. Why is this? (Discussion on accuracy of drawing, accuracy of measuring, protractor is marked only in degrees, not minutes or seconds).

4. Now tear off each corner (tearing is better than cutting as it produces a jagged edge which makes the vertex easier to see) and position them with the vertices (pointed ends) together (Fig. 3).

5. They will form a straight angle measuring 180º (Fig. 4). Place a straight edge against it to see this. This will work no matter what size triangle you draw. What does this tell us about the measure of the three angles of a triangle?

CLASSROOM MANAGEMENT IMPLICATIONS:

Students can work individually on the above exercise but should then be encouraged to share and discuss their results with their peers as the exercise goes on. In addition, if time is an issue, the teacher can give a demonstration of the result on his/her own.

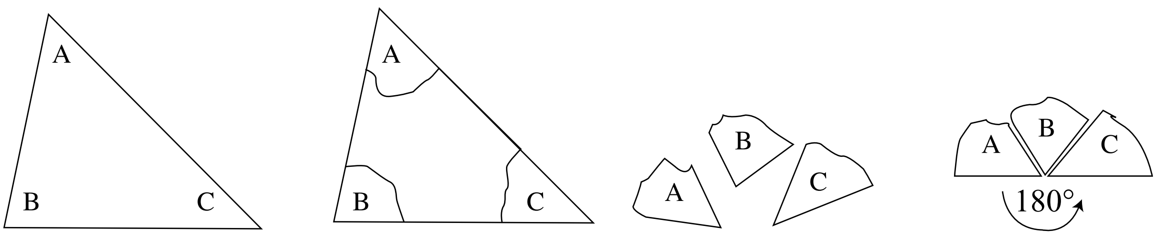


Figure 1 Figure 2 Figure 3 Figure 4

GEOMETRY LESSON IDEA 5

TITLE: AN EXTERIOR ANGLE OF A TRIANGLE EQUALS THE SUM OF THE TWO INTERIOR OPPOSITE ANGLES IN MEASURE

TOPIC: THEOREMS – AN EXPERIMENTAL APPROACH

AIM: That students will demonstrate how the exterior angle of a triangle equals the sum of the two interior opposite angles in measure.

RESOURCES:

Paper or card, scissors, straight-edge.

METHOD:

1. Use a straight-edge to draw a triangle onto a sheet of paper or card (Fig. 1).
2. Cut it out carefully. Label the three corners A, B, and C (Fig. 2).
3. Lay a straight-edge along the base of the triangle to produce the external angle D (Fig. 2).
4. Now tear off angles A and B (Fig. 3).
5. Position them with the vertices (pointed ends) together into angle D. They should fit exactly (Fig. 4).

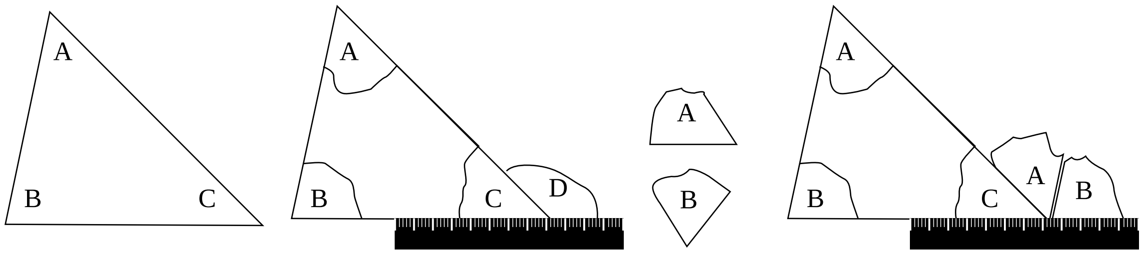


Figure 1 Figure 2 Figure 3 Figure 4

CLASSROOM MANAGEMENT IMPLICATIONS:

Students can work individually on the above exercise but should then be encouraged to share and discuss their results with their peers as the exercise goes on. In addition, if time is an issue, the teacher can give a demonstration of the result on his/her own.

GEOMETRY LESSON IDEA 6

TITLE: USING A PHYSICAL MODEL TO SHOW THAT THE MEASURE OF THE ANGLE AT THE CENTRE OF THE CIRCLE IS TWICE THE MEASURE OF THE ANGLE AT THE CIRCUMFERENCE, STANDING ON THE SAME ARC

TOPIC: THEOREMS – AN EXPERIMENTAL APPROACH

AIM: That students will demonstrate, using concrete materials, how the measure of the angle at the centre of a circle is twice the measure of the angle at the circumference, standing on the same arc.

RESOURCES:

A wooden disc (hereinafter referred to as the bread board!) with small stud screws at the circumferenc at intervals of 10 or 20 degrees, rubber bands, scissors, coloured paper, protractor and set square. Many teachers of mathematics get their friendly materials technology colleagues to construct the wooden models.

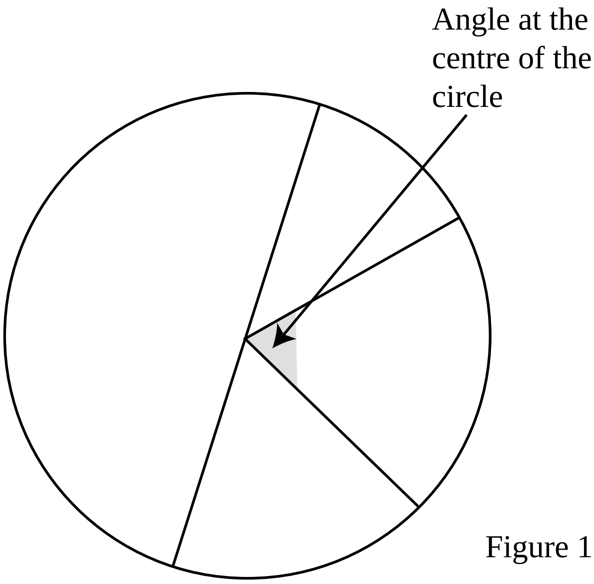


Figure 1

METHOD:

1. The first idea that students need to experiment with is the concept of what an angle at the centre is. The teacher can use one elastic band to represent the diameter and this should remain for all steps below. Two additional bands are then used to make an angle at the centre as shown in Fig. 1 opposite. Students can construct various angles at the centre using the wooden model.

2. Next, the teacher can demonstrate what an angle at the circumference is as shown in Fig. 2. Again, various students can be selected to demonstrate different examples, helping to consolidate this idea of an angle at the circumference.

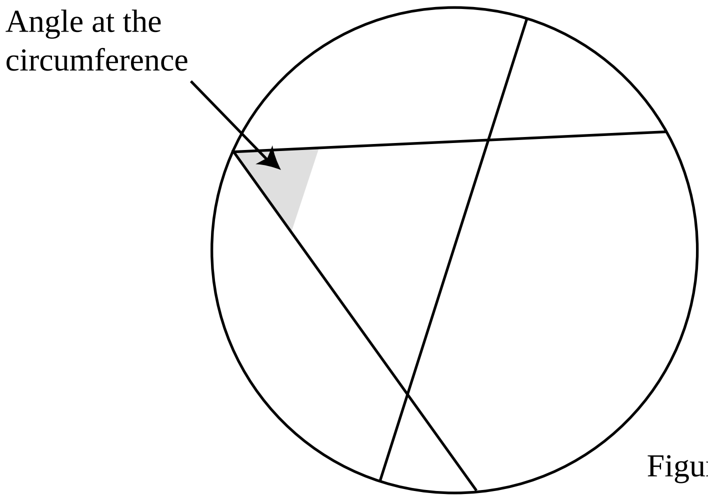


Figure 2

3. Now, the teacher can demonstrate the notion of both angles standing on the same arc ab as shown in Fig.3. Using the model, many different examples of this can be found by students. After each example, students should be strongly encouraged to turn the bread board around, showing that the diameter is not a fixed horizontal, or that the angle at the circumference is not fixed at the top, etc. This dynamic nature of the bread board is a huge advantage over the static model on the blackboard.

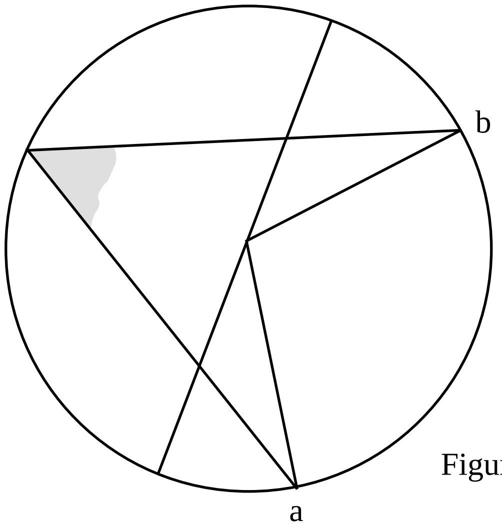


Figure 3

4. Two students can be asked to measure the angle at the centre with a protractor and then to cut this angle from coloured paper and place it at the centre angle as shown in Fig. 4. Pupils can then be asked then to explore how this angle at the centre compares with the angle at the circumference (both standing on the same arc ab).
   It is most useful, and makes quite an impact, if the angle at the centre is then taken, folded over, and superimposed on the angle at the circumference. Students can readily see in a concrete fashion that the latter angle is half the angle at the centre, thus laying a good foundation for the formal proof of this theorem.

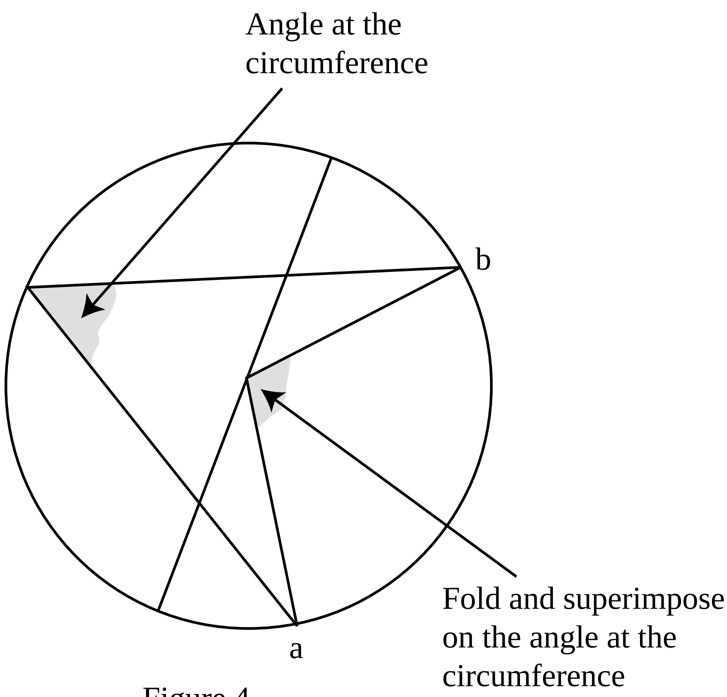


Figure 4

CLASSROOM MANAGEMENT IMPLICATIONS:

If only one bread board is available the above steps can be effectively carried out in demonstration mode using different students for various steps. Alternatively, if several wooden models are available students can work in small groups of two or three but should then be encouraged to record, share and discuss their results with their peers as the exercise goes on. In addition, if time is an issue, the teacher can give a demonstration of the result on his/her own. Teachers who restrict this exercise to a demonstration often retain their cut-out coloured angles and keep them in a small paper pouch attached to the reverse side of the bread board for future use and indeed for revision purposes to consolidate the ideas learnt.

NOTE:

Students may use a similar experimental approach for the three associated deductions.

GEOMETRY LESSON IDEA 7

TITLE: THE THEOREM OF PYTHAGORAS – A DEMO!

TOPIC THEOREMS – AN EXPERIMENTAL APPROACH

AIM That students will demonstrate the Theorem of Pythagoras.

RESOURCES:

Paper or card, scissors, straight-edge, overhead projector.

METHOD:

1. Construct two squares of any size side by side (Fig. 1). Let the larger square have side length *a*, and the smaller square have side length *b*. The area of this figure is thus $a^2 + b^2$.

2. Mark off a section of length *b* along the base of the larger square. Construct lines as shown, of length *c*. *c* is the length of the side opposite the right angle of a right angled triangle with other sides of length *a* and *b* (Fig. 2).

3. Cut out the two triangles thus constructed and label them X and Y. Rotate X anti-clockwise and rotate Y clockwise (Fig. 3).

4. Continue to rotate these pieces until a square with side *c* and area c^2 is formed (Fig. 4).

5. The initial area of $a^2 + b^2$ was preserved, so $a^2 + b^2 = c^2$ where *a, b, c* are sides of a right-angled triangle, c being opposite the right angle.

CLASSROOM MANAGEMENT IMPLICATIONS:

This is a complicated procedure and would probably be best done as a demonstration by the teacher using an overhead projector.

NOTE:

Once students have been led through one or two of these demonstrations in geometry lesson ideas 4-7, they can be given a result to demonstrate for homework. The resulting mound of cut-outs often proves interesting, particularly if students are given the opportunity to "explain" their demonstration to other members of their group or to the whole class. This gives students a chance to use geometrical language.

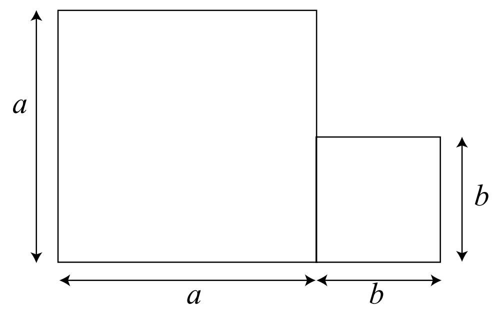


Figure 1

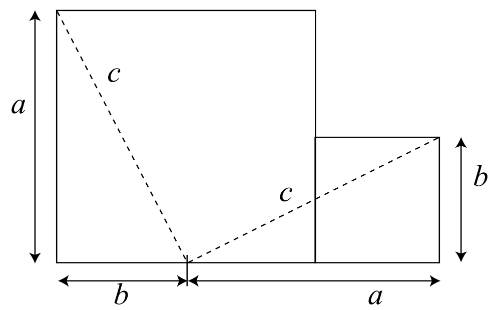


Figure 2

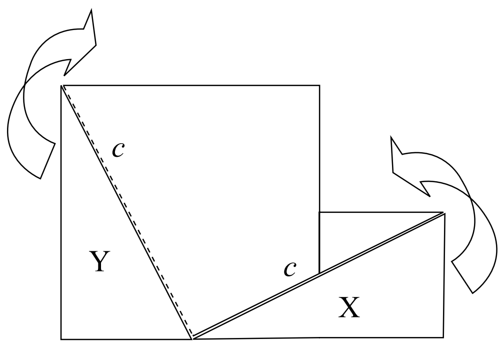


Figure 3

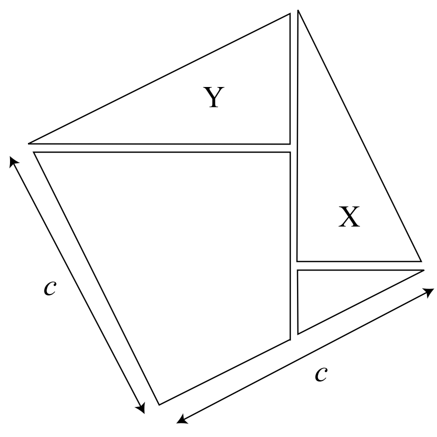


Figure 4

GEOMETRY LESSON IDEA 8

TITLE: DISCOVERING PYTHAGORAS

TOPIC: THEOREMS – AN EXPERIMENTAL APPROACH

AIM: To encourage students to try to "discover" what Pythagoras proved, by experimenting with squares of different areas.

RESOURCES:

Coloured cardboard sheets, graph paper, straight-edge and scissors.

METHOD:

1. Students (or the teacher) should construct a range of squares of side 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 units from different coloured cardboard sheets.
2. In groups (or individually, if resources permit) students have to try to make right-angled triangles from the given squares as shown in Figure 1.
3. It can be helpful to use a sheet of graph paper to confirm that the triangle is right-angled (see Figure 2).
4. Students can record their findings in a table such as this:

Length of sides of three squares	Forms a Right-Angled Triangle?
6, 8, 9	NO

5. Students should discover that squares of side 3, 4, 5 and 6, 8, 10 form right-angled triangles (the graph paper is a good help). Students then calculate the area of each of these squares and record their work as shown in figure 2.

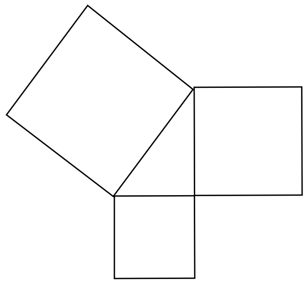


Figure 1

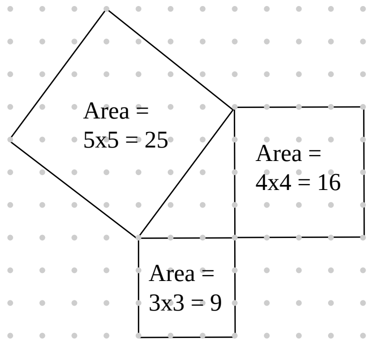


Figure 2

NOTE:

Follow-up work could include a class discussion along the following lines:

Is there a pattern to the answers 9, 16, 25 and 36, 64, 100?

Students are encouraged to discover that 9+16 = 25 and 36+64 = 100.

Now add another square of side 13 to the squares already in use. Can students find one more right-angled triangle (5, 12, 13)? Does the pattern still hold?

Describe the pattern for the sort of squares that form a right-angled triangle.

Finally, this activity can be revised and assessed by the worksheet such as that used in geometry lesson idea 9.

CLASSROOM MANAGEMENT IMPLICATIONS:

Students can work individually (if resources permit) or in small groups.

GEOMETRY LESSON IDEA 9

TITLE: THEOREM OF PYTHAGORAS – RIGHT-ANGLED OR NOT?

TOPIC: THEOREMS – AN EXPERIMENTAL APPROACH

AIM: That students will apply the converse of the theorem of Pythagoras to check if triangles are right-angled or not.

This lesson idea requires that lesson idea 8 has already been used in class.

RESOURCES:

The worksheet below.

METHOD:

Recap on how the "3,4,5" triangle obeys the theorem of Pythagoras:

The area of the square on the side measuring 4 units was $4 \times 4 = 4^2 = 16$ units squared.

The area of the square on the side measuring 3 units was $3 \times 3 = 3^2 = 9$ units squared.

The area of the square on the side measuring 5 units was $5 \times 5 = 5^2 = 25$ units squared.

Then it was noticed that the two smaller areas add up to the bigger area:

$$4^2 + 3^2 = 5^2$$

$$16 + 9 = 25$$

By calculating the area of the squares on each side of the triangles in the worksheet find out which are right-angled and which are not. Here is an example.

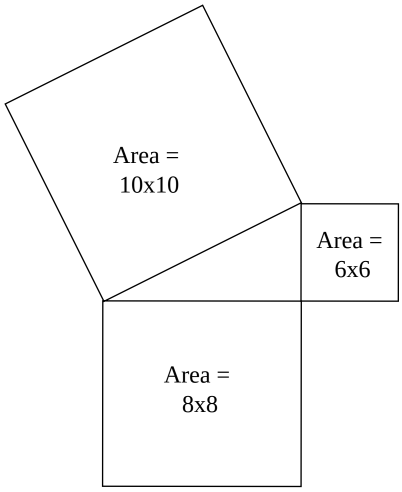
EXAMPLE:

Area of square on side length 6 is $6 \times 6 = 6^2 = 36$
Area of square on side length 8 is $8 \times 8 = 8^2 = 64$
Area of square on side length 10 is $10 \times 10 = 10^2 = 100$

Check:

Is $36 + 64 = 100$? YES

So this triangle IS a right-angled triangle.



CLASSROOM MANAGEMENT IMPLICATIONS:

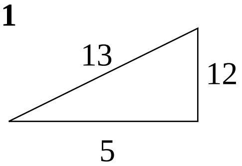
Students can work individually on the above exercise but should then be encouraged to share and discuss their results with their peers as the exercise goes on. The calculator can be used if the calculations become too cumbersome.

Student Worksheet

Find out which of these triangles are right-angled, using the converse of the theorem of Pythagoras.

(Note that the triangles below are not drawn to scale. Remember, the fact that a triangle LOOKS right-angled does not mean it IS right-angled.)

1



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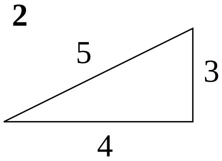

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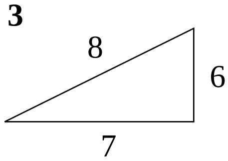

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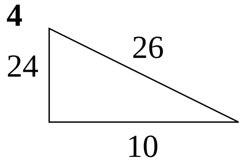

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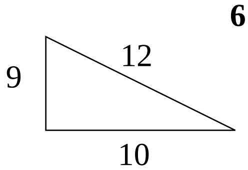

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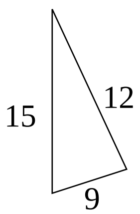

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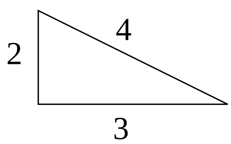

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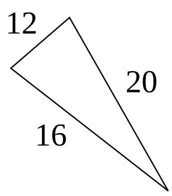


GEOMETRY LESSON IDEA 10

TITLE: PYTHAGOREAN TRIPLES

TITLE: THEOREMS – AN EXPERIMENTAL APPROACH

AIM: That students will revise and apply their knowledge of the theorem of Pythagoras.

RESOURCES:

The calculator can be used to accelerate the learning process.

METHOD:

1. From previous exercises and activities, Pythagorean triples like (3, 4, 5), (6, 8, 10) and (5, 12, 13) may well have been discovered by the students themselves. If so, they will remember them better.
2. Break the class into teams of "table quiz" type and see how many Pythagorean triples they come up with in a specified length of time, for example 7 minutes.

CLASSROOM MANAGEMENT IMPLICATIONS:

The class should be organised into small groups of three or four for the table quiz.

NOTE:

It is possible for teachers with a basic knowledge of a spreadsheet package to construct a spreadsheet file using elementary algebraic formulae, and so generate any desired number of Pythagorean triples. The columns in the spreadsheet – and the calculations to be performed on the chosen numbers *x*, *y* (with $x > y$) entered in the first two columns – are given by:

x	y	a	b	c
		$x^2 - y^2$	$2xy$	$\sqrt{a^2 + b^2}$

The final three columns yield Pythagorean triples.

GEOMETRY LESSON IDEA 11

TITLE: PYTHAGORAS AND WALLS!

TOPIC: THEOREMS – AN EXPERIMENTAL APPROACH

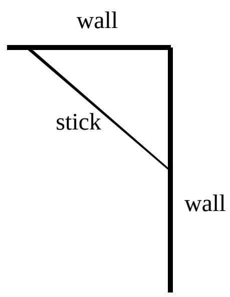
AIM: That students will verify the theorem of Pythagoras with the aid of walls in their classroom.

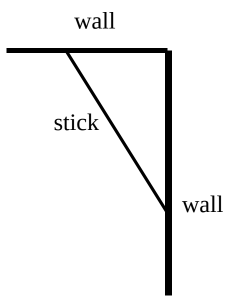
RESOURCES:

One or two lengths of timber, a measuring tape and calculator.

METHOD:

1. By varying where they position the length of timber, teams of students can take turns to measure the three sides of the triangle formed between the piece of timber and the two walls.
2. Using the calculator, the students can verify that the theorem of Pythagoras holds true.
3. This exercise can be related to workers who build a house and who want to check for right angles in the corners. Likewise the workers who do the internal plastering often have a large right-angled triangle with them to check that the walls form right angles at the corners.





CLASSROOM MANAGEMENT IMPLICATIONS:

Small teams of three students can be chosen for this activity, taking their turn at measuring and calculating. The activity can be repeated a number of times.

GEOMETRY LESSON IDEA 12

TITLE: SIMILARITY V. CONGRUENCE AND RATIO

TOPIC: GEOMETRY

AIM:

1. To give students an opportunity to discover the difference between similar and congruent triangles.
2. To give students an opportunity to calculate ratios and get a feel for the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides in the same ratio.

RESOURCES:

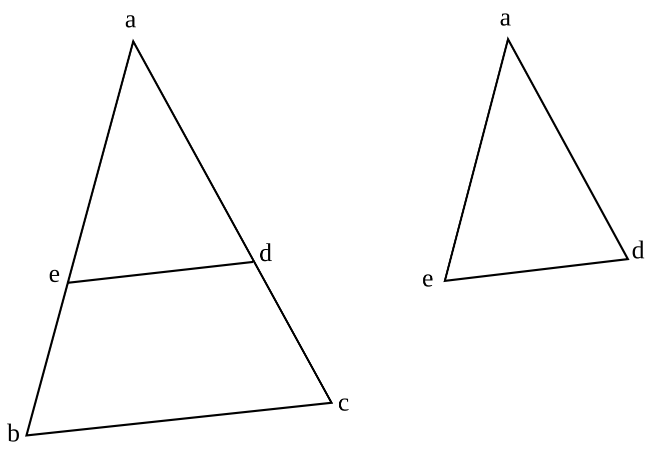
Two A4 sheets of coloured card (different colours), compass, scissors, calculator, paper and pencil.

METHOD:

1. On one coloured A4 sheet students are asked to draw a scalene triangle, the larger the better. Next, they have to draw a line parallel to one side and cutting the other two (see diagram). There are many different ways of drawing parallel lines and much debate as to the best method ensues.

2. Now the students have to make *an exact replica* of the upper (smaller) triangle from the second coloured A4 sheet. Again, many methods might be used but the unsophisticated method of "pinning through" the vertices of the small triangle with the compass point onto the other sheet placed underneath works well. The new triangle is then cut out.

3. Placing the new triangle on top of the original one can show corresponding angles and emphasises "similarity" v. "congruence". Students should be allowed to discover, as they position the new triangle in each corner, that the "other side is always parallel". This configuration arises time after time in the mathematics programme and the students should be helped to recognise it instantly.



4. The ratio of the divided lines can now be investigated. The students need to learn to measure accurately in millimetres and to make sensible use of the calculator. They should be asked to investigate the link between the length of the corresponding sides of the small and large triangles. In the diagram that is

$$\frac{|ab|}{|ae|} = \frac{|ac|}{|ad|} = \frac{|bc|}{|ed|}$$

   Within reasonable tolerances the ratios should be the same. Reasonable measurements, and sensible decimal places decided from the calculator result, will be the issues for debate in the final part of this exercise.

5. The new triangle can be repositioned and the process repeated. (It is perhaps easier for the students to *see* the equal ratios if the smaller value is divided into the larger.) As each student will have a differently sized scalene triangle the generalisability of this "rule" will be quite obvious, and the visual impact of the models will help the students to remember both the rule and the configuration.

NOTE:

From this work a number of mathematical terms can become part of the students' lexicon. These include congruence, similarity, scalene, vertex, ratio, proportion, corresponding, parallel, enlargement, reduction, translation, rounding up/down, accuracy, and estimation.

CLASSROOM MANAGEMENT IMPLICATIONS:

Students can work individually or in pairs when doing the investigative work.

GEOMETRY LESSON IDEA 13

TITLE: HIT OR MISS! [THIS ACTIVITY IS BASED ON THE CHILDREN'S GAME OF *BATTLESHIPS*, WHICH USES A COORDINATED GRID OF LETTERS AND NUMBERS.]

TOPIC: COORDINATE GEOMETRY

AIM: That students will learn an entertaining way to draw and read coordinate points.

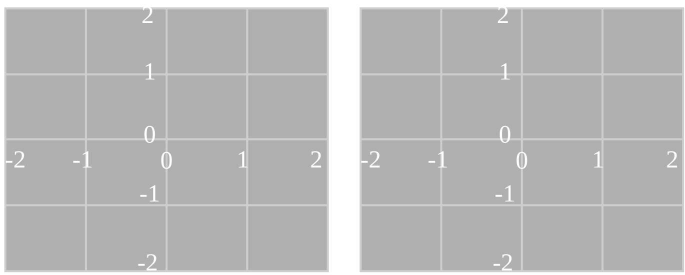
RESOURCES:

Graph paper.

METHOD:

1. Students work in pairs for the game. Each student draws two grids as shown. One grid is to display the student's own battleships and the other to record guesses of the opponent's positions.

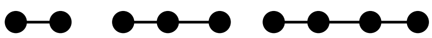
Home Fleet | Away Fleet



Hit or Miss

Guess	Hit/Miss	Guess	Hit/Miss

2. All students fill in their fleet of ships on their home grid. Each student has a fleet of 1 two-dot, 1 three-dot and 1 four-dot ships. Each ship can be placed on the grid in a horizontal or vertical position.

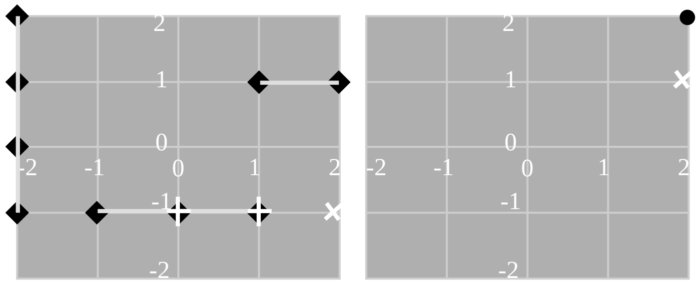


3. One student guesses a pair of coordinates and records them on the away grid and in the table of values. If this is a hit, the same student guesses again. When a hit is made, it is normal for the opponent to indicate if the battleship is a two-dot, three-dot or four-dot ship. If it is a miss, the opponent takes a turn.

Sample Game of Hit or Miss

I go first. I guess (2,2). It is a hit and I record this with a heavy dot (see table and away grid). I guess (2,1). It is a miss and I record this information with an x (see table and away grid).

Home Fleet | Away Fleet



Hit or Miss

Guess	Hit/Miss	Guess	Hit/Miss
(2,2)	Hit		
(2,1)	Miss		

My opponent guesses (0,-1) and I say that it is a hit and put a + onto my home grid. The next guess is (1,-1). Another hit. The third guess is (2,-1) – a miss which I record with an x on my home fleet grid.

I guess (1,2), which is a hit, and (0,2), another hit. These are recorded by two more heavy dots on the away grid. I have now sunk my opponent's 3-dot ship.

And so the game continues until one person has sunk all of the opponent's ships. The grids ensure that if a student is confusing (2,1) with (1,2), the opponent can check that the guesses correspond with correctly reported hits or misses once the game has finished.

CLASSROOM MANAGEMENT IMPLICATIONS:

Students play this game in pairs. If the grids are drawn in pen and the coordinates in pencil, the grids can be re-used.

NOTE:

1. If playing properly, students will check each other's knowledge of reading and plotting co-ordinate points.
2. The rules of the game can be changed to make the game move along more quickly or slowly as required.
3. It also works well with a positive number grid rather than the integer grid used above and this might be a more useful starting point with some students.

GEOMETRY LESSON IDEA 14

TITLE: CONGRUENT TRIANGLES

TOPIC: SYNTHETIC GEOMETRY

AIM: To introduce students to the idea of congruency with concrete materials. To give students practice at constructing triangles, given any of the following four sets of data: SSS, SAS, ASA, or RHS.

RESOURCES:

Light cardboard, scissors, geometry set and straight-edge.

METHOD:

1. The idea of congruent triangles can be conveyed to the students by getting them to construct triangles of light cardboard (such as that in cereal boxes), using given data, and cutting them out. All the triangles of the same measurements can be compared and seen to be identical.

2. For example, for homework, get each of the students to draw a triangle with side lengths 13cm, 12cm and 10cm (it needs to be reasonably big) on cardboard and bring it to class the next day. The teacher may have an accurate "template" against which to compare the triangles, and may discard inaccurate attempts. When all the accurate triangles are placed on top of each other, the students can see that it is impossible to draw a triangle with these measurements which is not identical to all the others. So SSS is seen to be enough to establish congruence.

3. The same can be repeated for three other triangles given SAS, ASA, and RHS.

4. The advantage of this method is that while students are improving their skills in constructing triangles, they are also becoming convinced of the fact that congruence in triangles can be established, given any of the four conditions.

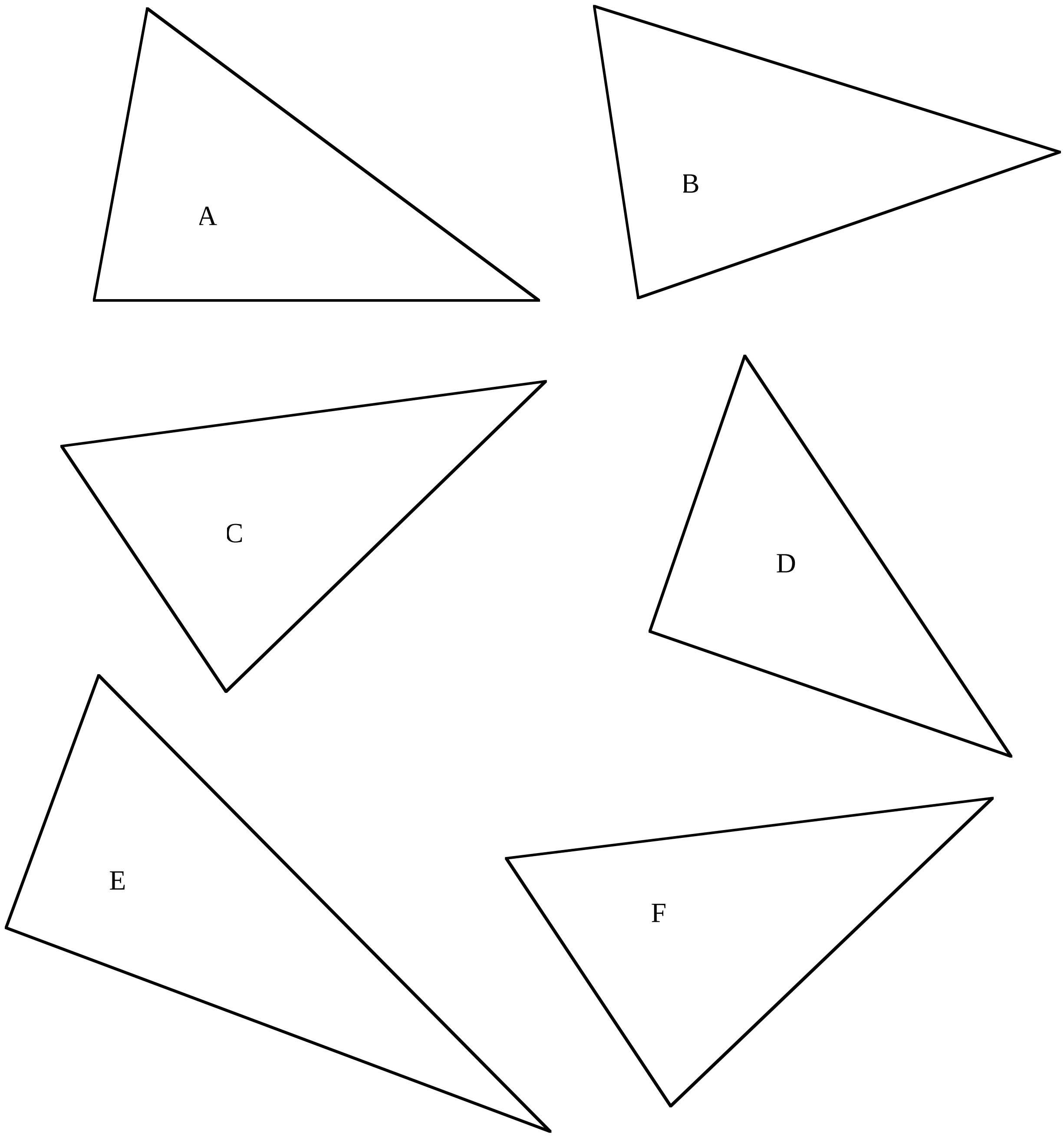
5. A variation on the above exercise involves students being presented with a page of triangles on a worksheet and a cut-out triangle of light cardboard. The students are asked to compare the given cut-out triangle with those on the sheet and to test for congruency. Any computer drawing package can be used to prepare the cut-out triangle. The copy and paste commands can then be used to produce identical images which can be rotated into different positions. Finally, a number of non-congruent triangles can be added to the sheet before printing. This can be repeated a number of times using different triangles. A sample worksheet is presented opposite.

CLASSROOM MANAGEMENT IMPLICATIONS:

Students should be encouraged to work on their own with the worksheets and later to compare and discuss their results with their peers.

Student Worksheet for Testing Congruency

Test your cut-out triangle against each of the triangles below.



My cut-out triangle is congruent with triangles which have the following letters:

____

FURTHER TEACHING NOTES ON CONGRUENT TRIANGLES, EQUIANGULAR TRIANGLES, AREA AND PARALLELOGRAMS

NOTE 1:

Students may mix up the concepts of congruent triangles, triangles of equal area and equiangular triangles which they meet later. It should be emphasised that congruent triangles have equal areas but triangles of equal areas are not necessarily congruent.

"AAA" does not give congruence, obviously, as it is possible to draw two equiangular triangles of very different sizes (suggestion: give as an exercise to the students the task of drawing three different sized equiangular triangles).

Class Experiment:

Try to draw a triangle with sides of length 10cm, 3cm and 5cm. Why is this impossible?

NOTE 2:

The *distance from a point to a line* is the length of the perpendicular from that point onto the line.

NOTE 3:

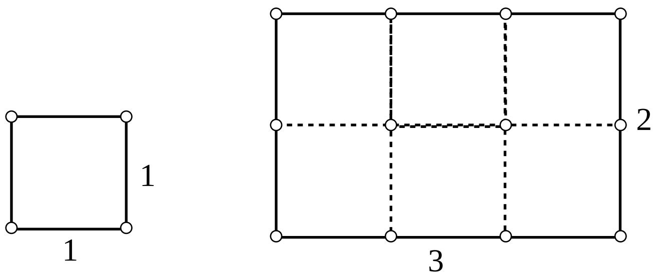
Any of the three sides of a triangle may be called a *BASE*. The *perpendicular height* of the triangle is the distance from the opposite vertex to a chosen base.

Similarly, any of the four sides of a parallelogram may be called a *BASE*, and the *perpendicular height* of the parallelogram is the distance to the chosen base from any point on the opposite side of the parallelogram.

NOTE 4:

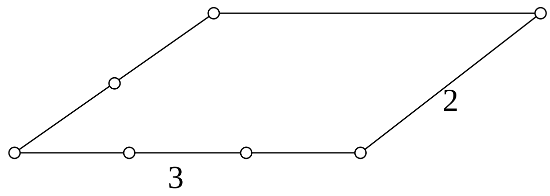
Why is the area of a rectangle = $length \times breadth$?

Example: A rectangle measuring 3 units by 2 units.

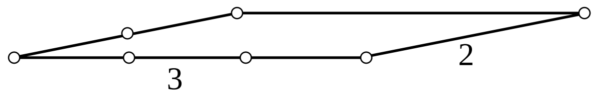


By counting, this has 6 square units of the size shown on the left. Since multiplication is a quick way to add, we get 3 times 2 = 6 square units.

Why then do we not get the area of a parallelogram by multiplying length by breadth? For example, is the area of the parallelogram below equal to or less than the area of the rectangle which has the same side lengths? The answer will not be obvious to the student.



But if we continue to "squeeze" the parallelogram downwards, it begins to look like this:



We can see now that the area is definitely smaller and could eventually become close to zero.

The area of a parallelogram can be arrived at in the following steps:

(a) Start with the "fact" that the area of a rectangle = $length \times breadth$.

(b) A triangle has half the area of a rectangle of the same height and base and therefore the area of a triangle = $\frac{1}{2} base \times height$.

(c) Since a diagonal bisects the area of a parallelogram and divides it into two triangles of equal area, the area of a parallelogram = $base \times height$.

NOTE 5:

The following properties of parallelograms may be interesting to investigate/prove:

- Opposite sides and angles are equal in measure
- Diagonals bisect each other (the proof is a nice exercise!)
- A diagonal bisects its area
- Area = $base \times height$.
- Any pair of adjacent angles sum to 180° (easy to prove)
- The diagonals divide the parallelogram into four triangles of equal area (prove)
- Only in the case of a rhombus are the diagonals perpendicular (prove)
- Only in the case of a rhombus does a diagonal bisect the angle(s) through which it passes – this deceives a lot of students – (prove).

4.10 TRIGONOMETRY

TRIGONOMETRY LESSON IDEA 1

TITLE: INTRODUCING THE TAN RATIO

TOPIC: TRIGONOMETRY

AIM: To give students an opportunity to find the angle of elevation of the sun from experimental field-work.

RESOURCES:

Poles of various lengths (e.g. sweeping brush, metre stick), measuring tape, graph paper, protractor, centimetre ruler, pencil, eraser, a sunny day and safety instructions about not looking at the sun!

METHOD:

Outside instructions

1. Students work in twos. One person holds the pole vertically on level ground so that its shadow can be clearly seen.
2. Record the date and time. Measure and record the length of the pole.
3. Measure and record the length of the shadow.
4. Draw a rough diagram.

Back in the classroom

1. Decide on a scale to use.
2. Draw an accurate diagram on graph paper.
3. Measure the angle of elevation of the sun (A) using a protractor.
4. Compare your answer with that of others in the class. Make up a table showing times and angles.

The results should yield (approximately) the same angle of elevation.

Fill in the table below. (The calculator can be used to evaluate tan$A$ so that results in columns 3 and 5 can be compared.)

Length of Pole	Length of Shadow	$\frac{\text{Length of Pole}}{\text{Length of Shadow}}$	Angle of elevation A	tan$A$

Students can experiment by drawing triangles with the same angle of elevation but various lengths on the horizontal and vertical, and then completing a similar table.

Length of Vertical line	Length of Horizontal line	$\frac{\text{Length of Vertical line}}{\text{Length of Horizontal line}}$	Angle of elevation A	tan$A$
			The same for all students!	

CLASSROOM MANAGEMENT IMPLICATIONS:

The initial field-work can be done in pairs but then students can be encouraged to compare results with their peers.

TRIGONOMETRY LESSON IDEA 2

TITLE: TRIGONOMETRY AND THE USE OF THE EYE!

TOPIC: TRIGONOMETRY

AIM: To show students how to calculate sides of a triangle using alternative definitions.

RESOURCES:

None.

METHOD:

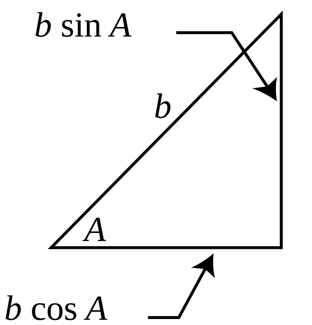
1. Start with the standard trigonometric ratios (based on a right-angled triangle):

   Sine of angle = Opposite/Hypotenuse
   Cosine of angle = Adjacent/Hypotenuse
   Tangent of angle = Opposite/Adjacent

2. Derive the following from the standard formulae:

Hypotenuse × Sine of angle = Opposite
Hypotenuse × Cosine of angle = Adjacent
Adjacent × Tangent of angle = Opposite

3. This approach enables students to determine the other two sides of the triangle when provided with the hypotenuse b and the base angle A.

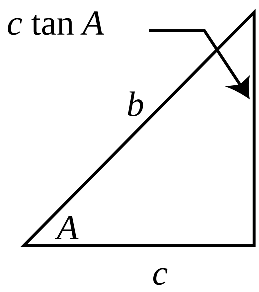


4. Frequently, students are asked to determine the height of a building. This involves standing back from the building and measuring

   i. the angle of elevation A
   ii. the horizontal distance c to the base of the building.

   Using a calculator, the students can key in the data to yield an immediate result:

   height = c tan A



CLASSROOM MANAGEMENT IMPLICATIONS:

None

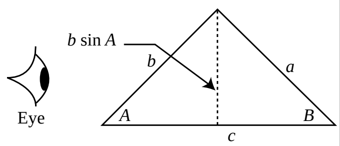
Additional, follow-up work is given below, showing how the Sine Rule and the formula for the area of a triangle might be derived (the proofs are not examinable).

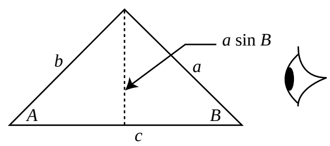
NOTE:

1. There is little doubt but that students have difficulty with applying trigonometric ideas. Much of the problem is with handling orientation in space. An eye is drawn on the blackboard to indicate the side from which the diagram is viewed. In the example given below the task is to find the perpendicular height.

   The use of the eye is helpful in that it can be seen clearly that

Viewed from the left:	Viewed from the right:
perpendicular = b sin A	perpendicular = a sin B
HYPOTENUSE × SINE = OPPOSITE	HYPOTENUSE × SINE = OPPOSITE





   Equating the two expressions for the perpendicular gives: a sin B = b sin A

   and this leads to the Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$

2. Equally, if the triangle is viewed from the left then the area of the triangle can be given by:

   $$\text{Area} = \tfrac{1}{2}\ \text{base} \times \text{perpendicular height}$$
   $$= \tfrac{1}{2}\ c \times b \sin A$$
   $$= \tfrac{1}{2}\ bc \sin A$$

   Viewing from the right produces:

   $$\text{Area} = \tfrac{1}{2}\ \text{base} \times \text{perpendicular height}$$
   $$= \tfrac{1}{2}\ c \times a \sin B$$
   $$= \tfrac{1}{2}\ ac \sin B$$

TRIGONOMETRY LESSON IDEA 3

TITLE: THE CLINOMETER AND TRIGONOMETRY

TOPIC: TRIGONOMETRY

AIM:

1. To give students an opportunity to make a clinometer and to measure elevation and declination.
2. To allow students the opportunity to calculate the heights of structures with the help of the clinometer in experimental fieldwork.
3. To give students an opportunity to experience mathematical modelling.

RESOURCES:

Stiff cardboard (280 gsm or heavier), paper, pencil, black thread, paper fasteners, plasticine and scissors.

METHOD:

1. The model *clinometer* shown in Figure 1 should be made from stiff card (280gsm or heavier). The viewing-tube can be made from a drinking straw or by rolling writing paper around a pencil. The "plumbline" should be made from black thread and attached to the centre of the semi-circle by a paper fastener and finally the weight might be plasticine. The degree markings should be made using a protractor and set at 5º spacings; in use, smaller measurements might have to be estimated. One of the many interesting problems for the students is whether to mark 0º to180º around the semicircle or to mark 0º to 90º on two halves, so eliminating the subtraction of readings in order to calculate angle α.

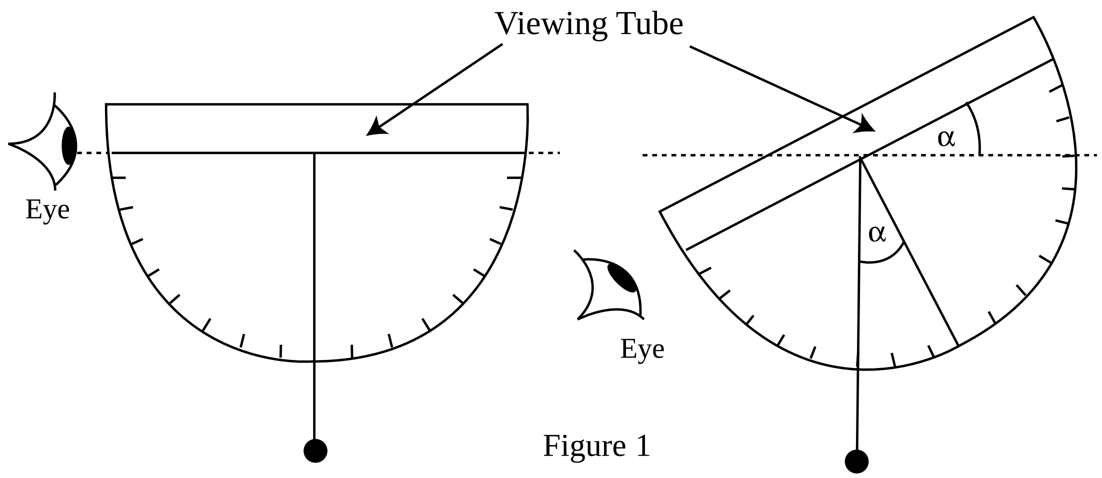
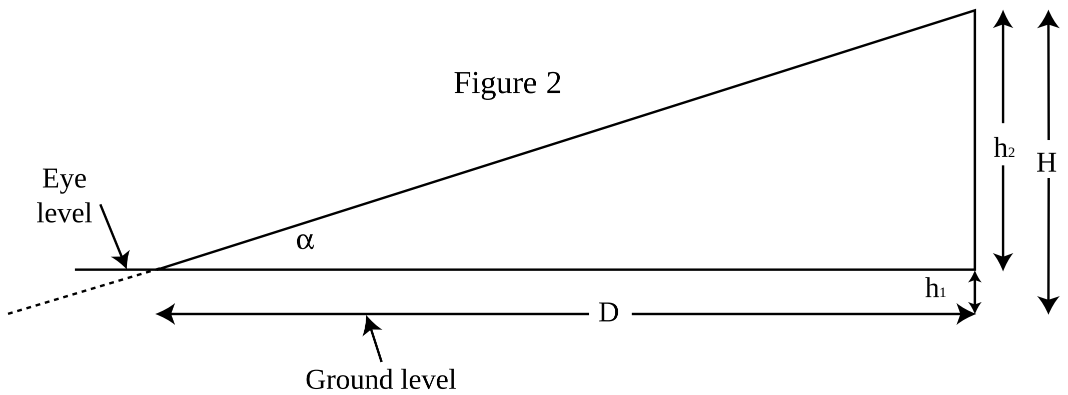


Figure 1

2. Figure 2 shows, schematically, the measurements students are expected to make: the distance D from the structure, h_1 (their eye-level), and finally α, the angle of elevation. It is surprising how many problems arise with simple measuring. Many students will never have used metre sticks and can often have trouble marrying metres, centimetres and millimetres into the correct written decimal form.

Figure 2



3. Students can then be introduced to the idea of "mathematical modelling" of the situation when they use the formula to determine H, the height of the structure:

$$H = h_1 + h_2$$

The value of h_2 can be found graphically by scaling the whole exercise on graph paper.

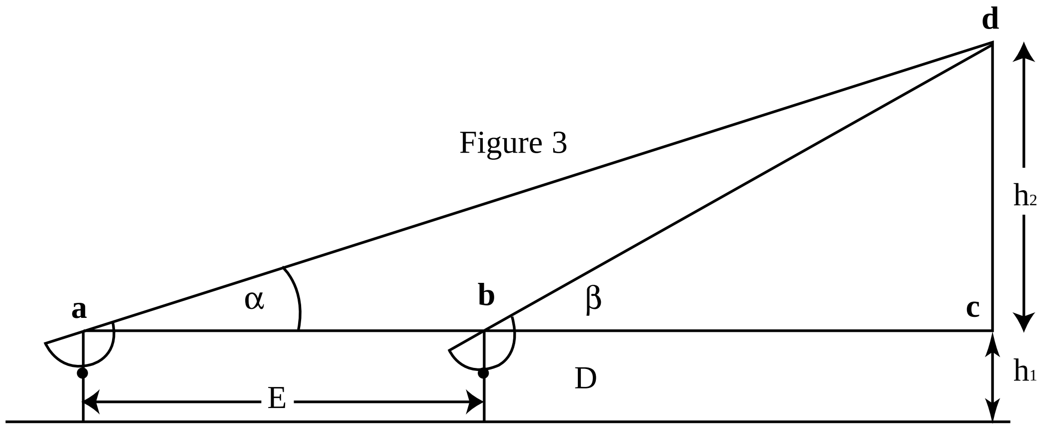
4. The more able students can perhaps find the value of h_2 by using:

$$\tan\alpha = \frac{h_2}{D}$$

The latter formulation should be tried eventually by all students, since it gives them a concrete example of what "tan" is in the case of right-angled triangles and hence how it can be used to calculate the lengths of the sides of a triangle.

5. For the more able mathematicians Figure 3 shows how, if the measurement D to the base of the structure cannot be measured, then determining two angles α and β from two measured points (distance E apart) will allow use of the sine rule in triangle abd in order to find bd, the hypotenuse of triangle bcd. This, in turn, allows the use of tanβ to calculate h_2 in triangle dbc.

Figure 3



NOTE:

The variety of mathematical words and ideas that occur when undertaking the making and using of the clinometer is immense. Many skills are employed by students: psychomotor, organisational and graphical skills; skills of measurement, scaling diagrams, recording values, and estimating accuracy. All these are allied to the algebraic skills of manipulating linear and trigonometrical expressions and formulae.

CLASSROOM MANAGEMENT IMPLICATIONS:

Students can work in small groups of two or three for the fieldwork.

4.11 FUNCTIONS AND GRAPHS

FUNCTIONS AND GRAPHS LESSON IDEA 1

TITLE: THE FUNCTION MACHINE

TOPIC: FUNCTIONS AND GRAPHS

AIM:
1. To give students an intuitive idea of functions using the analogy of a function machine.
2. To give students the opportunity to complete a graph table by reading the ordered pairs from the graph.
3. To show how the calculator can be used as a learning tool.

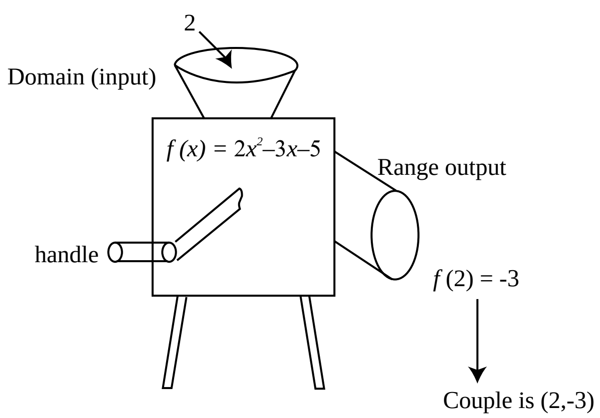
RESOURCES:

A copy of the "function machine" for display using an overhead projector.

METHOD:

1. Ask students to consider the function $f(x) = 2x^2 - 3x - 5$ in the domain $-3 \leq x \leq 3$, $x \in R$. The image of 2 is -3. A similar pattern exists for the other elements of the domain. Each input element is coupled with its output image in the form (x, y), for example (2,-3).
2. In passing, students could be shown how to use the PLAYBACK and the DEL functions on a suitable calculator to substitute the 2 by another number and get a new result or output.
3. While constructing the traditional long form of the table is foolproof, in many respects the logic behind how functions operate can get somewhat lost. What is required ultimately is a set of couples that can be graphed. This is where the "function machine" can help.

The Function Machine

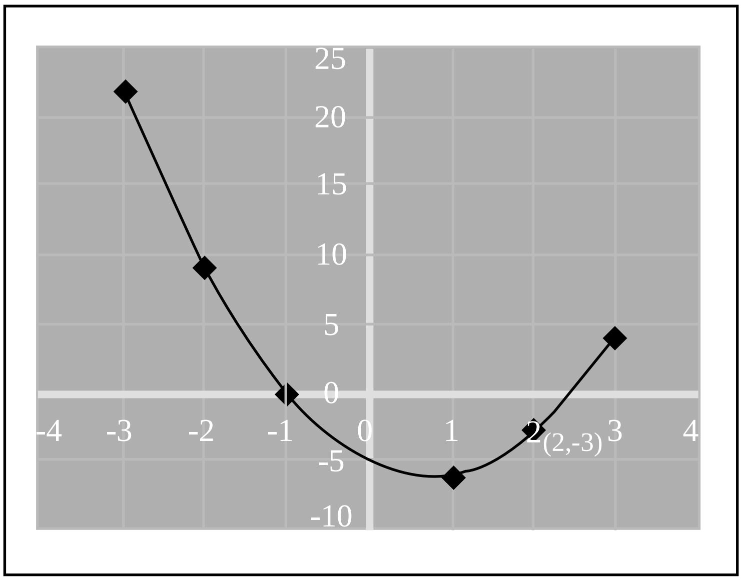


As the couples are found they can be filled into a table (the calculator can help with the calculations).

x	-3	-2	-1	0	1	2	3
$f(x)$						-3	

4. Explain to students that the elements which are fed into the top of the function machine are plotted along the x-axis, and the elements which correspond to the output are plotted along the y-axis.
5. Now, questions such as "Find the value of x such that $f(x) = -3$" take on the meaning "For what values of x inserted at the top of the function machine will an output of -3 be produced?" Students readily relate to this type of treatment and see the concept of a function as an operation (turning the handle) on a set of elements.
6. It is instructive to drop $-\frac{1}{2}$ into the top of the machine and to observe that the image is also -3, giving the couple $(-\frac{1}{2},-3)$. Students soon observe that the "processing" or number crunching takes place inside the machine and corresponds to "adding up" the values in the graph table normally used.
7. The reverse approach to graph drawing is also of educational benefit. Working from a graph such as that illustrated overleaf, the student is asked to complete the table to the best of his/her ability by reading the ordered pairs from the graph. These pairs can be recorded in a table similar to that used at step 3.

x	-3	-2	-1	0	1	2	3
$f(x)$						-3	



CLASSROOM MANAGEMENT IMPLICATIONS:

None.

NOTE:

To create a graph such as the one shown, enter the x and y values into a spreadsheet (e.g. Excel) in the following format:

x	-3	-2	-1	0	1	2	3
y	22	9	0	-5	-6	-3	4

Select the table and click on the chart wizard. Choose XY scatter and select the scatter with data points connected by smoothed lines. More detailed changes to the graph can be made by using the option tags presented as the graph is produced.

FUNCTIONS AND GRAPHS LESSON IDEA 2

TITLE: MAXIMUM AND MINIMUM VALUES FOR QUADRATIC FUNCTIONS

TOPIC: FUNCTIONS AND GRAPHS

AIM: To show students the method of "completing the square" for finding the maximum and minimum values of quadratic functions.

RESOURCES:

None.

METHOD:

1. Consider the following question.

   Find the minimum value of the function:

   $f(x) = x^2 - 6x + 13, x \in R$

   $f(x) = x^2 - 6x + 13$

   $\Rightarrow f(x) = x^2 - 6x + 9 - 9 + 13$

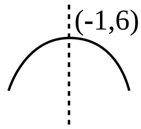
   $= x^2 - 6x + 9 + 4$

   $= (x - 3)^2 + 4$

   Therefore the minimum value is 4 and this will occur when $x = 3$.

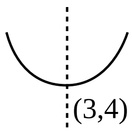
   In passing, it could be pointed out to students that $x = 3$ is the axis of symmetry of the graph

(-1,6)



$x = -1$

$x = 3$



(3,4)

2. Similarly:

   Find the maximum value of the function:

   $f(x) = 5 - 2x - x^2, x \in R$

   $f(x) = 5 - 2x - x^2$

   $\Rightarrow f(x) = 5 + 1 - 1 - 2x - x^2$

   $= 6 - (1 + x)^2$

   giving a maximum value of 6, which occurs when $x = -1$

CLASSROOM MANAGEMENT IMPLICATIONS:

None

NOTE:

The syllabus (p.16) mentions the "maximum and minimum values of quadratic functions estimated from graphs" for Higher Level students.
It would be desirable if the algebraic method of "completing the square" for evaluating the maximum or minimum was treated at least once for students taking the Higher level.

4.12 MATHEMATICAL STORIES

The revised Junior Certificate Mathematics syllabus (page 4) states that students "should be aware of the history of mathematics and hence of its past, present and future role as part of our culture". A number of mathematical stories now follow which can be used to enliven a topic or as an attention grabber at the start of a new topic.

INTRODUCING IRRATIONAL NUMBERS– THE MURDER STORY

Pre-requisite knowledge: Theorem of Pythagoras.

MURDER MOST FOUL

Excerpt from *Fermat's Last Theorem* by Simon Singh (Fourth Estate, London, 1997, p.55)

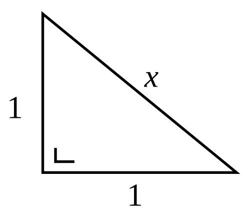
> For Pythagoras, the beauty of mathematics was the idea that rational numbers (whole numbers and fractions) could explain all natural phenomena. This guiding philosophy blinded Pythagoras to the existence of irrational numbers and may have led to the execution of one of his students. One story claims that a young student by the name of Hippasus was idly toying with the number $\sqrt{2}$, attempting to find the equivalent fraction. Eventually he came to realise that no such fraction existed…. Hippasus must have been overjoyed by his discovery, but his master was not…. Pythagoras was unwilling to accept that he was wrong, but at the same time he was unable to destroy Hippasus' argument by the power of logic. To his eternal shame he sentenced Hippasus to death by drowning.

NOTE

$\sqrt{2}$ is related to the theorem of Pythagoras as follows:

Consider an isosceles right-angled triangle as shown. By the theorem of Pythagoras:

$$x^2 = 1^2 + 1^2$$
$$\Rightarrow x^2 = 2$$
$$\Rightarrow x = \sqrt{2}$$



which we know today as an irrational number – one that cannot be expressed as a fraction!

A SPHERICAL STORY!

Archimedes was one of the all-time great mathematicians, especially when one realises that all he had access to was a pencil, a ruler and a compass. He is credited with many inventions including giant catapults, solar ray guns, and giant lever systems. However he was very proud of the following equation:

$$V = \frac{4}{3}\pi r^3$$

Archimedes proved that a sphere is exactly two-thirds as big as the smallest cylinder into which it will fit. So, a solid ball which would only just fit inside a tin can (with the lid on) takes up exactly two thirds of the space inside the can. There is a small sign consisting of a sphere in a cylinder on his gravestone in memory of this discovery. When the Romans invaded his town one night, the Roman general Marcellus had especially ordered that the 75-year-old Archimedes should be spared. However, a soldier found him doodling in the sand and Archimedes upset him by saying, "Do not disturb my diagrams", so the soldier killed him.

IMPORTANCE OF USING THE CORRECT UNITS – AN EXPENSIVE LESSON

The $125 million Mars Climate Orbiter, launched on 11 December 1998, was lost when it came too close to planet Mars. It had probably come too deep into the atmosphere of Mars, because it had been erroneously navigated on a trajectory bringing it down to only 50 km above the surface, and was very likely destroyed – its safe altitude would have been about 80 km. The error was due to a *confusion of metric and imperial units* between different collaborating teams.

Arthur Stephenson, chairman of the Mars Climate Orbiter Mission Failure Investigation Board said:

> *The "root cause" of the loss of the spacecraft was the failed translation of English (imperial) units into metric units in a segment of ground-based, navigation-related mission software….*

For further details of this story see
http://www.seds.org/~spider/spider/Mars/ms98mco.html

http://www.astronomynow.com/mars/mco/991110findings/index.html

PRIME NUMBERS – NOW THAT'S BIG!

On 1 June 1999, the team of Nayan Hajratwala, George Woltman, Scott Kurowski et. al. discovered a new record prime number: $2^{6972593}-1$. This is the 38th *known* Mersenne prime (there may be smaller ones as not all previous exponents have been checked). This is the fourth record produced by GIMPS (the Great Internet Mersenne Prime Search) in four years! This prime number has 2 098 960 digits (it would take over 400 typed pages to write it down!!) and is the largest known prime number in the world – until someone discovers the next one, that is. The discoverers got a $50,000 award for finding it!

NOTE

A prime is any integer greater than 1 which has only 1 and itself for positive divisors. The first few primes are 2, 3, 5, 7 and 11. Mersenne primes are those which are a power of two, minus one. For example, the first few are $2^2-1=3$, $2^3-1=7$, $2^5-1=31$, $2^7-1=127$. These first few were known to the ancient Greeks several hundred years before Christ.

For more information see
http://www.utm.edu/research/primes/largest.html#largest

Finally, mathematics teachers in search of suitable stories to weave into their mathematics classes will find useful resources on the following web pages.

History of Mathematics Archive:
http://www-groups.dcs.st-and.ac.uk/~history/index.html

Female Mathematicians:
http://www-history.mcs.st-and.ac.uk/history/Indexes/Women.html

Earliest known uses of some of the words of mathematics:
http://members.aol.com/jeff570/mathword.html

Earliest uses of various mathematical symbols:
http://members.aol.com/jeff570/mathsym.html

4.13 COMMUNICATING MATHEMATICS

Several lesson ideas have deliberately focused on the importance of getting students to communicate their mathematics both verbally and in written form in pairs or in small groups. This is an important feature emphasised in the revised syllabus. The usefulness of building up a lexicon of mathematical terms in a gradual fashion is mentioned a number of times in these *Guidelines*.

One possible intervention in this regard is mathematics journal writing. This can help students enormously in communicating their mathematical learning experiences. In addition the teacher can use the insights gained to check for student understanding and where difficulties occur. Journal writing can take place over a period from one week up to a month or longer if so desired. Students are invited to think about what they are doing in class and then at the end of class (or for homework) to complete open-ended questions under such headings as: summary of lesson, questions to ask the teacher and reflection. Students need to know that the aim of journal writing is to help them to understand more clearly what they are doing in class. In the section on reflection, students could be presented with a number of suggestions including:

- Describe your favourite mathematics class
- Describe how you feel about mathematics
- How could we use mathematics class to best advantage?
- Where do the rules of mathematics come from?
- Can you understand what I do?
- What can I do to help you with your mathematics?
- Can you relate what we are doing to other topics that we have covered?

Practising teachers who use journal writing regularly with their classes comment favourably on the honesty of student feedback and on how it contributes to improving their mathematical understanding. Thus, there are benefits for the teacher, student and the student/teacher relationship.

Here is a comment from one practising teacher on the merits of journal writing.

> The benefits to me as teacher were enormous. Through reading the students' journals I was in a better position to evaluate and offer remediation to individual students. I received unique feedback that I would not have had access to previously. The students were telling me that they liked my use of examples but I also realised that they were becoming dependent on them and that this was leading them to learning by rote. I put them working in pairs and I quickly realised that what they were learning collaboratively today they will learn independently tomorrow. I became more aware of the tasks that I was setting and I began to scaffold and fade exercises according to their needs. Journal writing is a very useful tool in the context of improving communication skills and student understanding as you the teacher engage in a unique and continuous dialogue with each individual student in the class.

4.14 MATHEMATICS AND INFORMATION AND COMMUNICATION TECHNOLOGIES

The National Council for Educational Technology (NCET) have identified six major ways in which information and communication technologies (ICTs) can provide opportunities for students learning mathematics. These include

- learning from feedback: provided by the computer in a fast, reliable, non-judgemental and impartial manner
- observing patterns: based on a computer's ability to produce many examples in a short time
- seeing connections: between formulae, tables and graphs
- working with dynamic images: allows students to manipulate geometrical diagrams
- exploring data: students can interpret and analyse real data in a variety of representations
- "teaching" the computer by means of an algorithm: encourages the student to express their commands unambiguously and in the correct order.

This section gives a brief overview of some of the work that is currently being undertaken in Irish mathematics classrooms with the aid of various software packages. It is by no means an exhaustive list. It concentrates on packages that are currently available to most schools and on some of the initiatives that are being run as part of the Schools Integration Project (SIP) under Schools IT 2000. More information on these projects can be obtained from the National Centre for Technology in Education (NCTE).

Many Education Centres have been offering training to teachers of mathematics on the software mentioned below and are a source of further information for interested teachers.

A set of ICT Guidelines to be issued to secondary schools will include a discussion of the issues, challenges and opportunities offered by ICT for teaching and learning.

DYNAMIC GEOMETRY SOFTWARE

In recent years a number of software packages which are collectively described as dynamic geometry packages have been developed. These include Geometer's Sketchpad, Cabri-Géomètre and JavaSketchPad. More information and samples can be viewed on the Internet by searching for the phrase "dynamic geometry". These packages are having a significant impact on the teaching and learning of geometry in very many countries. They have the potential to provide students with a new and fundamentally different geometry learning experience. They can be used in a variety of ways from a versatile demonstration aid for the teacher to a powerful tool for student-centred investigative learning.

The Geometer's Sketchpad is a visualisation tool which emphasises spatial reasoning and logical abstraction. It allows for discussion, co-operative learning, and improved communication skills in mathematics. Students can discover properties and test conjectures by *dragging* parts of their constructions. As constructed relationships remain valid while one *drags*, geometry becomes dynamic. Sketchpad is a dynamic geometry environment. Translations, rotations and reflections can be performed to analyse the effects of motion in the plane. Much useful work in coordinate geometry can be achieved by getting students to construct points and lines and asking the program to find the mid-point and slope and hence the equation of lines. Interactive scripting records and generalises the steps of a construction so that it can be repeated on another figure or played again and again to create more complex figures. Teachers and students can add comments, thus making scripts an ideal medium for communicating mathematically.

Geometer's Sketchpad is also the focus of a Schools Integration Project (or SIP) under the auspices of the NCTE. There are four schools involved and the project commenced in the 1999/2000 school year. The current target group is students of Junior Certificate mathematics. The synthetic geometry section of the revised syllabus is the main focus of the project. Preliminary work concentrates on constructing line segments and half-lines. To date, lesson plans have been developed in the area of Geometry theorems relating to angles, parallelograms, circles and triangles. Additional work has also been undertaken in the areas of Coordinate Geometry and Transformation Geometry. The lesson plans for all of these topics will be disseminated via Scoilnet (http://www.scoilnet.ie) at the conclusion of the project. A sample lesson plan idea on how the teaching of the *bisector of an angle and incircle construction* might be approached is provided in Geometry lesson idea 3 to give mathematics teachers an indication of the possibilities which the Geometer's Sketchpad offers. The package is available in Education Centres, from where further information can be obtained.

COMPUTER ALGEBRA SOFTWARE

Computer algebra systems, such as Maple and Mathematica, have been significantly changing the manner in which many mathematicians, scientists and engineers go about their professional work. These packages are not designed for mathematics education per se, but rather as powerful tools to aid the professional in his or her work. Nevertheless, it is useful for mathematics teachers to become familiar with such software for several reasons. First, many of the students who pass through our care in schools will actually continue their mathematical experiences at third level and beyond and will thus be exposed to this type of software. Secondly, mathematics teachers are often looking for ways to maintain their levels of enthusiasm, and this kind of software offers opportunities for them to pursue their mathematical interests in new ways. Finally, despite the fact that these packages were not designed as educational ones, they still offer significant potential for use as such. One such package which has received exposure in Irish schools recently is MathView (now renamed LiveMath).

LiveMath is the focus of one of the innovative SIP projects under the auspices of the NCTE. There are four schools involved and the project commenced in the 1999/2000 school year. Although the current target group is Transition Year students there are many possibilities for teachers and students of Junior Certificate Mathematics. LiveMath is a computer algebra package which allows students to interact with the computer and hence improve their understanding of mathematics. The package also has graphic abilities which are noteworthy. The SIP project has exploited many of these features. To date, lesson plans have been developed in each of the following topic areas: Algebra, Co-ordinate Geometry (Line and Circle), Functions and Graphs.

The lesson plans for these topics will be disseminated via Scoilnet (**http://www.scoilnet.ie**) at the conclusion of the project. A sample lesson plan idea on Algebra is provided in Algebra lesson idea 2 to give mathematics teachers an indication of what is involved. The package is available in Education Centres, from which further information can be obtained. *Mathematics and Science with MathView* is a book by a practising Irish second level mathematics teacher which describes additional ways of using LiveMath; a reference to this book is made in the Resources section. LiveMath is now web-enabled and by visiting the website (**http://www.livemath.com**) it is possible to download a plug-in and interact with lesson plans from other users.

MATHEMATICAL SYMBOLS FOR EXAMINATION PAPERS AND OTHER DOCUMENTS

One of the ways in which computers are being used by teachers of all subjects is in the production of notes and examination papers. One problem for the mathematics teacher in this regard is a perceived difficulty with mathematical type-setting. There are a number of packages which can be used by mathematics teachers to overcome this problem. These include MathType, LaTeX and Equation Editor. Equation Editor is an easy-to-use package which comes free with Microsoft Word, and which is sufficiently powerful to meet all of the needs of the mathematics teacher in regard to the production of notes, articles for journals, examination papers and solutions and so forth. It is possible over a couple of years to build up a number of examination papers using the computer and it then becomes very easy to produce examination papers subsequently by combining and making adjustments to parts of old ones.

To verify that equation editor has been loaded from the Microsoft Word disk on a PC, within Word click on Insert, select Object, and search the list for Microsoft Equation. If it does not appear as an option, you need to load it from the Microsoft Word disk supplied by your dealer.

Here are some examples of questions produced using Equation Editor.

(i) Evaluate $(4.37)^2 + \frac{1}{2.05} \times \sqrt{50.9}$ correct to two decimal places.

(ii) Simplify $\frac{5^2 \times 25^{\frac{1}{2}}}{125^{\frac{2}{3}} \times 5^3}$

(iii) Write the following as a single fraction

$$\frac{3x-4}{5} - \frac{2x+1}{3}$$

SPREADSHEETS

Spreadsheets are one of the most commonly available types of application, designed primarily for a wide range of business uses. Almost all modern computers come with at least one spreadsheet application as standard "bundled" software, so that this is one tool which is available to any teacher in a school with computers. Spreadsheets lend themselves to being used very effectively in mathematics learning, and their use forms part of the mathematics curriculum in many countries. Even within a didactic model of teaching, they can be used to free the student from tedious calculations that may stand in the way of conceptual understanding in standard numerical procedures. In a more investigative and student-centred learning environment, their power and ease of use provide a facility to tackle material which would be difficult or impossible to approach in this way without a computer. Furthermore, the use of spreadsheets provides a solid aid to the understanding of some basic algebraic concepts (such as variables and functions).

An example of how a spreadsheet can be used in the context of mathematics teaching is provided in Functions and graphs lesson idea 1.

CALCULATORS

The issue of calculators has been addressed in the context of changes in content (see section 3.3). A set of guidelines on calculators is being produced, in which a pedagogical rationale for their use is outlined. The purchase of suitable machines for schools is also discussed. It is worth noting again that the Department of Education and Science has set up a research project to monitor numeracy-related skills (with and without calculators) over the period of introduction of the revised courses.

4.15 EFFECTIVE MATHEMATICS TEACHING

The concept of an effective mathematics teacher is a problematic one. Nevertheless, research identifies critical areas of concern ranging from skill acquisition to teaching style:

- sound subject knowledge
- planning, preparation and management process skills
- provision for a variety of teaching methods ranging from exposition to practical, project and investigational work
- emphasis on problem-solving and the relevance and application value of mathematics
- the ability and willingness to be proactive, to self-evaluate and to take responsibility for one's own professional development.

One technique being employed by a number of practising teachers of mathematics to analyse their effectiveness is the completion of a teacher's log/diary. This can take place on a daily basis for short period of time. Typically, this activity takes no more than one or two minutes and involves teachers reflecting and writing down brief notes on *good* and *bad* lessons, unusual incidents and students' difficulties.

Feedback from teachers using this technique indicates that the most successful mathematics lessons are characterized by:

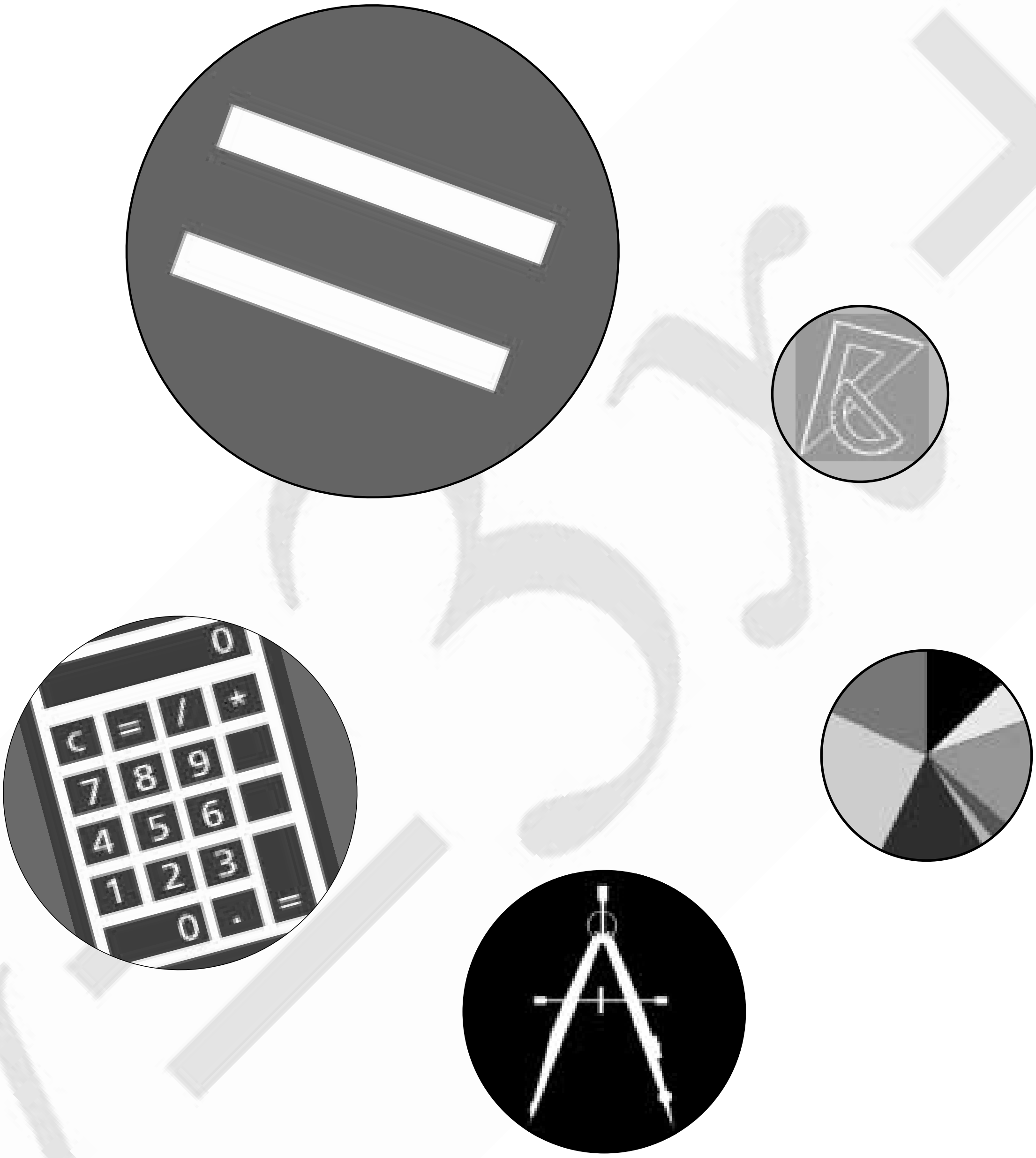
- careful planning and preparation
- good relationship with students
- positive attitudes of students
- the teacher's own enthusiasm
- variety of teaching aids
- fortunate remarks and interventions by students.

The least successful lessons were characterized by:

- inadequate preparation
- unclear explanations and interventions
- lack of motivational materials
- poor personal relationships with classes.

Other techniques being employed by teachers to look at the effectiveness of their teaching include action research and peer observation on a reciprocal basis.

Assessment



5.1 INTRODUCTION

Assessment is an integral part of teaching and learning. It is intended to "support the learning of important mathematics, and furnish useful information to both teachers and students" (NCTM, 2000). Assessment is traditionally categorised as being either *formative* or *summative*: formative assessment focuses on providing feedback which helps students to learn, while summative assessment describes the levels they have reached at the end of some section of their education. Obviously the two roles are not entirely distinct, but the former is of daily concern, whereas the latter is associated chiefly with national certification and hence (for the junior cycle) the Junior Certificate examinations. Formative assessment is discussed in Section 5.2. The remaining sections consider assessment for the Junior Certificate: choice of mode, design and marking of examination papers, and development of grade criteria.

5.2 FORMATIVE ASSESSMENT

Formative assessment is chiefly the business of the classroom teacher. Throughout the junior cycle, students need feedback to help them monitor their progress and develop their concepts and skills. They receive this feedback in a multitude of ways – perhaps most obviously in being told whether their answers to questions are right or wrong, but more importantly in being helped to understand *why*. In this way they not only correct errors and improve procedural skills, but also develop their understanding of concepts and their ability to apply them to the solution of problems. Moreover, they can be affirmed and encouraged, enabling them not only to enhance their mathematical expertise but also to grow as people.

There are many ways in which formative assessment can be carried out. International studies of mathematics education indicate that we have a strong tradition in this country with regard to some of these ways. In particular, we often give students exercises to do on their own in class, and we set frequent short homework tasks; these typically provide reinforcement of the students' procedural skills. Class tests and school examinations are other tools familiar in an Irish setting – though examinations, in particular, lose much of their formative role if teachers do not provide students with appropriately detailed feedback on their performance. Tests and examinations tend to focus chiefly on procedural skills. Our strong tradition as regards assessment of these skills may be complemented by putting more emphasis on encouraging reflection, discussion and exploration in the mathematics classroom. These are activities which help students to build concepts, identify and clarify their misconceptions, and (in however small a way) create mathematics for themselves. In terms of the *syllabus objectives*, therefore, perhaps our formative assessment has tended to emphasise the lower-order ones (objectives A and B), reflected in classes in which the students engage busily in doing routine exercises; but perhaps it has paid less heed to those of higher order (objectives C, D, E, F and H), concerned with the students communicating mathematically, connecting and extending ideas, exploring consequences and becoming actively involved in their own learning.

Students accustomed to "busywork" classrooms find it difficult to take control of their own learning. They find the transition to a reflective and analytical learning style quite hard to achieve. However, they can eventually learn more meaningfully and with greater enjoyment, and may achieve the kind of independence to which we aspire on their behalf. *The activities suggested in Section 4 of this document provide opportunities for students to develop these higher-order behaviours and for teachers to assess their development.* Many of the objectives can be assessed by observing the students as they work, by listening to their discussions among themselves, and by talking to them individually, encouraging them to communicate their insights and difficulties. These processes are time-consuming and demanding for teachers. However, they can provide crucial information, complementing that obtained from the routine correction of classwork, homework, tests and examinations. In particular, the assessment of *relational understanding* – the aspect of learning to which so much of these *Guidelines* is devoted – is greatly enhanced by the use of such processes.

It is appropriate here to mention *diagnostic assessment:* assessment which aims to identify specific areas of difficulty (and strength) for a given student. Teachers carry this out informally when they use their professional expertise to identify areas in which individual students are having problems with their mathematics. More formal approaches to diagnostic assessment are outside the scope of these *Guidelines.* However, it would be inappropriate to ignore the contribution that such assessment can make to identifying and catering for the needs of individual students.

5.3 ASSESSMENT FOR CERTIFICATION: SCOPE AND CONSTRAINTS

The point was made in Section 2.4 that *summative* assessment in Mathematics – assessment for the Junior Certificate – at present takes place solely by means of a terminal examination. Forms of summative assessment other than terminal examinations are unfamiliar in mathematics education in Ireland.

Unfortunately, not all skills are easily assessed by terminal examinations. The syllabus points to the consequences of this.

> Written examination at the end of the Junior Cycle can test the following objectives (see section 1.3 [of the syllabus document]): objectives A to D, G and H, dealing respectively with recall, instrumental understanding, relational understanding and application, together with the appropriate psychomotor (physical) and communication skills.

As pointed out in Section 2.4 of these *Guidelines*, this limits the assessment objectives to those specified in the syllabus (A, B, C, D, G and H). In the context of a general review of assessment in the junior cycle, the list of assessment objectives might be extended. *Such a move might necessitate a substantial revision of the rest of this section of the Guidelines.*

The Junior Certificate mathematics syllabus highlights the following points with regard to summative assessment.

> Assessment … is based on the following general principles:
>
> - candidates should be able to demonstrate what they know rather than what they do not know;
> - examinations should build candidates' confidence in their ability to do mathematics;
> - full coverage of both knowledge and skills should be encouraged.

If examinations are to satisfy the first two of these principles, they must allow students to show the conceptual knowledge and procedural mastery that they have developed. This can be done by asking students to answer questions many of which should be well within their scope. The third principle points to the importance of spanning all major content areas and as many types of skill as can validly be assessed under examination conditions (bearing in mind the age of the candidates and their inexperience in dealing with major examinations).

NOTE

Given the exclusion of some objectives from the summative assessment process, it is all the more important to ensure that these objectives are addressed during the students' mathematical education. The revised Primary School Curriculum gives a higher priority to these objectives than does its predecessor. Ongoing development of Junior Certificate mathematics might address ways in which they can receive some coverage even in formal examinations; there are models in other countries' assessment systems and in recent international studies (for example, the OECD Programme for International Student Assessment (PISA)). The limited brief given for the current revision prevented their inclusion at this stage. It may be noted here that Transition Year provides opportunities for students to broaden their range of learning styles in mathematics, and to bring multiple intelligences to bear on the subject. This can provide them with a stronger platform for moving on to the Leaving Certificate, as well as developing abilities greatly valued in the workplace: applying their work in real-life contexts, solving non-standard problems, and communicating their findings clearly and succinctly when required.

5.4 SPECIFICATIONS FOR THE DESIGN OF THE JUNIOR CERTIFICATE EXAMINATIONS

The formal specifications for the Junior Certificate examinations involve describing the number of examination papers and the time allocated to each, the format of each paper, the distribution of questions per topic on each paper and the marks allocated to individual questions, and finally the structure of each question. The specifications proposed by the NCCA Course Committee for Junior Certificate mathematics were reflected in the proposed sample assessment materials circulated in Autumn 2000. However, in the light of developments in assessment at junior cycle, some of these proposals may not be implemented. The sample examination papers, issued by the Department of Education and Science, will indicate the number of papers at each level, the time allocation per paper, and a typical distribution of questions for each topic. Comments can be made here about the format of the papers and the design of individual questions.

PROPOSED FORMAT OF THE PAPERS

It is proposed that the examination papers for at least some levels will be in *booklet form.* Instead of reading the questions from an examination paper and doing their work in a separate booklet, candidates will write their answers in the booklet that contains the questions. The booklet will be handed in at the end of the examination.

Where a booklet is used, each part of each question will be followed by a *solution box* designed to accommodate candidates' answers, along with supporting work where relevant. A special symbol (✍) will appear in solution boxes where work to support answers *must* be shown in order to earn full marks. When this symbol is absent, supporting work can be included at the discretion of the student. (Examples are given in Section 5.6 below.)

The change in format is intended to serve a number of ends. First, it will give a new, and hopefully more student-friendly, look to the examination papers. This is in keeping with the intended emphasis on making the subject more enjoyable for the student. Secondly, students may find it easier to complete questions when they do not have to transfer attention from examination paper to script and back again at frequent intervals. They can also be helped to work through individual question parts, where appropriate, by the inclusion of "prompts". Less able students, in particular, may find it easier to keep working in the more structured setting that the booklets will provide. Thirdly, the requirement that students show their working in at least some cases can support the drive for *learning with understanding* and can test *communication skills.* It should be noted that the introduction of booklet-style examination papers in other subject areas in the Junior Certificate has been deemed very worthwhile from the student's point of view.

DESIGN OF INDIVIDUAL QUESTIONS

Questions will typically have the three-part structure – "part (a)", "part (b)", and "part (c)" – currently used for the Leaving Certificate. Again as for the Leaving Certificate, questions will display a "gradient of difficulty", leading students through easier work to more difficult tasks.

With regard to the objectives, typically:

- part (a) of the question will test recall (objective A), very simple manipulation (objective B) or basic relational understanding (objective C);
- part (b) will test the choice and execution of routine procedures or constructions (objective B), or various aspects of relational understanding (objective C);
- part (c) will test application (objective D).

The characteristics of the various "parts" are considered in more detail below, in the context of indicating grade criteria at each level of the examination. It should be noted that this proposed format is a change for the Foundation level.

NOTE

For the remainder of this section of the Guidelines it is taken as a working assumption that the Ordinary level and Foundation level papers will be in booklet form, but that the Higher level papers will be of traditional format (with the students presenting their work on examination scripts as heretofore).

5.5 THE MARKING SCHEME

The marking scheme is crucial in implementing the examination design and in ensuring that the assessment objectives are appropriately tested. In particular, to reflect the aims and objectives of the revised syllabus, objectives C and H – those concerned with *relational understanding* and *communication* – need to be given greater emphasis than in the past. In order to make such demands reasonable, the marking scheme must be appropriately transparent and familiar to teachers (and indeed students), and the terminology used on the examination papers must be well understood.

In this section, therefore, relevant features of the marking scheme are presented and the meanings to be ascribed to frequently-used terms and expressions are set out. The following section relates the marking scheme and the assessment objectives to the development of grade criteria.

AWARD OF MARKS

Typically, but not inevitably, the marks for the three parts of the question will be in the ratio 1:2:2. Thus, for a fifty-mark question, there will be ten marks for a part (a), twenty for a part (b), and twenty for a part (c).

Marking schemes are designed so as to give maximum reward to candidates whose answering meets the requirements of the questions asked. This includes adhering to instructions that are embedded in questions. For example, the use of a protractor may not be allowed, answers may be required in a certain form or an earlier result may have to be used to perform a task. For the foreseeable future, it is envisaged that candidates' work will be marked in the traditional way: that is, with marks usually being deducted for "slips" (minor errors) and "blunders" (major errors) but awarded for "attempts" where relevant, though some questions are scored on a "hit or miss" basis. Procedural mistakes tend to be categorised as slips and conceptual ones as blunders, but no general rule can be given, as the distinction is heavily dependent on the context. Details are specified in published marking schemes for individual Junior Certificate papers; schemes from past examinations are available on the Department of Education and Science website (http://www.irlgov.ie/educ).

Where a booklet format is used, the proposed provision of solution boxes will allow the examination paper to offer candidates considerable guidance as to what type of answer is required. In particular, the presence of the ✍ symbol is a clear indication that supporting work is essential and that a correct answer on its own will not be awarded full marks. Similarly, the absence of the ✍ symbol in a solution box will reassure candidates that they will not be penalised for presenting a correct answer without work. In general, however, incorrect answers standing alone are worthless. It is therefore in candidates' interest to show their working where possible; this enables examiners to identify slips and blunders, deduct only such marks as should be lost, and award the remainder. Thus, work leading to answers should be clearly presented wherever it is practical to do so, even in the absence of the ✍ symbol.

Also in general, all correct mathematical solutions are accepted for full marks. For example, an Ordinary level candidate may solve a quadratic equation using the formula even though this is not in the Ordinary level syllabus. However, there is one exception to this rule. With regard to the synthetic geometry at Higher level, it should be noted that *what constitutes a correct mathematical proof is dependent upon the context within which it is presented, in this case the particular system of geometry specified in the syllabus.* It must be emphasised again (see Section 3.3) that proofs using transformations are not acceptable. In proving any theorem, candidates must build their arguments upon the theorems and facts that precede the one concerned, as set out in the syllabus. Further details are supplied in Appendix 2.

TERMINOLOGY

There are some terms and expressions that occur frequently in examination questions and that carry particular meanings and expectations during the marking process. The principal ones are briefly explained with a view to helping students in their answering and to facilitating understanding of published marking schemes.

- *"Construct ..."*

  "Construct" means to draw according to specific requirements, usually with instruments such as ruler, compass and protractor. Accurate measurements are required and construction lines such as arcs should be shown clearly. Free-hand drawings are not acceptable. The marking of constructions involves measuring of the candidates' work by examiners. For each measurement a small tolerance is allowed without penalty.

- *"Draw the graph ... / Graph ..."*

Graphs are likely to be required in questions on co-ordinate geometry, statistics and functions. Where relevant, they should be presented in solution boxes, in the spaces in which gridlines are provided – or, if necessary, on separate sheets of graph paper. They should be distinctly drawn and sufficiently large to ensure clarity. Axes should be perpendicular and clearly labelled. Appropriate scales should be chosen and indicated. Graduations should be marked clearly on both axes. Plotted points should be accurately positioned and identified. Where the points are to be joined this should be done in the appropriate manner; for example, a smooth curve is necessary for a quadratic function whereas line segments are required for a trend graph.

It should be noted that an ogive (cumulative frequency curve) is a smooth curve. In examination answers it should always start on the X-axis, since its initial point will represent the value below which it is known there are no data.

- *"Estimate .../Show how to calculate an approximate value of ..."*

Questions involving estimation or approximation require candidates to use their own judgement regarding the level of rounding that is appropriate in order to lead to a value close enough to the exact calculation to be useful. In a solution box, the layout of solutions may be prompted by the provision of appropriate blank spaces, and candidates should be alert to this source of guidance. Examiners will focus on the depth of understanding of the approximation process displayed by candidates as well as on their ability to perform the mechanical steps. This means that flexibility will be exercised in the marking of the numerical results presented and that usually a specific estimate will not be required for full marks. For example, two acceptable estimates of

$$\frac{36.2}{12.4}-\frac{24.9}{50.4}$$

could be $$\frac{40}{15}-\frac{25}{50}\approx 2.7-0.5=2.2$$

or $$\frac{36}{12}-\frac{25}{50}=3-0.5=2.5$$

as either approach is clearly a sound guide to arriving at the actual value (2.425307…).

- *"Give your answer in the form ..."*

For full credit, candidates must adhere closely to instructions of this type. For example, an answer of 1.87 will be penalised if the question requests a result to one place of decimals. In the same way, 25 will not suffice if candidates are told to give the answer in the form 5^n.

- *"Hence…"*

The word "hence" is generally used to connect two tasks which the candidate is expected to perform, one after the other, with the outcome of the first helping the second. It points candidates to the method or approach which examiners expect. For example, consider:

" ... show that $y=\dfrac{p-2q^2}{q}$

Hence, evaluate y when p = 30 and q = 3."

It is important to note that when "hence" is used is this way, candidates may be penalised if the first result is not used in order to perform the second task.

More commonly, the phrase "hence, or otherwise" is used. This indicates that any approach of the candidate's choosing can be taken to the second task. However, a helpful lead-in is always provided by the first part, and candidates usually fare better if they follow this rather than make a fresh start at the second part.

- "Prove ..."

In accordance with the syllabus, the idea of proof will be addressed only in Higher level questions. Candidates may be required to prove the theorems marked with asterisks in the syllabus as well as "cuts" arising from the results (theorems and "facts").

All the steps in proofs must be written down in logical order. Each assertion in the proof should be accompanied by a reason. Further details are given in Appendix 2.

Proofs should be accompanied by diagrams wherever they serve to clarify the argument being presented. However, it should be noted that information marked on diagrams will *not be* accepted as a substitute for written steps.

- *"Show ..."*

  Normally, when students are asked to show a result, any correct mathematical method is awarded full marks assuming that it is properly applied. One exception is that measurement from diagrams using a ruler, protractor, or other instrument is not accepted unless this approach is specifically requested. In cases where a particular method is required, the question will give clear directions – for example, "Show, by calculation, that $|ab| = |bc|$" – and these directions must be followed.

- *"Sketch …"*

  When a sketch is required, diagrams are not expected to conform to specific measurements. Examiners will be assessing candidates' intuitive feel for the task at hand. For example, in sketching images of shapes under transformations, examiners will be looking for evidence that the shape has the correct orientation and is in roughly the correct location.

- *"Use your graph to show that ..."*

  To earn full marks, candidates must display evidence that they have extracted their answers from their graphs. It is not acceptable to use other methods of arriving at the required results even if the alternative methods are mathematically correct and accurately applied. For example, candidates who successfully solve the equation $2x^2 - 3x - 5 = 0$ using the quadratic formula cannot be awarded marks if the instruction given is "Use your graph to estimate the roots of the equation $2x^2 - 3x - 5 = 0$".

- *"Verify that ..."*

  Verifying a solution of an equation in algebra involves substituting the value into the given equation and showing that the result is a true statement. It is important to note that solving the equation is *not* acceptable if verification is sought.

5.6 QUESTIONS, OBJECTIVES AND STANDARDS: THE ROUTE TO GRADE CRITERIA

Knowledge and skills displayed by the students can be related to standards of achievement, as reflected in the different grades awarded for the Junior Certificate examinations. The three-part design of questions (with the typical relationship to objectives described at the end of Section 5.4 above), taken together with the marking scheme which allocates marks approximately in the ratio 1:2:2 to the three parts, is intended to operationalise the *grade criteria.* Thus:

- recall alone (accounting for about twenty per cent of the marks) should not be enough to enable a candidate to achieve a D grade;
- recall together with some instrumental and relational understanding – execution of familiar and well-learnt techniques, and grasp of concepts – together with the ability to communicate the results, should be necessary for a D grade;
- recall together with good instrumental and relational understanding – diligent and accurate execution of familiar and well-learnt techniques, and grasp of concepts – together with the ability to communicate the results (accounting jointly for some sixty per cent of the marks), should be required for a C grade;
- the ability to apply, or to execute more difficult examples of familiar exercises, or to demonstrate understanding at the upper level of the syllabus, is needed for a higher grade;
- evidence of abstraction and/or better application, with good communication, is needed for a top grade.

It remains to give fuller descriptions of the characteristics of the three question parts, to provide examples of questions suitable for each part at each level, and to indicate key aspects of the solutions required to obtain full marks. *It is important to note that the solutions given here are sample solutions, provided in order to illustrate the type of detail and level of communication expected. They are not the only (or necessarily the best!) possibilities.* In view of the assumption that the Higher level papers will be of traditional format, as indicated in Section 5.4, it should be noted that the "boxes" shown for the Higher level examples are not solution boxes; they represent portions of the student's examination script. *It should be stressed that students are encouraged to show detailed working in order to obtain maximum marks.*

The examples also serve to indicate the style and depth of coverage expected in some areas of the syllabus. Moreover, they indicate ways in which the candidates' relational understanding and communication skills can be demonstrated and rewarded.

CHARACTERISTICS OF A "PART (a)"

A "part (a)" is intended to have the following characteristics.

- It should allow candidates to demonstrate what they know, and also permit them to "limber up" for the remainder of the question. As indicated above, questions typically test facts, very straightforward skills, or basic understanding.
- It should be presented as straightforwardly as possible, so that the required mathematics is tested directly. For example, candidates with poor reading skills should not be handicapped.
- Ideally, *all* (credible) candidates should get "part (a)" right – except for the slips that can strike even the best candidates in examinations.

EXAMPLES OF "PART (a)" QUESTIONS

"a" 1: Foundation level

Find the value of 3(x+y) when x = 2 and y = 1.

3(3) = 9

[Note: although no work is required, it is often natural and advisable that candidates show some.]

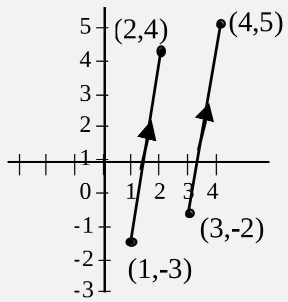
"a" 2: Ordinary level

Find the image of the point (3, -2) under the translation (1, -3) → (2, 4).

(1, -3) → (2, 4) *or*

add 1 to *x*

add 7 to *y*



So, (3, -2) → (4, 5)

"a" 3: Ordinary level

Find the values of x for which $5+2x \leq 13, \quad x \in N$.

$$2x \leq 13-5$$
$$2x \leq 8$$
$$x \leq 4$$
x could be $0, 1, 2, 3$ or 4

"a" 4: Ordinary level

VAT at 21% is added to a bill of €130. Calculate the total bill.

€130 × 21% = €27.30

€130 + €27.30 = €157.30

or

€130 × 1.21 = €157.30

"a" 5: Ordinary level

Divide 1506 by 0.6 and express your answer in the form $a \times 10^n$, where $1 \leq a < 10$ and $n \in \mathbf{Z}$.

$$\frac{1506}{0.6} = 2510$$

$$2510 = 2.51 \times 10^3$$

"a" 6: Ordinary level

Given that tan$A$ = 3.3544 find the value of A to the nearest degree, where $A < 90°$.

A = 73.399844…

A = 73°

"a" 7: Higher level

Given $y = ax + a^3$ and $x = 3 - 2a^2$

(i) express y in terms of a and simplify the result

(ii) evaluate y when a = 2.

(i) $y = a(3-2a^2) + a^3$
$= 3a - 2a^3 + a^3$
$= 3a - a^3$

(ii) When a = 2, y = 6 – 8 = -2

"a" 8: Higher level

Simplify $(2+\sqrt{7})(3-\sqrt{7})$.

$(2+\sqrt{7})(3-\sqrt{7})$
$= 2(3-\sqrt{7}) + \sqrt{7}(3-\sqrt{7})$
$= 6 - 2\sqrt{7} + 3\sqrt{7} - \sqrt{49}$
$= 6 + \sqrt{7} - 7$
$= -1 + \sqrt{7}$

CHARACTERISTICS OF A "PART (b)"

A "part (b)" is intended to have the following characteristics.

- It should allow the candidates to demonstrate their ability to execute diligently practised procedures, or to display non-trivial understanding (for example, interpreting a mathematical statement, reading and using information from graphs, recognising the solution of an equation, or – for Higher level candidates – writing out a proof).
- In some cases the "part (b)" may be divided into sub-parts, containing (say) slightly easier and slightly harder "sums" in the required area.
- The candidate who *just* deserves a D grade on a particular question should be able to get around half of the marks available for this part of the question. For example, in the case of a "part (b)" with two sub-parts, the candidate may get the first sub-part correct except for slips, and may earn an attempt mark on the second sub-part.
- Candidates deserving a C grade for a particular question should be able to get the "part (b)" right (except perhaps for slips).

It should be noted that, although "part (b)" questions are not intended to test the higher-order skills associated with *problem-solving*, they may be formulated as simple "word problems". Thus, in accordance with the thrust of the revised syllabus, information may be presented in a *context.* The skills of identifying and interpreting the relevant information, and then processing it, should involve relational and instrumental understanding rather than genuine problem-solving. A hallmark of the latter is that the method of solution should not be immediately apparent. For "part (b)" questions, however, candidates – having duly understood the principles involved in such work and practised many examples – should be able to see at once how they should tackle the question.

EXAMPLES OF "PART (b)" QUESTIONS

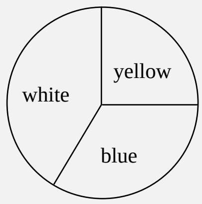
"b" 1: Foundation level

60 students were asked to choose a colour to paint the school hall. 15 said yellow, 20 said blue and the rest said white.

(i) How many said white?

(ii) Draw a pie chart to show this information.

(i) Number saying white = 60 -15 - 20
= 60 - 35
= 25



(ii) 60 people "share" 360°
1 person gets 6°
15 people get 90° …… yellow
20 people get 120° ……blue
25 people get 150° ……white

"b" 2: Ordinary level

Given that cos*A* = 0.5, find the value of sin*A* and the value of tan*A*. Give your answers correct to two places of decimals.

A = 60°
sin*A* = 0.866025 …..
tan*A* = 1.723205 ….
sin*A* = 0.87
tan*A* = 1.72

"b" 3: Ordinary level

Síle is six years older than Seán. The sum of their ages is 30 years.

(i) Letting x = Seán's age write down an equation in x to represent this information.

(ii) Hence, find Síle's age.

(i) Seán's age = x
Síle's age = Seán's age plus 6 = $x + 6$

$x + x + 6 = 30$

(ii) $2x + 6 = 30$
$2x = 30 - 6$
$2x = 24$
$x = 12$

Síle's age = 12 + 6 = 18 years.

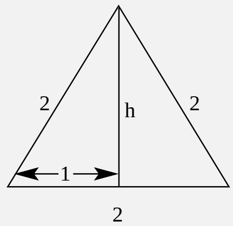
"b" 4: Ordinary level

(i) Sketch an equilateral triangle with sides of length 2 units.

(ii) Calculate the perpendicular height in surd form.

(iii) Hence, find sin60°.

(i) Sketch:



(ii) Calculate perpendicular height:

$h^2 + 1^2 = 2^2$

$h^2 = 4 - 1 = 3$

$h = \sqrt{3}$

(iii)

$\sin 60° = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{3}}{2}$

"b" 5: Higher level

It takes 4 hours and 20 minutes to travel a journey at an average speed of 120 km/hr. How many hours and minutes will it take to travel the same journey if the average speed is reduced to 100 km/hr?

Distance = speed × time
= 120 km/hr × 4 hr 20 mins

$= 120 \times 4\frac{1}{3}$ km

$= 120 \times \frac{13}{3}$ km

$= 40 \times 13$ km

= 520 km

Time $= \frac{\text{Distance}}{\text{Speed}}$

$= \frac{520 \text{ km}}{100 \text{ km/hr}}$

= 5.2 hr
= 5 hr 12 mins (since 1hr = 60 mins, 0.1 hour = 6 mins, 0.2 hr = 12 mins).

"b" 6: Higher level

By putting the smallest number first, place the following numbers in order:

$$\sqrt{0.25},\quad \frac{1}{\sqrt{2}},\quad 0.3,\quad \frac{\pi}{2},\quad \frac{\sqrt{3}}{2}$$

$$\sqrt{0.25} = 0.5$$

$$\frac{1}{\sqrt{2}} = 0.707106....$$

$$\frac{\pi}{2} = 1.570796....$$

$$\frac{\sqrt{3}}{2} = \frac{1.7320...}{2} = 0.86602....$$

Approximate values of numbers are: 0.5, 0.7, 0.3, 1.6, 0.9

When ordered these become: 0.3, 0.5, 0.7, 0.9, 1.6

Answer: $0.3,\quad \sqrt{0.25},\quad \frac{1}{\sqrt{2}},\quad \frac{\sqrt{3}}{2},\quad \frac{\pi}{2}$

"b" 7: Higher level

(i) Write the following as a single fraction:

$$\frac{3}{x+2} + \frac{6}{x-4}, \qquad x \neq -2, \quad x \neq 4.$$

(ii) Evaluate your answer when $x = 2$.

(i) $$\frac{3}{x+2} + \frac{6}{x-4}$$

$$= \frac{3(x-4) + 6(x+2)}{(x+2)(x-4)}$$

$$= \frac{3x - 12 + 6x + 12}{(x+2)(x-4)}$$

$$= \frac{9x}{(x+2)(x-4)}$$

(ii) When $x = 2$ the result is:

$$\frac{9(2)}{(2+2)(2-4)}$$

$$= \frac{18}{(4)(-2)}$$

$$= \frac{18}{-8} \qquad = -\frac{9}{4}$$

CHARACTERISTICS OF A "PART (c)"

A "part (c)" is intended to have the following characteristics.

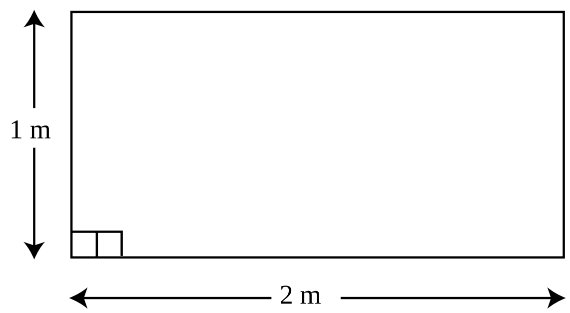
- It should provide a challenge, so that good candidates (relative to the level they are taking) can demonstrate their ability to apply their mathematics, solve problems, and so forth, and to cope with the subject-matter that is at the highest conceptual level on the relevant syllabus.
- To obtain a safe C grade on a given question, candidates should be able to attempt the "part (c)" (or at least the first sub-part of the "part (c)", if it is divided into sub-parts) for most of the questions on the paper. However, full marks on a "part (c)" may be gained in general only by very competent candidates.

Obviously, "word problems" presented in a "part (c)" will be more complex than those appearing as a "part (b)". Typically they involve application (objective D). Thus, information presented verbally may have to be translated (non-trivially) into mathematical form, and suitable approaches and techniques chosen. However, in examination conditions, it would not be fair to present information in very unfamiliar guise, or in highly complex fashion. Moreover, the solution must be obtainable at the end of a few minutes' work. The type of problem-solving involved is therefore rather limited.

EXAMPLES OF "PART (c)" QUESTIONS

"c" 1: Foundation level

A rectangular space on a bathroom wall measures 1 m by 2 m. It is to be covered with square tiles, each of which measures 10 cm by 10 cm. How many tiles will be needed?



Lower edge measures 2 m or 200 cm

Number of tiles needed along lower edge is $\frac{200}{10} = 20$

Left edge of space is 1 m or 100 cm

Number of tiles needed along left edge is $\frac{100}{10} = 10$

Total number needed to fill rectangular area is 20×10 tiles

Answer: 200 tiles

[Candidates may support this work with a sketch of the rectangle.]

"c" 2: Ordinary level

A solid sphere made of lead has radius 6 cm.

(i) Calculate its volume in terms of π.

This sphere is melted down and all of the lead is used to make smaller solid spheres each of radius 3 cm.

(ii) How many of these spheres are made?

(i) Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3$

$= \frac{4}{3}\pi \times 6 \times 6 \times 6$

$= 4\pi \times 2 \times 36$

$= 288\pi \text{ cm}^3$

[Note: Penalty if value of π used.]

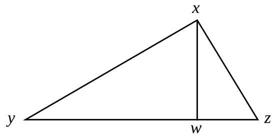
(ii) Volume of smaller sphere $= \frac{4}{3}\pi(3)^3$

$= \frac{4}{3}\pi \times 3 \times 3 \times 3$

$= 36\pi \text{ cm}^3$

Number of smaller spheres $= \frac{288\pi}{36\pi} = 8$

"c" 3: Higher level



(i) State the theorem of Pythagoras

(ii) In the triangle *xyz*, $xw \perp yz$

Prove that $|xy|^2 + |wz|^2 = |yw|^2 + |xz|^2$.

(i) In a right angled triangle the square built on the hypotenuse has the same area as the areas of the two squares built on the other two sides when they are added together.

(ii) The triangle *xwy* is right angled.
So, by Pythagoras: $|xy|^2 = |xw|^2 + |wy|^2$ *

The triangle *xzw* is also right angled.

So, by Pythagoras: $|xz|^2 = |xw|^2 + |wz|^2$

This is the same as $|wz|^2 + |xw|^2 = |xz|^2$

Re-arranging gives $|wz|^2 = |xz|^2 - |xw|^2$ *

Adding the two results marked with * gives:

$$|xy|^2 + |wz|^2 = |xw|^2 + |wy|^2 + |xz|^2 - |xw|^2 = |yw|^2 + |xz|^2$$

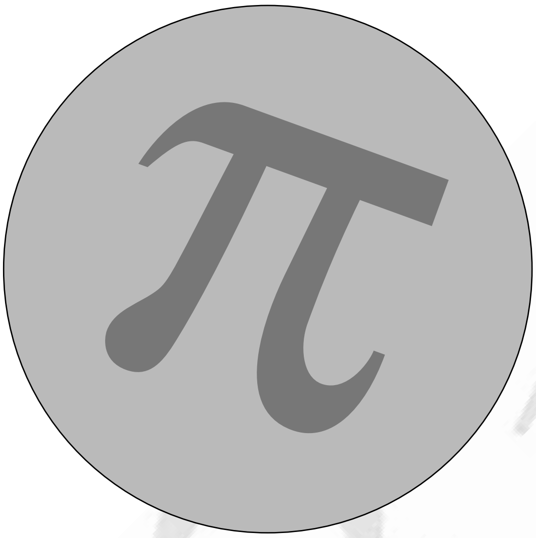
…. the required result.

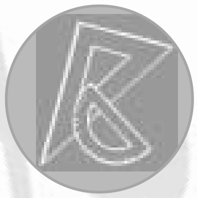
NOTE

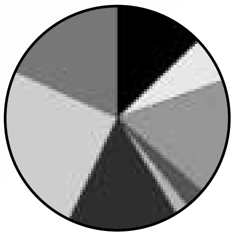
Use in certificate examinations of the three-part questions, with the different parts aiming to test different objectives and with the question as a whole displaying an appropriate gradient of difficulty, has been a positive development in mathematics education in Ireland. If students are sitting for a paper at a level appropriate to them, they can be confident that they will be able to tackle the earlier parts of the questions and so earn their reward for good understanding and diligent work.

However, there is a danger that some students may focus unduly on the first two parts of the questions, targeting a middle grade in the examinations. While this may sometimes be appropriate, or inevitable, in examination conditions, it would be unfortunate if the skills associated with a "part (c)" were to be regarded as a "bolt-on extra" in the teaching and learning of mathematics. Just as for the currently non-examinable objectives (see Section 5.3), objectives such as *application* and appropriate *problem-solving* should be addressed by all students in their mathematical education.

Appendices







APPENDIX 1
CORRECTIONS, CLARIFICATIONS AND CHANGES

INTRODUCTION

This Appendix serves three purposes. First, it points out four *misprints* in the syllabus document. Secondly, it aims to *clarify* the status (included or excluded) of some topics or techniques. Thirdly, it summarises the *changes* from the 1987 syllabus and indicates the status of some topics about which there have been questions.

CORRECTIONS

The following *misprints* occur in the syllabus document:

1. On p. 9, in *"Number systems" paragraph 4,* in the list of rules for indices: $(a^p)a^q$

   should read: $(a^p)^q$

2. On p.11, in *"Algebra" paragraph 5,* the equation

$$\frac{a}{bx+c} \pm \cdots \pm \frac{p}{qx+r} = \frac{d}{e}$$

   should read $$\frac{a}{bx+c} \pm \frac{p}{qx+r} = \frac{d}{e}$$

   that is, manipulation is restricted to *two* terms of the given form.

3. On p. 21, in *"Applied arithmetic and measure "paragraph 2,*

   time(s)

   should read time (s)

4. On p. 21, in *"Applied arithmetic and measure" paragraph 3,* the final "note" in the right-hand column is badly positioned; it should be opposite the entry "Application to problems."

CLARIFICATIONS

No syllabus document can specify the exact scope of the content or depth of treatment intended (and liable to be tested in state examinations). For a syllabus that has undergone only minor revisions, the nuances of the previous syllabus are deemed to guide the interpretation of the present one except where otherwise stated. The intended changes are flagged in the table in the following section of this appendix. This table also aims to clarify the status of some topics which were not mentioned explicitly in the 1987 syllabus but were deemed to be included, and of other topics which were dropped from the syllabus when it was revised in 1987 but have continued to be taught to at least some students.

- An example of a topic not actually mentioned in the 1987 syllabus – or in its predecessor of 1973 – but deemed to be part of the intended syllabus throughout the period is *perimeter.* Perimeter is mentioned explicitly in the present syllabus, but this does not constitute a change of content.

- An example of an item excluded from the syllabus in 1987, but apparently still used in a number of classrooms, is the notation N_0 for the set of natural numbers excluding 0. The notation was used in the 1973 syllabus but was removed from the syllabus in 1987. *It is worth emphasising here, since international practice varies, that the set **N** includes 0.*

- Of course, the fact that a topic is excluded from the syllabus does not mean that it cannot be taught in the classroom. It does indicate, however, that knowledge of the topic is not necessary for answering questions in the Junior Certificate examination.

While the inclusion or exclusion of *topics* can be tabulated, the intended *depth of coverage* is not easily captured in a succinct tabular summary. The set of proposed sample assessment materials, drawn up and circulated to schools by the Department of Education and Science, provides one indicator of the required depth for various topics. Naturally, however, these only *sample* the topics and techniques which might be examined. Further instances are provided in Section 5.6, where some typical "question parts" are presented.

Additional guidance is offered with regard to the *limits* in the depth of coverage in a number of cases. The following examples are outside the scope of the syllabus.

1. The expression of repeating decimals as fractions.

2. The division of compound surds.

3. The formation of a quadratic equation from given roots.

4. The solution of two equations one of which is linear and the other of which is quadratic, such as

$$2x - y - 1 = 0$$
$$xy = 6$$

5. The solution of an equation of form such as

$$2(x+3) = \frac{1}{x+1} \quad \text{or} \quad \frac{4}{x-1} - \frac{3}{x} = \frac{5}{x+2}$$

6. The difference of two squares where "squared" terms are written as squares (or higher powers), as in

   $a^4 - b^4$ or $x^4 - 81$

7. Calculation of the volume of, say, a hut with a pitched roof or a bar of chocolate with triangular cross-section. (The syllabus limits problems on this topic to objects with rectangular cross-section.)

8. Uses of linear and quadratic graphs that require manipulation of the algebraic expressions in addition to graph-reading skills. For example, determine from a graph of a function f(*x*) the values of *x* for which f(*x*) > 6.

9. Solution of inequalities with non-integer coefficients, such as

   $\frac{x}{3} + 1 \leq 4$.

INS AND OUTS: SUMMARY OF SIGNIFICANT CHANGES AND CLARIFICATIONS

Material excluded shown in Roman script; *material included shown in Italic script.*

Course	Higher	Ordinary	Foundation
Sets	Symmetric difference removed Closure removed Cartesian product removed		Examples with three sets excluded
Number systems	Notation N_0 excluded *Prime factorisation included* *HCF included* *Estimation included* *Rounding for any number of decimal places included; significant figures included but restricted to integers* Use of Tables removed Logarithms removed Division applied to $a \pm \sqrt{b}$ **removed**	Notation N_0 excluded *Prime factorisation included* *HCF included* *Estimation included* *Rounding for any number of decimal places included;* significant figures excluded Index notation for square root included Use of Tables removed Scientific notation: $n \in$ **Z\N** **removed**	*Order for N included* Notation N_0 excluded Nesting of brackets excluded *Estimation included* General division of fractions removed Fraction-decimal conversion with calculator expanded and division of decimals included Use of Tables removed
Applied arithmetic and measure	Rates removed *Percentage profit of CP or SP (as specified)* Simple Interest excluded (Sub)multiples limited to specified list *Perimeter included* *Surface area of rectangular solids included*	Rates removed *Percentage profit of CP or SP (as specified)* Simple Interest excluded (Sub)multiples limited to specified list *Perimeter included* *Surface area of rectangular solids included* π not necessarily accepted as 3.14 or 22/7	*Percentage profit of CP or SP (as specified)* (Sub)multiples limited to specified list *Percentage increase included* *"Use of scales"* replaces "drawing to scale *Perimeter included* *Area of square and rectangle included* π not necessarily accepted as 3.14 or 22/7
Algebra	Sum and difference of cubes removed *Rational expressions with numerical denominator included* *Simple rational expressions with variable in denominator included*	Division of expressions removed Rearrangement of formulae removed *Rational expressions with numerical denominator included* but those with variable in denominator removed Factorisation of quadratics restricted to those with coefficient of x^2 unity Difference of two squares restricted to $x^2 - y^2$ Coefficients and solutions for simultaneous equations restricted to Z Formula for solution of quadratics excluded	Coefficients and values of *x* in expressions restricted to N *Elementary simplification included* *Extra example of equation 4(x – 1) = 12 included*

Course	Higher	Ordinary	Foundation
TOPIC			
Statistics (For Foundation course: Statistics and data handling)	*Collecting, recording and tabulating data included*	*Collecting, recording and tabulating data included*	*Collecting, recording and tabulating data included* *Pictogram included* Pie chart restricted to angles multiples of 30° and 45° ***Additional material** on tables of data and on relationships between these and graphs; see Section 4.8*
Geometry			
Synthetic geometry:	Different treatment, with fewer proofs to be examined; see Section 3. 3 and Appendix 2	Different treatment; see Section 3.3 and Appendix 2	Different presentation and treatment; see Section 3.3 and Appendix 2
Transformation geometry:	Intuitive approach	Intuitive approach	Translation removed; central symmetry included; constructing non-rectilinear figures excluded
Co-ordinate geometry:	Parallel projection and equation of image of line under translation removed Line formula $ax + by + c = 0$ *included* Area of triangle removed	Diagrams restricted to same scale on each axis *Coordinates of image points for specified simple examples included* Format $y = mx + c$ removed, but intersection with axes retained (using algebraic methods)	
Trigonometry	Angles outside range 0° – 360° excluded *Use of triangles for surd form of ratios for specified angles included* Use of Tables removed Proof of sine rule and area formulae excluded	Trig. functions of angles greater than 90° excluded Minutes removed Use of Tables removed Compass directions removed	
Functions and graphs (For Foundation course: Relations, functions and graphs)	Relations and arrow diagrams removed Composition and inverse of functions removed	*Codomain included* Relations and arrow diagrams removed Graphing inequalities of forms such as $a < x < b$ removed	*For plotting points, non-integral coordinates included*

****Use of calculators included; use permitted in Junior Certificate examinations****

APPENDIX 2
NOTES ON GEOMETRY

CONTEXT

The purpose of this appendix is to outline the logical structure of the geometry section of the course, in order to provide background information for teachers and to clarify what will be expected of Higher Level students in the examination, vis-à-vis proofs of theorems.

Difficulties have been experienced in the past, owing to the fact that the geometry on the 1987 syllabus consists of a mixture of a transformation-based approach with a traditional one based on congruence. The revised syllabus eliminates this dual approach from the formal treatment of geometry. Transformations are removed from the formal treatment, so that the system adopted is a congruence-based one. It is largely built upon the same ideas as those used by Euclid, but supplemented, as is common in modern treatments, by the use of measure (length/distance, angle measure, and area).

It should be noted that the geometry section of the syllabus is the vehicle by which students are first introduced the ideas of formal deductive reasoning. It is therefore important that by the time they finish the course, they will have gained an appreciation of the manner in which results are built up in a coherent and logical way within a formal system. It is hoped that the revised version of the syllabus facilitates this to a greater extent than its predecessor.

Many people rightly express enormous appreciation for the monument that Euclid left to humankind in producing *The Elements*. That it was one of the greatest triumphs in the development of mathematical thinking, however, does not necessarily imply that it is the most suitable course for second level students to follow. By starting with a sparse set of assumed results (axioms), Euclid had a long task ahead in building up logically to the geometric results familiar to us all. Pythagoras' Theorem, for example, is Proposition 47 in his first book, and the ratio results for triangles do not appear until Book 6. In providing a reasonable modern course for second level students, it is desirable to reach some interesting, familiar and applicable results within a reasonable timeframe. After all, along with the formal proofs of theorems, we want the students to be able to demonstrate the ability to apply the results to new problems. This is best achieved when the students have available to them a varied battery of interesting theorems with which to work. Because of this, the geometry course adopted is one involving a highly redundant axiom system. In other words, we assume far more results than are strictly necessary. We still demand, however, the intellectual rigour required in using these results and these results only (along with some assumed general properties) to build up towards the remaining results on the course.

The syllabus lists a number of geometric statements as "facts". The word "fact" is to be taken here as synonymous with the word *axiom.* It was decided that an appreciation of the exact meaning of the term *axiom* in a formal system is not accessible to all students at this level. The term "fact" reflects the idea that it is sufficient for the students to conceive of these statements as ones which we accept as being true without requiring any proof. It should of course be possible for students to appreciate that since all proofs rely only on results already proved, then one cannot get started at all if one is not prepared to assume something.

As mentioned elsewhere, it is most important that students encounter the proofs of all of the theorems, including those that are not examinable. Otherwise, there is a great risk of failure to appreciate the building up of results in a logically sound way.

NOTES ON THE PRELIMINARY CONCEPTS AND ASSUMED PROPERTIES

This section provides supplementary material to that in the syllabus, in order to clarify the intended definitions and assumed properties of the geometric objects considered. The extent to which teachers address the details explicitly in class is a matter for their own discretion. *Note that the terminology and detail below are not designed for student consumption;* many teachers, however, want clarity of definitions for their own benefit. Whether or not they are addressed explicitly in class, these properties nonetheless constitute those that it is legitimate to assume in presenting proofs at examination.

No attempt is made to define the terms *point, line, plane*. It is expected that an intuitive understanding of these will be acquired by analogy and in other ways (for example, "the plane is like a page that stretches on for ever in all directions"). It should of course be noted that the plane is an infinite set of points, that lines are subsets of the plane, and so forth. Students need to be aware that a line goes on for ever in both directions. Teachers generally have little difficulty in making clear the distinction between lines, segments and half-lines, and an intuitive understanding of terms such as "between" and "on the same side as" can be assumed in order to define these if required. For the present, lower case letters will continue to be used to denote points and upper case letters to denote lines and circles. (Lines of course may also be referred to by giving two points.) Notation for segments and half-lines also remains the same as before. Thus: line *ab*, line segment [*ab*], and half-line [*ab*. Three or more points that lie on the same line are called *collinear.*

It should be noted that, unlike Euclid, we are in a position to take advantage of the power of real numbers, the properties of which have been established on a solid logical foundation independent of geometry. Accordingly, we have a concept called *distance* or *length* and a concept called *angle measure*. The properties associated with these concepts are as follows.

Given any two points *a* and *b*, there is a real number called the *distance* from *a* to *b*, or the *length* of the line segment [*ab*]. By observation, distance has the following properties (for any *a,b,c*).

- $|ab| = |ba|$.
- If $a = b$ then $|ab| = 0$. Otherwise $|ab| > 0$.
- If *b* lies on [*ac*] then $|ac| = |ab| + |bc|$.
- If *b* does not lie on [*ac*] then $|ac| < |ab| + |bc|$.
- Given any half-line [*ab* and any positive real number *k*, there is a unique point *c* on [*ab* such that $|ac| = k$.

Without ever making reference to these properties, note that students will certainly assume the first three of them without even thinking about them, as they already have an intuitive understanding of length. The fourth property is perhaps not so immediate and hence is listed as a "fact" in the syllabus document (third "fact" on page 13). The fifth property simply states that one can measure and mark off a certain distance along a line from any point; once again students assume it unquestioningly.

By observation, we note the following basic properties of angles and angle measure.

- When two half-lines [*ab* and [*ac* have the same initial point *a*, two angles are formed. (It may be helpful to think of the two angles as the two [closed] regions of the plane.)
- Every angle has a *measure*, which is a real number of degrees in the range [0°, 360°].
- The two angles formed by [*ab* and [*ac* have measures that sum to 360°.
- If a is between *b* and *c*, the two angles are called *straight angles*, and the measure of each is 180°. Otherwise, one has measure less than 180°; this angle is referred to as $\angle bac$. The other has measure greater than 180° and is referred to as $\angle bac$ reflex. (In the case of straight angles, the context is usually sufficient to determine which angle is being referred to as $\angle bac$.)
- If the angle $\angle bac$ contains the half-line [ad, then $|\angle bac| = |\angle bad| + |\angle dac|$. (This can also be formulated with more case-by-case detail to cover reflex angles in the required way.)
- Given any half-line [*ab* and any real number *k* in the range [0°,180°), there exists a unique half-line [*ac* on each side of *ab* such that $|\angle bac| = k°$.
- If the angle $\angle bac$ contains the half-line [*ad* and if $|\angle bac| < 180°$ then [*ad* intersects [*bc*].

Note that if students have an intuitive understanding of angles and their measurement, then the above properties are already known and used, although not verbalised. There is no need to address them unless the students raise the issue.

Note also that one of these properties, the fact that a straight angle measures 180°, is listed in the syllabus (as the first "fact"). This is done so that the first theorem can refer to it directly, to assist in developing the idea of building logically from assumed facts to theorems. Teachers may wish to discuss the rationale behind this "fact", referring to the idea that some unit has to be chosen to measure angles and that historically somebody decided to do this by dividing a full angle into 360 parts. This provides a nice opening into the possibility of some investigation of the history involved. It also lends itself well to the discussion about simply having to agree to start somewhere. There is the possibility too of investigating other angle measures with the calculator (gradient and radian).

Having sorted out the properties of angle measure, acute, right and obtuse angles are definable accordingly.

We take the following assumed properties of lines, points and circles and the following definitions. (Once again, the students will assume these properties anyway).

- Through any two distinct points there exists a unique line.
- Any two distinct lines have either one or no point in common.

- Two lines that have no point in common are called *parallel*. Also, for any line L we take $L \parallel L$.

- Given any point p and any line L, there is exactly one line through p parallel to L and exactly one line through p perpendicular to L. (*Perpendicular* is defined via angle measure. The distance from p to L is defined as the distance from p to the point at which L intersects the perpendicular to L through p.)

- The definitions of parallel and perpendicular are extended in the obvious way to cover half-lines and line segments also, so that we may refer to line segments being parallel to each other, and so forth.

- Any given line and any given circle have 0, 1 or 2 points in common. (A suitable definition of circle is given below.)

- If a line and a circle have exactly one point in common, the line is said to be a *tangent* to the circle, and the point is called the *point of contact.*

DEFINING THE PLANE FIGURES

The rectilinear figures listed in the syllabus could be defined precisely as various intersections of half-planes. This is probably not be the most suitable approach with students, so non-rigorous definitions will suffice (for example, a triangle as any three-sided figure; a quadrilateral as any four-sided figure; a convex quadrilateral as one whose interior angles all measure less than 180°.) As *convex* is not a term in use at this level heretofore, teachers may wish to note that a region in the plane is called *convex* if, given any two points in the region, the line segment joining them is contained in the region. Intuitively, however, it is sufficient to understand that a convex quadrilateral is a quadrilateral that does not have any corners "sticking inwards". A few examples and counter-examples will clarify this with ease for students. Note that re-entrant (i.e. non-convex) quadrilaterals are not on the course.

Unlike in the classical world, modern definitions of geometric shapes are usually *inclusive*. In other words, a square is a rectangle, a rectangle is a parallelogram, and a parallelogram is a quadrilateral. A square is also a rhombus. The following definitions could be used:

A *parallelogram* is a quadrilateral in which opposite sides are parallel.

A *rhombus* is a parallelogram with all its sides equal in length.

A *rectangle* is a parallelogram with a right angle at each vertex.

A *square* is a rectangle with all its sides equal in length.

Note also that triangles and quadrilaterals are *regions*. Hence they have areas (see below). A circle, on the other hand, is not a region but a curve. A circle does not have an area (although it encloses a disc that has an area). Hence, one can (somewhat loosely) say: "the area enclosed by the circle" but not "the area of the circle". Note also that the *length of a circle* is the distance around the circle, whereas the length of a rectangle is not the distance around the edge of the rectangle, but rather the length of one of its sides.

A *circle* is the set of all points that are a given distance from a given point (the centre). Any line segment joining the centre to a point of the circle is called a *radius*. The given distance (the common length of all the radii of a circle) is called the *radius-length*. Where confusion would not arise, *radius* may be used instead of *radius-length*, as is common practice (e.g. "a circle of radius 5 cm").

AREA

Note that in this respect the syllabus differs significantly from the Leaving Certificate Ordinary Level syllabus introduced in 1992. In the latter syllabus, area is not an assumed concept. Rather, the theorems on that course build towards, among other things, establishing a definition of area. On this syllabus, however, area is taken to be an assumed concept; i.e. it is assumed that a plane figure has such a thing as an area and that this idea of area has certain properties. The properties assumed for area are as follows.

- Each rectilinear figure (triangle, quadrilateral, ...) has an area. The area is a positive real number. (We do not consider degenerate cases, so area is not zero.)

- Congruent triangles have equal area.

- If a rectilinear figure can be decomposed into two non-overlapping rectilinear figures, then its area is equal to the sum of their two areas. (Non-overlapping means that their intersection consists of, at most, boundary line segments.)

- The area of a rectangle is equal to its length multiplied by its breadth.

The last two of these properties are listed as "facts" in the syllabus.

It is recognised that this approach to area via rectangles introduces a potential inconsistency into the system (which does not in fact materialise), and that the Leaving Certificate approach is logically more satisfactory. However, the approach adopted here facilitates to a greater extent the transfer of the understanding of area developed in the measure section. The conceptual development of area in that section usually involves investigations that include counting squares inside a region and similar activities; it thus relies heavily on rectangles.

LEGITIMATE PROOFS OF THEOREMS

A proof cannot be seen in isolation from the assumptions upon which it depends. When proofs of theorems are required in the examination, the context is the system of geometry as laid out in the syllabus and expanded upon here. Accordingly, proofs must rely only on the assumed properties, the facts listed in the syllabus and any theorems listed earlier in the syllabus than the one being proved.

In particular, teachers should note the following.

- Transformation techniques are not valid, since the definitions and properties of transformations are not in the system. Hence, one cannot use an axial symmetry to prove the isosceles triangle theorem, nor use an isometry in the similar triangles theorem.
- Results not on the syllabus cannot be parachuted in. For example, the proof – usually used in the Leaving Certificate course – that a line drawn parallel to one side of a triangle divides the other two sides in the same ratio involves drawing a set of parallel lines and relies on another result concerning transversals and parallels. This latter result is not on the Junior Certificate course, and hence that proof is not valid here. (Conversely, the envisaged proof on this course, which involves areas of triangles, is not valid in the context of the Leaving Certificate course.)
- Proofs need to be laid out in a clear and logical fashion. Proofs should be illustrated by well-labelled diagrams but such illustrations are not a substitute for written lines in a proof. In particular, they do not convey the sequence of the assertions, and are often not as precise in their meaning as the written form.
- Where practicable, each assertion made in a proof should have an associated reason given (reference to a fact or previous theorem or to how the earlier lines are being used). Words and logical connectives should be used as appropriate, so that when the written proof is verbalised, it makes sense as piece of language. Commonly used abbreviations are of course legitimate, for example, SAS for the *Side Angle Side* congruence rule, and so forth.

Consider, for example, the following two versions of the same proof of the isosceles triangle theorem.

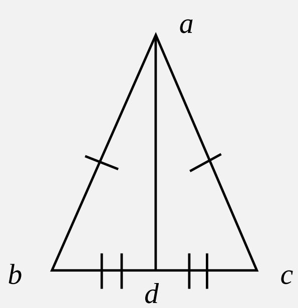
Prove that if two sides of a triangle are equal in measure, then the angles opposite these sides are equal in measure.

Given: Δabc, $|ab| = |ac|$

To Prove: $|\angle abc| = |\angle acb|$

Construction: [ad]

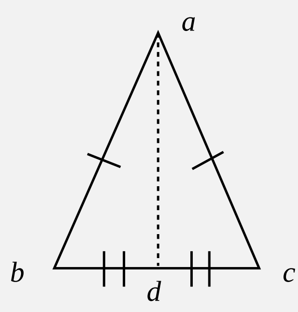
Proof:
$|ab| = |ac|$
$|ad| = |ad|$
$|db| = |dc|$
SSS
$|\angle abc| = |\angle acb|$



Given: Δabc, with $|ab| = |ac|$

To Prove: $|\angle abc| = |\angle acb|$

Construction: Construct $[ad]$, where d is the midpoint of $[bc]$



Proof: $|ab| = |ac|$ (given).
$|ad| = |ad|$ (since they are the same segment).
$|db| = |dc|$ (since d is the midpoint of [bc]).
$\therefore$ Δabd is congruent to Δacd (by the SSS rule)
$\therefore$ $|\angle abc| = |\angle acb|$ (These are corresponding parts of the congruent triangles.)

The second version is more complete and is a more effectively communicated argument, and therefore has more merit. There are, of course, other valid proofs of this theorem.

FINAL NOTE

It is important to appreciate that this appendix has concerned itself with the logical detail of the system from the perspective of formal proofs. Many other arguments, justifications and explorations of the results have a valuable role to play in teaching and learning of the material, even though they are not valid proofs for examination. For example, as detailed in Section 4, justification of Pythagoras' Theorem by a variety of dissection methods is both beneficial and interesting. Explorations and informal reasoning based on paper-folding and on transformations are similarly to be encouraged. However, in proving the results in examination, candidates must be able to operate within the logical system.

APPENDIX 3
RESOURCES

MATHEMATICS TEACHERS' ASSOCIATIONS

Irish Mathematics Teachers Association

Information can be found at **http://www.imta.ie**

Other associations

National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 20191-1593; email nctm@nctm.org, website address **http://www.nctm.org**

Mathematical Association, 259 London Road, Leicester, LE2 3BE, England

Association of Teachers of Mathematics, 7 Shaftesbury Street, Derby, DE23 8YB, England

These three associations are among the best sources of teaching materials, posters, and so forth.

MATHEMATICAL WEBSITES

The following is a sample selection of mathematical websites. Each is accompanied by a short review.

Cornell Theory Center Math and Science Gateway

Website address:
http://www.tc.cornell.edu/Edu/MathSciGateway/math.html

Review:

A very comprehensive gateway site (a site that categorises and links to lots of other sites). It contains sections on

- General Topics
- Geometry
- Fractals
- History of Mathematics
- Tables, Constants and Definitions
- Mathematical Software.

Mathematics Teaching Resource Centre

Website address:
http://www.qesn.meq.gouv.qc.ca/mapco/index.htm

Review:

This Canadian site has a notice board which tells what is new to the site, a framework for "Improving Student Performance in Mathematics", a resource toolkit, a staff lounge and many other mathematical links. To read most of the material on this site, a programme called Acrobat Reader is required. It can be downloaded from the site, but be prepared to wait a while.

Great Math Programs

Website address:
http://xahlee.org/PageTwo_dir/MathPrograms_dir/mathPrograms.html

Review:

This site gives a description of a wide range of recreational and educational mathematical software available mostly for the Macintosh platform rather than for PCs.

BBC Education

Website address:
http://www.bbc.co.uk/education/megamaths/

Review:

Some fun numeracy games here can be played online. A good way to practice tables against the clock!

Internet Mathematics Library

Website address: **http://forum.swarthmore.edu/library/**

Review:

This is part of one of the most famous sites for mathematics education: the Math Forum at Swarthmore. There are many categorised links to other pages.

Mathematics with Alice

Website address:
http://library.thinkquest.org/10977/

Review:

This is a quirky site based on the Lewis Carroll stories.

Yahoo Mathematics

Website address:
http://dir.yahoo.com/science/mathematics/

Review:

This is a search engine and yields Yahoo's categorisation of many mathematical sites.

Center of Excellence for Science and Mathematics Education

Website address:
http://cesme.utm.edu/MathLinks/mathlinks.htm

Review:

This is another resource with many links to other mathematical sites. Following some of the links brings the surfer to a whole page of links dedicated to helping students with solving word problems.

A selection of additional web-site addresses

Overview of the history of mathematics, with links to many interesting topics:

http://www-history.mcs.st-and.ac.uk/history/HistTopics/History_overview.html

Development of the symbols we use for numbers:
http://www.islam.org/Mosque/ihame/Ref6.htm

Fibonacci numbers and the Golden Ratio:
http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fib.html
http://math.holycross.edu/~davids/fibonacci/fibonacci.html

The Four-Colour Problem:
http://129.237.247.243/wizard/pages/01391.html

Search for previously unknown Mersenne primes:
http://www.mersenne.org/prime.htm

Biographies of female mathematicians (note the members of the Boole family):
http://www.agnesscott.edu/lriddle/women/women.htm

The American Mathematical Society's pages on what is new in mathematics (rather advanced, but may be of interest):
http://www.ams.org/new-in-math/
(see for example a reference to William Rowan Hamilton:
http://www.ams.org/new-in-math/cover/dna-abc2.html)

Logo:
http://www.terrapinlogo.com/
(for instance, follow the route: FOR EDUCATORS/Why use Logo?)
http://www.softronix.com
(and look at information and on-line books on MSW Logo)
http://www.lcsi.ca/
(and for instance look at "links")

Many enjoyable activities and puzzles:
http://www.mathsyear2000.org/

This is just a short selection of some of the sites to give mathematics teachers a flavour of what is available. They contain, among other things, class plans, chat groups, mathematical games and puzzles, free software, reviews of programs, interesting projects and links with schools. See also **http://www.scoilnet.ie** for ideas and references submitted by Irish teachers.

BOOKS

Lesson ideas

The books in this category contain ideas for teaching mathematics (in some cases alongside discussion of mathematics or mathematics education).

Bolt, B. *Mathematical Activities: a Resource Book for Teachers.* Cambridge: Cambridge University Press, 1982 [and subsequent books of activities by Bolt].

Brophy, Tim. *Mathematics and Science with MathView.* John F Marshall, 1997.

Cotton, David. *Mathematics Lessons at a Moment's Notice.* London: Foulsham, 1986.

Creative Publications. *Algebra with Pizazz and Middle School Math with Pizazz*. [See the website: http://www.creativepublications.com]

Jones, Lesley, ed. *Teaching Mathematics and Art.* Cheltenham: Stanley Thornes, 1991.

Kjartan, Poskitt. *Murderous Maths.* Hippo Publications, 1997.

Mitchell, Merle. *Mathematical History: Activities, Puzzles, Stories and Games.* Reston, VA: NCTM, 1978.

Pappas, Theoni. *Mathematics Appreciation.* San Carlos, CA: Wide World Publishing / Tetra, 1987. [Postal address: PO Box 476, San Carlos, CA 94070]

Sawyer, W. W. *Vision in Elementary Mathematics.* Harmondsworth: Penguin, 1964.

Shan, Sharan-Jeet, and Bailey, Peter. *Multiple Factors: Classroom Mathematics for Equality and Justice.* Stoke-on-Trent, England: Trentham Books, 1991. [This book contains lesson ideas from many cultures.]

Sharp, R. M., and Metzner, S. *The Sneaky Square and 113 Other Math Activities for Kids.* Blue Ridge Summit, PA: Tab Books, 1990.

Thyer, Dennis. *Mathematical Enrichment Exercises: A Teacher's Guide.* London: Cassell, 1993.

General interest

The books in this section are chiefly books about mathematics, and give scope for enjoyable browsing. Most are aimed at a general audience, but may be particularly interesting for teachers. Some books could be relevant for students also.

Bondi, Christine, ed. (for the Institute of Mathematics and its Applications). *New Applications of Mathematics.* Harmondsworth: Penguin, 1991.

Cosgrave, John B. *A Prime for the Millennium.* Roundstone: Folding Landscapes, 2000.

Davis, Philip J., and Hersh, Reuben. T*he Mathematical Experience.* Boston: Birkhauser, 1980; also Harmondsworth: Pelican Books, 1983.

Devlin, Keith. *Mathematics: the Science of Patterns.* New York: Scientific American Library, 1994.

Eastway, Rob, and Wyndham, Jeremy. *Why Do Buses Come in Threes? The Hidden Mathematics of Everyday Life.* London: Robson Books, 1998, 1999.

Flannery, Sarah, with Flannery, David. *In Code: a Mathematical Journey. London:* Profile Books, 2000.

Houston, Ken, ed. *Creators of Mathematics: the Irish Connection.* Dublin: UCD Press, 2000.

Jacobs, Harold R. *Mathematics: a Human Endeavor: a Book for Those who Think They Don't Like the Subject.* 2nd ed. San Francisco: Freeman, 1982.

Pappas, Theoni. *The Joy of Mathematics: Discovering Mathematics All Around You.* San Carlos, CA: Wide World publishing / Tetra, 1986, 1989.

Pappas, Theoni. *More Joy of Mathematics.* San Carlos, CA: Wide World Publishing / Tetra, 1991 .

Singh, Simon. F*ermat's Last Theorem.* London: Fourth Estate, 1997 [paperback 1998].

Smullyan, Raymond. *What is the Name of This Book? The Riddle of Dracula and Other Logical Puzzles.* Harmondsworth: Penguin, 1981.

Struik, Dirk J. A *Concise History of Mathematics,* 4th ed. Toronto: Dover Publications, 1987.

Vorderman, Carol. *How Mathematics Works.* London: Dorling Kindersley, 1996, 1998.

Wells, David. *The Penguin Dictionary of Curious and Interesting Numbers.* Harmondsworth: Penguin, 1986, 1987.

Mathematics education

The first book listed below is aimed chiefly at beginning teachers, but is mentioned here because its philosophy is so close to that of the one underlying the methodology discussed in these *Guidelines.* The second book is a very valuable resource containing many practical ideas presented against a background of research.

Backhouse, J., Haggarty, L., Pirie, S., and Stratton, J. *Improving the Learning of Mathematics.* Children, Teachers and Learning Series. London: Cassell, 1992.

National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics.* Reston, VA: NCTM, 2000.

SUGGESTED RESOURCES AND EQUIPMENT FOR A MATHEMATICS CLASSROOM

Active teaching and learning are often facilitated by the use of various kinds of material. Useful items include the following:

- Large geometry compasses, protractor and setsquares.
- Set of scissors (the "blunt-nosed" variety!)
- Coloured paper or thin cardboard
- Paper fasteners
- Trundle wheel
- Dominoes
- Clinometers
- Boxes of 2D and 3D shapes
- Graduated cylinders
- Set of calculators
- Set of log tables
- The polydron kit
- Jim Marsden's activity kits
- Calculator hex board game
- Dart mathematics game
- Dice
- Counters
- Unifix cubes
- Mathematical posters
- Geoboards, pegboards and pegs

Many of these items can be bought in stationers' shops. References to others can be found in the journals or on the websites of mathematics teachers' associations.

APPENDIX 4
TABLES FOR PLANNING AND RECORDING

This appendix contains tables that can be used to facilitate planning of the topics and sub-topics given in the syllabus, for each of the three levels. Also included is a template for developing further lesson ideas, along the lines of those contained in chapter 4 of these *Guidelines*. Teachers are encouraged to use this template to plan, record and share lesson ideas with colleagues.

HIGHER COURSE: PLANNING AND RECORD SHEET

	1st year	2nd year	3rd year	Notes
Sets				
1. Basic concepts				
2. Venn Diagrams				
3. Set operations (NOT three sets)				
Set operations: three sets				
4. Properties etc.				
Number systems				
1. Basic concepts				
Operations/estimation and approximation				
2. Z, operations etc. in Z				
[Revision throughout year]				
3. Rationals				
Rationals as decimals				
Operations/estimation and approximation				
Ratio and proportion				
4. Powers: first 2 rules				
Second two rules				
Remaining rules				
Square roots, reciprocals				
Scientific notation				
5. Reals				
Irrationals, surds				
6. Properties				
Applied arithmetic and measure				
1. Bills etc. (social arithmetic: basics)				
Ditto: problems				
2. SI units, physical units				
3. Perimeter / area /volume (rectangular solids)				
Circles etc.				
Applications including Pythagoras				
Algebra				
1. Basic concepts, evaluation				
2. Manipulation of expressions (to multiplication)				
Division of expressions				
Rearrangement of formulae				
Addition and subtraction of algebraic fractions.				

	1st year	2nd year	3rd year	Notes
3. Factors: simple				
Factors: trinomials				
Factors: two squares				
4. Linear equations				
Simultaneous equations: basic				
Simultaneous equations: problems				
Quadratic equations: factor method				
Quadratic equations: formula				
5. Equations involving algebraic fractions				
6. Inequalities				
Statistics				
1. Diagrammatic representation of data				
2. Frequency table and histogram				
Mean and mode				
Mean of grouped frequency				
Cumulative frequency, ogive etc.				
Geometry				
1. Synthetic geometry: elementary concepts				
Basic work up to congruency				
Area and parallelogram (etc.) results				
Circles and circle theorems				
Similarity theorems etc.				
Pythagoras & converse				
2. Transformation geometry: basics (NOT rotation)				
Rotation				
3. Co-ordinate geometry: basics				
Images of points				
Distance, midpoint				
Slope etc.				
Equation of line				
Intersection of lines				
Trigonometry				
1. Basics (up to 90 degrees)				
2. Solving right-angled triangles				
Above for larger angles where relevant				
Sine rule, area				
Functions and graphs				
1. Basic concepts, couples etc.				
2. Function notation, graphs				
3. Maxima and minima from graphs				
4. Solution sets of inequalities on number line				
5. Graphs for simultaneous equations				
TOTALS				

ORDINARY COURSE: PLANNING AND RECORD SHEET

	1st year	2nd year	3rd year	Notes
Sets				
1. Basic concepts				
2. Venn Diagrams				
3. Set operations (NOT three sets)				
Set operations: three sets				
4. Properties etc.				
Number systems				
1. Basic concepts				
Operations/estimation and approximation				
2. Z, operations etc. in Z				
[Revision throughout year]				
3. Rationals, rationals as decimals				
Operations/estimation and approximation				
Ratio and proportion				
4. Powers: first 2 rules				
Remaining rule and concepts				
Square roots, reciprocals				
Scientific notation				
5. Reals				
6. Properties				
Applied arithmetic and measure				
1. Bills etc. (social arithmetic: basics)				
Ditto: problems				
2. SI units, physical units				
3. Perimeter / area /volume (rectangular solids)				
Circles etc.				
Applications				
Algebra				
1. Basic concepts, evaluation				
2. Manipulation of expressions (to multiplication)				
Addition and subtraction of algebraic fractions.				
3. Factors: simple				
Factors: trinomials				
Factors: two squares				
4. Linear equations				
Simultaneous equations: basic				
Simultaneous equations: problems				
Quadratics equations				
5. Equations involving algebraic fractions				
6. Inequalities				

	1st year	2nd year	3rd year	Notes
Statistics				
1. Diagrammatic representation of data				
2. Frequency table				
Mean and mode				
Geometry				
1. Synthetic geometry: elementary concepts				
Basic work up to congruency				
Area and parallelogram (etc.) results				
Circle and circle results				
Pythagoras & converse				
2. Transformation geometry				
3. Co-ordinate geometry: basics				
Images of points				
Distance, midpoint, slope				
Equation of line				
Intersection of lines with axes				
Trigonometry				
1. Basics				
2. Solving right-angled triangles				
Functions and graphs				
1. Basic concepts, couples etc.				
2. Function notation, functions				
3. Solution sets of inequalities on number line				
4. Graphs for simultaneous equations				
TOTALS				

FOUNDATION COURSE: PLANNING AND RECORD SHEET

	1st year	2nd year	3rd year	Notes
Sets				
1. Basic concepts				
2. Venn Diagrams				
3. Set operations				
4. Properties etc.				
Number systems				
1. Basic concepts				
Operations/estimation and approximation				
2. Z, addition in Z				
[Revision throughout year]				
3. Positive rationals				
Fractions: operations/estimation and approximation				
Decimals: operations/estimation and approximation				
Percentages: operations/estimation and approximation				
Equivalence				
4. Squares, square roots				
5. Properties				
Applied arithmetic and measure				
1. Bills etc. (social arithmetic: basics)				
Ditto: problems				
2. SI units, physical units				
3. Maps				
4. Perimeter / area /volume (rectangular solids)				
Circles etc.				
Statistics and data handling				
1. Diagrammatic representation of data				
Relationships expressed by graphs and tables				
2. Frequency table				
Mean and mode				
Algebra				
1. Basic concepts, evaluation				
2. Simplification of expressions				
3. Linear equations with natural solutions				
Relations, functions and graphs				
1. Basic concepts, arrow diagrams for relations				
2. Plotting points, line				
3. Drawing and interpreting linear grahs				

	1st year	2nd year	3rd year	Notes
Geometry				
1. Synthetic geometry: elementary concepts				
Line segments				
Angles and triangles				
Right-angled triangles				
Rectangles, area etc.				
2. Transformation geometry				
Constructions				
TOTALS				

LESSON IDEAS TEMPLATE

Teachers are encouraged to document lesson ideas which they have found useful and effective for a particular topic. The template given below, which follows the style of the lesson ideas in these *Guidelines*, may be used for this purpose.

TITLE:

TOPIC:

AIM:

RESOURCES:

METHOD:

CLASSROOM MANAGEMENT IMPLICATIONS:

Published by The Stationery Office
To be purchased directly from:
Government Publications Sales Office,
Sun Alliance House,
Molesworth Street, Dublin 2.
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