FACTORS

(i) **Two term factorisation**

$2a^2b - 8ab^2 = 2ab(a-4b)$

(ii) **Four term factorisation**

A: $2x^2 + 3x - 10xy - 15y = x(2x+3) - 5y(2x+3)$

$= (x-5y)(2x+3)$

B: $ax^2 - bx^2 - ay^2 + by^2$

$= x^2(a-b) - y^2(a-b)$

$= (x^2 - y^2)(a-b)$

$= (x-y)(x+y)(a-b)$

(iii) **Quadratic factorisation**

$10x^2 - x - 2 = 10x^2 - 5x + 4x - 2$

$= 5x(2x-1) + 2(2x-1)$

$= (5x+2)(2x-1)$

(iv) **Difference of two squares**

$x^2 - y^2 = (x-y)(x+y)$

EXAMPLES

A: $25x^2 - 49y^2 = (5x-7y)(5x+7y)$

B: $\left(\sqrt{x+1} - \sqrt{x-1}\right)\left(\sqrt{x+1} + \sqrt{x-1}\right)$

$= \left(\sqrt{x+1}\right)^2 - \left(\sqrt{x-1}\right)^2$

$= x + 1 - (x-1) = x + 1 - x + 1 = 2$

C: $3x^2 - 12 = 3(x^2 - 4)$

$= 3(x-2)(x+2)$

D: $(x+y)^2 - z^2 = (x+y-z)(x+y+z)$

(v) **Sum of two cubes**

$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

EXAMPLES

A: $8x^3 + 125 = (2x)^3 + 5^3$

$= (2x+5)((2x)^2 - (2x)5 + 25)$

$= (2x+5)(4x^2 - 10x + 25)$

B: $x^4 + x = x(x^3 + 1)$

$= x(x+1)(x^2 - x + 1)$

(vi) **Difference of two cubes**

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

EXAMPLES

$27a^3 - 8b^3 = (3a - 2b)(9a^2 + 6ab + 4b^2)$

(vii) Difference of two fourth powers

$$x^4 - y^4 = (x^2)^2 - (y^2)^2$$
$$= [(x^2) - (y^2)][(x^2) + (y^2)]$$
$$= (x - y)(x + y)(x^2 + y^2)$$

EXAMPLE

$$x^4 - 81 = x^4 - 3^4$$
$$= (x^2 - 3^2)(x^2 + 3^2)$$
$$= (x - 3)(x + 3)(x^2 + 9)$$

FRACTIONS

Adding and subtracting – use common denominators.

EXAMPLES

A: $$\frac{7}{2y+1} - \frac{6}{2y-1} = \frac{7(2y-1) - 6(2y+1)}{(2y+1)(2y-1)}$$
$$= \frac{14y - 7 - 12y - 6}{(2y+1)(2y-1)}$$
$$= \frac{2y - 13}{(2y+1)(2y-1)}$$

B: $$\frac{7}{x+3} + \frac{4}{x^2 + 4x + 3} = \frac{7}{x+3} + \frac{4}{(x+3)(x+1)}$$
$$= \frac{7(x+1) + 4}{(x+3)(x+1)}$$
$$= \frac{7x + 7 + 4}{(x+3)(x+1)}$$
$$= \frac{7x + 11}{(x+3)(x+1)}$$

SURDS

Breaking down surds

$\sqrt{28} = \sqrt{4}\sqrt{7} = 2\sqrt{7}$

Using the quadratic formula

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXAMPLE

$$\left.\begin{matrix} ax^2+bx+c=0 \\ x^2-6x+4=0 \end{matrix}\right\} \quad a=1,\ b=-6,\ c=4$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(4)}}{2(1)}$$

$$x=\frac{6\pm\sqrt{20}}{2}=\frac{6\pm\sqrt{4}\sqrt{5}}{2}=\frac{6\pm2\sqrt{5}}{2}=3\pm\sqrt{5}$$

Rationalising the denominator

EXAMPLES

A: $\frac{6}{\sqrt{2}}=\frac{6}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}=\frac{6\sqrt{2}}{2}=3\sqrt{2}$

B: $\frac{5\sqrt{2}}{(\sqrt{2}-1)}=\frac{5\sqrt{2}}{(\sqrt{2}-1)}\frac{(\sqrt{2}+1)}{(\sqrt{2}+1)}=\frac{5\sqrt{2}\sqrt{2}+5\sqrt{2}}{(\sqrt{2})^2-1}$

$$=\frac{10+5\sqrt{2}}{2-1}=10+5\sqrt{2}$$

SURD EQUATIONS

(i) **Isolate** the surd

(ii) **Square** both sides using the formula
SQUARE OF A SUM
$x^2+2xy+y^2=(x+y)^2$
square the first + twice the product + square the second

SQUARE OF A DIFFERENCE
$x^2-2xy+y^2=(x-y)^2$
square the first – twice the product + square the second

(iii) Repeat the above if there's still a surd

(iv) Simplify

(v) **Solve** the quadratic or two term (you can use factors **or** the minus-b formula.)

(vi) **Check** your solutions

EXAMPLE

$\sqrt{2x+1} = x-1 \Rightarrow \left(\sqrt{2x+1}\right)^2 = (x-1)^2$

$2x+1 = x^2 - 2x + 1$

$0 = x^2 - 2x + 1 - 2x - 1$

$0 = x^2 - 4x = x(x-4)$

$\Rightarrow x = 0$ or $x = 4$

CHECK: $\sqrt{2x+1} = x-1$

$\sqrt{2(0)+1} = 0-1 \Rightarrow 1 = -1$ FALSE

$\sqrt{2(4)+1} = 4-1 \Rightarrow 3 = 3$ TRUE

ANSWER: $x = 4$

REARRANGING FORMULAE

- Use the rule of opposites.
- Be careful of fractions with two terms in the denominator.

EXAMPLE

Make t the subject of the formula: $x = \dfrac{2t+1}{t-3}$

ANSWER

$x = \dfrac{2t+1}{t-3} \Rightarrow x(t-3) = 2t+1$

$\Rightarrow xt - 3x = 2t + 1 \Rightarrow xt - 2t = 1 + 3x$

$\Rightarrow t(x-2) = 1 + 3x \Rightarrow t = \dfrac{1+3x}{x-2}$