

$$2. \text{ (i) } \frac{\#\{1,3,5\}}{6} = \frac{1}{2} \quad \text{(ii) } \frac{\#\{1,2,3,6\}}{6} = \frac{4}{6} = \frac{2}{3}$$

$$\text{(iii) } \frac{\#\{1,2,4\}}{6} = \frac{3}{6} = \frac{1}{2}$$

$$4. \text{ (i) } \frac{16}{30} = \frac{8}{15} \quad \text{(ii) } \frac{14}{30} = \frac{7}{15} \quad \text{(iii) } \frac{3+2}{30} = \frac{5}{30} = \frac{1}{6}$$

$$\text{(iv) } \frac{3}{30} = \frac{1}{10}$$

$$5. \text{ (i) } \frac{13}{52} = \frac{1}{4} \quad \text{(ii) } \frac{26}{52} = \frac{1}{2} \quad \text{(iii) } \frac{4}{52} = \frac{1}{13}$$

$$\text{(iv) } \frac{13+26}{52} = \frac{39}{52} = \frac{3}{4} \quad \text{(v) } \frac{13+3}{52} = \frac{16}{52} = \frac{4}{13} \quad \text{(vi) } \frac{26+2}{52} = \frac{28}{52} = \frac{7}{13}$$

$$8. \text{ (i) } P(A) = \frac{7+3}{7+3+2+8} = \frac{10}{20} = \frac{1}{2}$$

$$\text{(ii) } P(B) = \frac{3+2}{7+3+2+8} = \frac{5}{20} = \frac{1}{4}$$

$$\text{(iii) } P(A \cap B) = \frac{2}{20} = \frac{1}{10}$$

$$\text{(iv) } P(A \cup B) = \frac{7+3+2}{20} = \frac{12}{20} = \frac{3}{5}$$

$$\text{(v) } P(A \setminus B) = \frac{7}{20}$$

$$\text{(vi) } P(A \setminus B \cup B \setminus A) = \frac{7+2}{20} = \frac{9}{20}$$

$$\text{(vii) } P(A') = \frac{2+8}{20} = \frac{10}{20} = \frac{1}{2}$$

$$\text{(viii) } P((A \cup B)') = \frac{8}{20} = \frac{2}{5}$$

$$P(A) - P(A \cap B) = \frac{10}{20} - \frac{3}{20} = \frac{7}{20} = P(A \setminus B)$$

$$\begin{aligned} P(A) + P(B) - P(A \cap B) &= \frac{10}{20} + \frac{5}{20} - \frac{3}{20} \\ &= \frac{12}{20} = P(A \cup B) \end{aligned}$$

11. There are 36 elements in the sample space:

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

(i)

total equal to four:

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$$\frac{\#\{(3,1), (2,2), (1,3)\}}{36} = \frac{3}{36} = \frac{1}{12}$$

(ii)

total of four or less:

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$$\frac{\#\{(1,1), (1,2), (2,1), (1,3), (2,2), (3,1)\}}{36} = \frac{6}{36} = \frac{1}{6}$$

(iii)

total greater than nine:

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$$\frac{\#\{(4,6), (5,5), (6,4), (5,6), (6,5), (6,6)\}}{36} = \frac{6}{36} = \frac{1}{6}$$

(iv)

total an even number

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$$\frac{1+3+5+5+3+1}{36} = \frac{18}{36} = \frac{1}{2}$$

(v)

total less than six or prime:

(NB: the primes between 1 and 12 are 2, 3, 5, 7 and 11)

First mark in red the ones which have a **prime** total.

Second mark in blue any remaining that total **less than six**.

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$$\frac{1+2+4+6+2+3}{36} = \frac{18}{36} = \frac{1}{2}$$

(vi)

Possible totals are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 or 12.

Totals that are twice any of these are: 4, 6, 8, 10, 12.

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$$\frac{3+5+5+3+1}{36} = \frac{17}{36}$$

(vii)

Total of ten or less i.e. all except (5,6), (6,5) and (6,6):

$$\frac{33}{36} = \frac{11}{12}$$

12.

$S = \{(H, H, H), (H, H, T), (H, T, H), (H, T, T),$
 $(T, H, H), (T, T, H), (T, T, H), (T, T, T)\}$

(i) $\#S = 8$

(ii) $P(3T) = \frac{1}{8}$

(iii) $P(\text{at least one head}) = P(\text{not TTT}) = 1 - P(\text{TTT}) = 1 - \frac{1}{8} = \frac{7}{8}$

(iv)

$$P(2 \text{ or more H}) = \frac{\#\{(H, H, H), (H, H, T), (H, T, H), (T, H, H)\}}{8}$$

$$= \frac{4}{8} = \frac{1}{2}$$

14. (i)

red	blue	green
1-10	11-16	17-20

$$P(\text{green}) = \frac{4}{20} = \frac{1}{5}$$

$$(ii) \quad P(\text{not blue}) = \frac{14}{20} = \frac{7}{10}$$

$$(iii) \quad P(\text{red or even}) = \frac{\#\{1,2,3,4,5,6,7,8,9,10,12,14,16,18,20\}}{20} = \frac{15}{20} = \frac{3}{4}$$

$$(iv) \quad P(\text{green or odd}) = \frac{\#\{1,3,5,7,9,11,13,15,17,18,19,20\}}{20} = \frac{12}{20} = \frac{3}{5}$$

$$(v) \quad P(\text{red and even}) = \frac{\#\{2,4,6,8,10\}}{20} = \frac{5}{20} = \frac{1}{4}$$

15.

yellow	green	blue
11	13	n

$$P(\text{blue}) = \frac{1}{4}$$

$$\Rightarrow \frac{n}{n+11+13} = \frac{n}{n+24} = \frac{1}{4}$$

$$\Rightarrow 4n = 1 \times (n+24) = n+24$$

$$\Rightarrow 3n = 24 \Rightarrow n = 8$$