1. (i) $P(\text{HH}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(ii) $P(\text{at least 1 T}) = P(\text{not HH}) = 1 - \frac{1}{4} = \frac{3}{4}$

2. (i) $P(\text{3 sixes}) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$

(ii) $P(\text{6,even,odd}) = \frac{1}{6} \times \frac{3}{6} \times \frac{3}{6} = \frac{1}{24}$

3. (i) $P(\text{4 heads}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$

(ii)

$$P(\text{at least one tail}) = P(\text{not all heads})$$
$$= 1 - \frac{1}{16} = \frac{15}{16}$$

4. (i) $P(\text{5 girls}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32}$

(ii)

$$P(\text{at least one boy}) = P(\text{not all girls})$$
$$= 1 - \frac{1}{32} = \frac{31}{32}$$

5. (i) $\frac{^{16}C_2}{^{30}C_2} = \frac{8}{29}$

(ii) $\frac{^{14}C_2}{^{30}C_2} = \frac{91}{435}$

(iii)

$$P(\text{at least one boy}) = P(\text{not both girls})$$
$$= 1 - \frac{91}{435} = \frac{344}{435}$$

6. (i) $P(\text{JQ}) = \frac{4}{52} \times \frac{4}{51} = \frac{4}{663}$

(ii) $P(\text{JJ}) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$

or

$P(\text{JJ}) = \frac{^{4}C_2}{^{52}C_2} = \frac{1}{221}$

(iii) $P(\text{neither a J}) = \frac{48}{52} \times \frac{47}{51} = \frac{188}{221}$

or

$P(\text{neither a J}) = \frac{^{48}C_2}{^{52}C_2} = \frac{188}{221}$

(iv)

$$P(\text{at least one J}) = 1 - P(\text{neither a J})$$
$$= 1 - \frac{188}{221} = \frac{33}{221}$$

7. (i) $\frac{{}^{4}C_3}{{}^{52}C_3} = \frac{1}{5525}$

(ii)

$$P(\text{at least one not a Q}) = P(\text{not QQQ})$$
$$= 1 - \frac{1}{5525} = \frac{5524}{5525}$$

8. (i) $P(\text{scores both}) = 0\cdot 6 \times 0\cdot 6 = 0\cdot 36$

(ii) $P(\text{scores neither}) = 0\cdot 4 \times 0\cdot 4 = 0\cdot 16$

(iii) $P(\text{at least once}) = 1 - P(\text{neither}) = 1 - 0\cdot 16 = 0\cdot 84$

(iv) $P(\text{one but not the other}) = 0\cdot 4 \times 0\cdot 6 + 0\cdot 6 \times 0\cdot 4 = 0\cdot 48$

9. $P(\text{on time}) = 1 - P(\text{late}) = 1 - \frac{1}{8} = \frac{7}{8}$

(i) $P(\text{on time 2}) = \frac{7}{8} \times \frac{7}{8} = 0\cdot 766$

(ii) $P(\text{on time 3}) = \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} = 0\cdot 670$

(iii) $P(\text{on time 5}) = \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} = 0\cdot 513$

10. (i) $P(\text{all 3 successful}) = \frac{1}{2} \times \frac{1}{4} \times \frac{1}{3} = \frac{1}{24}$

(ii) $P(\text{all 3 fail}) = \frac{1}{2} \times \frac{3}{4} \times \frac{2}{3} = \frac{1}{4}$

(iii)

$$P(\text{at least one succeeds}) = 1 - P(\text{all 3 fail})$$
$$= 1 - \frac{1}{4} = \frac{3}{4}$$

11. (i)

$P(\text{first out}) = 1 - 0\cdot 5 = 0\cdot 5$

$P(\text{second out}) = 1 - 0\cdot 8 = 0\cdot 2$

$P(\text{first out, second out}) = 0\cdot 5 \times 0\cdot 2 = 0\cdot 1$

(ii)

$P(\text{first out}) = 1 - 0\cdot 7 = 0\cdot 3$

$P(\text{second out}) = 1 - 0\cdot 7 = 0\cdot 3$

$P(\text{first out, second out}) = 0\cdot 3 \times 0\cdot 3 = 0\cdot 09$

12. (i) $\frac{12}{12}\times\frac{11}{12}\times\frac{10}{12}=\frac{55}{72}$

(ii)

$$P(\text{at least two same})=1-P(\text{all different})$$
$$=1-\frac{55}{72}=\frac{17}{72}$$

13. (i) $P(\text{five spades})=\frac{{}^{13}C_5}{{}^{52}C_5}=\frac{33}{66640}$

(ii)

$$P(\text{five same})=P(\text{five spades})+P(\text{five clubs})+P(\text{five hearts})+P(\text{five diamonds})$$
$$=\frac{{}^{13}C_5}{{}^{52}C_5}+\frac{{}^{13}C_5}{{}^{52}C_5}+\frac{{}^{13}C_5}{{}^{52}C_5}+\frac{{}^{13}C_5}{{}^{52}C_5}$$
$$=\frac{33}{16660}$$

14. (i) $P(\text{all different})=\frac{7}{7}\times\frac{6}{7}\times\frac{5}{7}\times\frac{4}{7}\times\frac{3}{7}=\frac{360}{2401}$

$$P(\text{two or more same})=1-P(\text{all different})$$
$$=1-\frac{360}{2401}=\frac{2041}{2410}$$

(ii) $P(\text{all different})=\frac{12}{12}\times\frac{11}{12}\times\frac{10}{12}\times\frac{9}{12}\times\frac{8}{12}=\frac{55}{144}$

$$P(\text{two or more same})=1-P(\text{all different})$$
$$=1-\frac{55}{144}=\frac{89}{144}$$

15. (i)

$$P(\text{all different})=\frac{365}{365}\times\frac{364}{365}\times\frac{363}{365}\times\frac{362}{365}$$
$$=\frac{365!}{(365-4)!}\times\frac{1}{365^4}$$
$$=0\cdot 98364$$
$$P(\text{two or more same})=1-P(\text{all different})$$
$$=1-0\cdot 9836=0\cdot 0164$$

(ii)

$$P(\text{two or more same})=1-P(\text{all different})$$
$$=1-\frac{365!}{(365-23)!}\times\frac{1}{365^{23}}=0\cdot 5073$$