

Oliver Murphy: Discovering Maths 4: EXERCISE 7H

1.

$x$	1	2	3	4	5
$f$	1	0	2	2	5

$$\begin{aligned}\mu &= \frac{\Sigma fx}{\Sigma f} = \frac{1 \times 1 + 0 \times 2 + 2 \times 3 + 2 \times 4 + 5 \times 5}{1 + 0 + 2 + 2 + 5} \\ &= \frac{40}{10} = 4\end{aligned}$$

$$\begin{aligned}\sigma &= \sqrt{\frac{\Sigma f(x - \mu)^2}{\Sigma f}} \\ &= \sqrt{\frac{1 \times (1 - 4)^2 + 0 \times (2 - 4)^2 + 2 \times (3 - 4)^2 + 2 \times (4 - 4)^2 + 5 \times (5 - 4)^2}{1 + 0 + 2 + 2 + 5}} \\ &= \sqrt{\frac{9 + 0 + 2 + 0 + 5}{10}} = \sqrt{\frac{16}{10}} = \sqrt{1.6} = 1.26\end{aligned}$$

2.

$x$	1	2	3	4	5
$f$	6	7	8	5	6

(i)

$$\begin{aligned}\mu &= \frac{\Sigma fx}{\Sigma f} = \frac{6 \times 1 + 5 \times 2 + 8 \times 3 + 5 \times 4 + 6 \times 5}{6 + 5 + 8 + 5 + 6} \\ &= \frac{6 + 10 + 24 + 20 + 30}{30} = \frac{90}{30} = 3\end{aligned}$$

(ii)

$$\begin{aligned}\sigma &= \sqrt{\frac{\Sigma f(x - \mu)^2}{\Sigma f}} \\ &= \sqrt{\frac{6 \times (1 - 3)^2 + 5 \times (2 - 3)^2 + 8 \times (3 - 3)^2 + 5 \times (4 - 3)^2 + 6 \times (5 - 3)^2}{6 + 5 + 8 + 5 + 6}} \\ &= \sqrt{\frac{24 + 5 + 0 + 0 + 5 + 24}{30}} = \sqrt{\frac{58}{30}} = 1.39\end{aligned}$$

(iii)

$x$	1	2	3	4	5
$f$	6	7	8	5	6

$$\mu - \sigma = 3 - 1.39 = 1.61$$

$$\mu + \sigma = 3 + 1.39 = 4.39$$

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$$1 \cdot 61 \leq x \leq 4 \cdot 39 \Rightarrow x = 2 \text{ or } 3 \text{ or } 4$$

$$\Rightarrow f = 5 \text{ or } 8 \text{ or } 5$$

$$\text{percentage} = \frac{(5+8+5)}{30} \times \frac{100}{1} = 60\%$$

3.

$x$	0	1	2	3	4
$f$	4	0	3	2	3

(i)

$$\begin{aligned} \mu &= \frac{\sum fx}{\sum f} = \frac{4 \times 0 + 0 \times 1 + 3 \times 2 + 2 \times 3 + 3 \times 4}{4 + 0 + 3 + 2 + 3} \\ &= \frac{0 + 0 + 6 + 6 + 12}{12} = \frac{24}{12} = 2 \end{aligned}$$

(ii)

$$\begin{aligned} \sigma &= \sqrt{\frac{\sum f(x - \mu)^2}{\sum f}} \\ &= \sqrt{\frac{4 \times (0 - 2)^2 + 0 \times (1 - 2)^2 + 3 \times (2 - 2)^2 + 2 \times (3 - 2)^2 + 3 \times (4 - 2)^2}{4 + 0 + 3 + 2 + 3}} \\ &= \sqrt{\frac{16 + 0 + 0 + 2 + 12}{12}} = \sqrt{\frac{30}{12}} = 1.58 \end{aligned}$$

4.

$x$	0	1	2	3	4
$f$	2	4	8	4	2

(i)

$$\begin{aligned} \mu &= \frac{\sum fx}{\sum f} = \frac{2 \times 0 + 4 \times 1 + 8 \times 2 + 4 \times 3 + 2 \times 4}{2 + 4 + 8 + 4 + 2} \\ &= \frac{0 + 4 + 16 + 12 + 8}{20} = \frac{40}{20} = 2 \end{aligned}$$

(ii)

$$\begin{aligned} \sigma &= \sqrt{\frac{\sum f(x - \mu)^2}{\sum f}} \\ &= \sqrt{\frac{2 \times (0 - 2)^2 + 4 \times (1 - 2)^2 + 8 \times (2 - 2)^2 + 4 \times (3 - 2)^2 + 2 \times (4 - 2)^2}{2 + 4 + 8 + 4 + 2}} \\ &= \sqrt{\frac{8 + 4 + 0 + 4 + 8}{20}} = \sqrt{\frac{24}{20}} = 1.095 \approx 1.10 \approx 1.1 \end{aligned}$$

(iii)

$x$	0	1	2	3	4
$f$	2	4	8	4	2

$$\mu - \sigma = 2 - 1 \cdot 1 = 0.9$$

$$\mu + \sigma = 2 + 1 \cdot 1 = 3.1$$

$$0.9 \leq x \leq 3.1 \Rightarrow x = 1 \text{ or } 2 \text{ or } 3$$

$$\Rightarrow f = 4 \text{ or } 8 \text{ or } 4$$

$$\text{total} = 4 + 8 + 4 = 16$$

5.

$x$	2	6	10	14	18
$f$	2	6	12	6	2

(i)

$$\begin{aligned} \mu &= \frac{\sum fx}{\sum f} = \frac{2 \times 2 + 6 \times 6 + 12 \times 10 + 6 \times 14 + 2 \times 18}{2 + 6 + 12 + 6 + 2} \\ &= \frac{4 + 36 + 120 + 84 + 36}{28} = \frac{280}{28} = 10 \end{aligned}$$

(ii)

$$\begin{aligned} \sigma &= \sqrt{\frac{\sum f(x - \mu)^2}{\sum f}} \\ &= \sqrt{\frac{2 \times (2 - 10)^2 + 6 \times (6 - 10)^2 + 12 \times (10 - 10)^2 + 6 \times (14 - 10)^2 + 2 \times (18 - 10)^2}{2 + 6 + 12 + 6 + 2}} \\ &= \sqrt{\frac{128 + 96 + 0 + 96 + 128}{28}} = \sqrt{\frac{448}{28}} = 4 \end{aligned}$$

6.

$x$	20	50	70	90	110
$f$	2	6	5	3	4

(i)

$$\begin{aligned} \mu &= \frac{\sum fx}{\sum f} = \frac{2 \times 20 + 6 \times 50 + 5 \times 70 + 3 \times 90 + 4 \times 110}{2 + 6 + 5 + 3 + 4} \\ &= \frac{40 + 300 + 350 + 270 + 440}{20} = \frac{1400}{20} = 70 \end{aligned}$$

(ii)

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$$\begin{aligned}\sigma &= \sqrt{\frac{\sum f(x-\mu)^2}{\sum f}} \\ &= \sqrt{\frac{2 \times (20-70)^2 + 6 \times (50-70)^2 + 5 \times (70-70)^2 + 3 \times (90-70)^2 + 4 \times (110-70)^2}{2+6+5+3+4}} \\ &= \sqrt{\frac{5000 + 2400 + 0 + 1200 + 6400}{20}} = \sqrt{\frac{15000}{20}} = 27 \text{ minutes}\end{aligned}$$

9.

$x$	2	4	6	20
$f$	5	8	8	9

(i)

$$\begin{aligned}\mu &= \frac{\sum fx}{\sum f} = \frac{5 \times 2 + 8 \times 4 + 8 \times 6 + 9 \times 20}{5 + 8 + 8 + 9} \\ &= \frac{10 + 32 + 48 + 180}{30} = \frac{270}{30} = 9\end{aligned}$$

$$\begin{aligned}\sigma &= \sqrt{\frac{\sum f(x-\mu)^2}{\sum f}} \\ &= \sqrt{\frac{5 \times (2-9)^2 + 8 \times (4-9)^2 + 8 \times (6-9)^2 + 9 \times (20-9)^2}{5+8+8+9}} \\ &= \sqrt{\frac{245 + 200 + 72 + 1089}{30}} = \sqrt{\frac{1606}{30}} = 7.317\end{aligned}$$

(ii)

$$x \rightarrow 10x$$

$$\Rightarrow \mu \rightarrow 10\mu = 90 \text{ and } \sigma \rightarrow 10\sigma = 73.17$$

(iii)

$$x \rightarrow x + 50$$

$$\Rightarrow \mu \rightarrow \mu + 50 = 59 \text{ and } \sigma \rightarrow \sigma = 7.317$$

(iv)

$$x \rightarrow 10x + 7$$

$$\Rightarrow \mu \rightarrow 10\mu + 7 = 97 \text{ and } \sigma \rightarrow 10\sigma = 73.17$$