7. There are 4! = 24 altogether.

(i) To be even the number must end in an even digit, here four.
So there are only three other numbers to be arranged. $3! = 6$

(ii) Take away all the even possibilities and the rest are odd,
i.e. $4! - 3! = 24 - 6 = 18$.

(iii) To be over 4000, the first digit must be either 4 or 5.
Thus the answer is $2 \times 3 \times 2 \times 1 = 12$

8. (i) LEAVING has 7 letters. Answer: 7! = 5040

(ii) $^7P_5 = \dfrac{7!}{(7-5)!} = 2520$

(iii) $^7P_2 = \dfrac{7!}{(7-2)!} = 42$

9. (i) MATRIX has 6 letters. Ans. 6! = 720

(ii) $\boxed{\text{T}} \times 5 \times 4 \times 3 \times 2 \times 1 = 120$

(iii) $\boxed{\text{T}} \times 4 \times 3 \times 2 \times \boxed{\text{M}} = 24$

(iv) There are two vowels, $2 \times 5 \times 4 \times 3 \times 2 \times 1 = 240$

(v) Of the 720 possibilities, 240 begin with a vowel.
Thus, $720 - 240 = 480$ do <u>not</u> begin with a vowel.

10. (i) MONSTER has 7 letters, thus 7! = 5040.

(ii) $2 \times 6! = 1440$

(iii) $5 \times 6! = 3600$

(iv) Consider OE as a single object. This gives 6! possibilities.
So does EO. Thus, $2 \times 6! = 1440$

(v) There are 5040 possibilities in total. 1440 of these have
the vowels together. Thus, $5040 - 1440 = 3600$ have them apart.

11. (i) DUBLINER has 8 letters, thus 8! = 40320.

(ii) $\boxed{\text{B}} \times 6 \times 5 \times 4 \times 3 \times 2 \times \boxed{\text{L}} = 120$

(iii) Consider UIE as one object. This, with the five other letters,
gives 6! = 24 possibilities.
But there are 3! ways the vowels can be together.
Answer: $3! \times 6! = 4320$

(iv) ER together, with the six other letters, gives 7! possibilities.
But there are 2 ways E and R can be together.
Answer: $2 \times 7! = 10080$

(v) There are 40320 possibilities in total.
10080 of these have E and R together.
Thus, $40320 - 10080 = 30240$ have them apart.

12. (i) TRIANGLES has 9 letters, thus 9! = 362880.

(ii) Treat the three vowels as one object, this gives
seven objects, including the other six letters.
There are 7! ways to arrange these.
There are 3! ways to arrange the three vowels.
Answer: $3! \times 7! = 30240$

(iii) Treat the 6 consonants as one object, this gives
4 objects, including the 3 other letters.
There are 4! ways to arrange these.
There are 6! ways to arrange the 6 consonants.
Answer: $4! \times 6! = 17280$.

(iv) Treat GR as one object. This gives 8 objects, including the other 7 letters.
Answer: 7! = 5040.

(v) There are six consonants. This gives 6 choices for first place and 5 choices for last place. There are 7 choices for the other places (three vowels and the remaining four consonants.)
Answer: $6 \times 7! \times 5 = 151200$.

13. (a)

(i) $\frac{10!}{9!} = 10$ **(ii)** $\frac{11!}{7!4!} = 330$

(iii) $\frac{23!}{22!} = 23$ **(iv)** $\frac{13!}{11!} = 13 \times 12 = 156$

(b)

(i) $\frac{n!}{(n-1)!} = n$ **(ii)** $\frac{(n+1)!}{n!} = n+1$

(iii)

$$n!\left[\frac{1}{(n-1)!} - \frac{1}{n!}\right] = n!\left[\frac{n}{n(n-1)!} - \frac{1}{n!}\right]$$
$$= n!\left[\frac{n}{n!} - \frac{1}{n!}\right]$$
$$= n-1$$

(c) (i) four digits, must start with 3, 5 or 7.
Answer: $3 \times 3 \times 2 \times 1 = 18$.

(ii) $3 \times 4 \times 4 \times 4 = 192$.

14. Numbers between 500 and 999 inclusive have three digits and must begin with 5, 6, 7, 8 or 9 i.e. five choices.
If they do not contain 0, 1, or 2 this leaves a choice of seven digits for each of the other two places.
Answer: $5 \times 7 \times 7 = 245$.

15. One-digit numbers: 3 possibilities.
Two-digit numbers: $3 \times 2 = 6$ possibilities.
Three-digit numbers: $3 \times 2 \times 1 = 6$ possibilities.
Answer: $3 + 6 + 6 = 15$.