**3. (i)**

$$\sin(225^\circ) = \sin(180^\circ + 45^\circ) = -\sin(45^\circ) = -\frac{1}{\sqrt{2}}$$

**(ii)**

$$\tan(300^\circ) = \tan(360^\circ - 60^\circ) = -\tan(60^\circ) = -\sqrt{3}$$

**(iii)**

$$\cos(330^\circ) = \cos(360^\circ - 30^\circ) = \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

**(iv)**

$$\sin(120^\circ) = \sin(180^\circ - 60^\circ) = \sin(60^\circ) = \frac{\sqrt{3}}{2}$$

**(v)**

$$\cos(150^\circ) = \cos(180^\circ - 30^\circ) = -\cos(30^\circ) = -\frac{\sqrt{3}}{2}$$

$$\sin(150^\circ) = \sin(180^\circ - 30^\circ) = \sin(30^\circ) = \frac{1}{2}$$

$$\cos(150^\circ) + \sin(150^\circ) = -\frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{1-\sqrt{3}}{2}$$

**(vi)**

$$\tan(30^\circ) + \tan(60^\circ) = \frac{1}{\sqrt{3}} + \sqrt{3} = \frac{1}{\sqrt{3}} + \frac{\sqrt{3}\sqrt{3}}{\sqrt{3}} = \frac{1+3}{\sqrt{3}} = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

**(vii)**

$$\cos(45^\circ) + \sin(45^\circ) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{1+1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

**(viii)**

$$\sin\left(\frac{7\pi}{4}\right) = \sin\left(\frac{7\times180^\circ}{4}\right) = \sin\left(315^\circ\right)$$

$$\sin(315^\circ) = \sin(360^\circ - 45^\circ) = -\sin(45^\circ) = -\frac{1}{\sqrt{2}}$$

**(ix)**

$$\tan\left(\frac{4\pi}{3}\right) = \tan\left(\frac{4\times180^\circ}{3}\right) = \tan\left(240^\circ\right)$$

$$\tan(240^\circ) = \tan(180^\circ + 60^\circ) = \tan(60^\circ) = \sqrt{3}$$

**(x)**

$$\cos\left(\frac{11\pi}{3}\right) = \cos\left(\frac{11\times180^\circ}{3}\right) = \cos(330^\circ)$$

$$\cos(330^\circ) = \cos(360^\circ - 30^\circ) = \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\sin(330^\circ) = \sin(360^\circ - 30^\circ) = -\sin(30^\circ) = -\frac{1}{2}$$

$$\cos(330^\circ) + \sin(330^\circ) = \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2}$$

**(xi)**

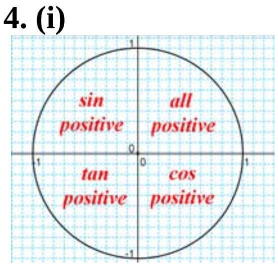
$$\sec\left(\frac{\pi}{3}\right) = \sec\left(\frac{180^\circ}{3}\right) = \sec(60^\circ) = \frac{1}{\cos(60^\circ)} = \frac{1}{1/2} = 2$$

**(xii)**

$$\cot\left(\frac{5\pi}{6}\right) = \cot\left(\frac{5\times 180^\circ}{6}\right) = \cot(150^\circ) = \frac{1}{\tan(150^\circ)}$$

$$= \frac{1}{\tan(180^\circ - 30^\circ)} = -\frac{1}{\tan(30^\circ)} = -\frac{1}{1/\sqrt{3}} = -\sqrt{3}$$

**4. (i)**

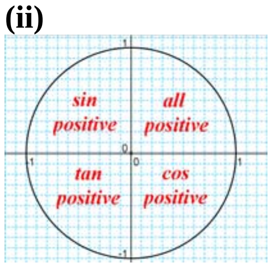



1st and 4th quadrants for positive cos $\Rightarrow$

$\cos(\theta) = 0\cdot 5 = \cos(60^\circ) = \cos(360^\circ - 60^\circ)$

$\Rightarrow \theta = 60^\circ$ or $\theta = 300^\circ$

**(ii)**



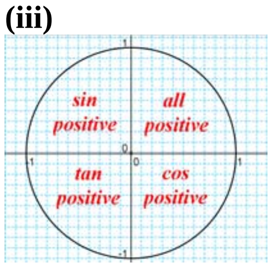

$\tan\left(45^\circ\right) = 1$ (tables)

Note: negative tan $\Rightarrow$ 2nd or 4th quadrant

$\tan\theta = -1 = \tan\left(180^\circ - 45^\circ\right) = \tan(360^\circ - 45^\circ)$

$\Rightarrow \theta = 135^\circ$ or $\theta = 315^\circ$

**(iii)**




$\sin(60^\circ) = \frac{\sqrt{3}}{2}$

3rd and 4th quadrants for negative sin $\Rightarrow$

$\sin(\theta) = -\frac{\sqrt{3}}{2} = \sin(180^\circ + 60^\circ) = \sin(360^\circ - 60^\circ)$

$\Rightarrow \theta = 240^\circ$ or $\theta = 300^\circ$

**(iv)**

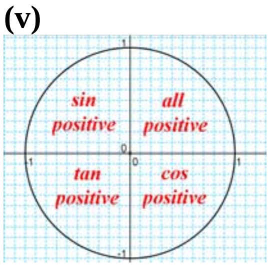

$\cos\left(30^\circ\right) = \dfrac{\sqrt{3}}{2}$ (tables)

Note: negative cos $\Rightarrow$ 2nd or 3rd quadrant

$\cos\theta = -\dfrac{\sqrt{3}}{2} = \cos\left(180^\circ - 30^\circ\right) = \cos(180^\circ + 30^\circ)$

$\Rightarrow \theta = 150^\circ$ or $\theta = 210^\circ$

**(v)**

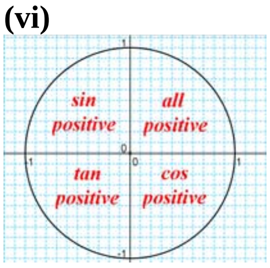



1st and 3rd quadrants for positive tan $\Rightarrow$

$\tan(\theta) = \sqrt{3} = \tan(60^\circ) = \tan(180^\circ + 60^\circ)$

$\Rightarrow \theta = 60^\circ$ or $\theta = 240^\circ$

**(vi)**



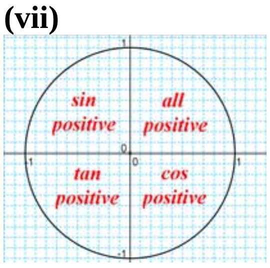

$\sin\left(30^\circ\right) = \dfrac{1}{2}$ (tables)

Note: negative sin $\Rightarrow$ 3rd or 4th quadrant

$\sin\theta = -\dfrac{1}{2} = \sin\left(180^\circ + 30^\circ\right) = \sin(360^\circ - 30^\circ)$

$\Rightarrow \theta = 210^\circ$ or $\theta = 330^\circ$

**(vii)**



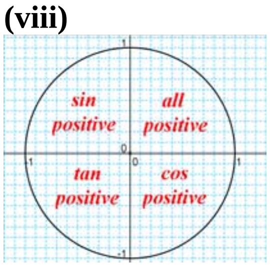

$$\cot\theta = \sqrt{3} = \frac{1}{\tan\theta} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

1st and 3rd quadrants for positive tan $\Rightarrow$

$$\tan(\theta) = \frac{1}{\sqrt{3}} = \tan(30^\circ) = \tan(180^\circ + 30^\circ)$$

$$\Rightarrow \theta = 30^\circ \text{ or } \theta = 210^\circ$$

**(viii)**




$$\text{cosec}\,\theta = -2 = \frac{1}{\sin\theta} \Rightarrow \sin\theta = -\frac{1}{2}$$

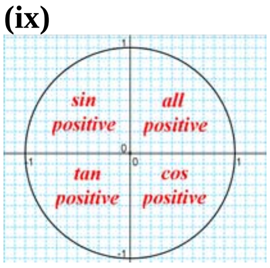
$$\sin(30^\circ) = \frac{1}{2}$$

3rd and 4th quadrants for negative sin $\Rightarrow$

$$\sin(\theta) = -\frac{1}{2} = \sin(180^\circ + 30^\circ) = \sin(360^\circ - 30^\circ)$$

$$\Rightarrow \theta = 210^\circ \text{ or } \theta = 330^\circ$$

**(ix)**




$$\sec\theta = -\sqrt{2} = \frac{1}{\cos\theta} \Rightarrow \cos\theta = -\frac{1}{\sqrt{2}}$$

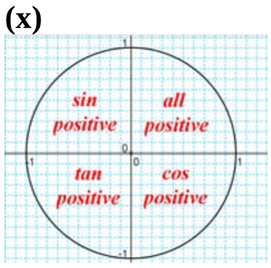
$$\cos(45^\circ) = \frac{1}{\sqrt{2}}$$

2nd and 3rd quadrants for negative cos $\Rightarrow$

$$\cos(\theta) = -\frac{1}{\sqrt{2}} = \cos(180^\circ - 45^\circ) = \cos(180^\circ + 45^\circ)$$

$$\Rightarrow \theta = 135^\circ \text{ or } \theta = 225^\circ$$

**(x)**




$\cot\theta = -1 = \dfrac{1}{\tan\theta} \Rightarrow \tan\theta = -1$

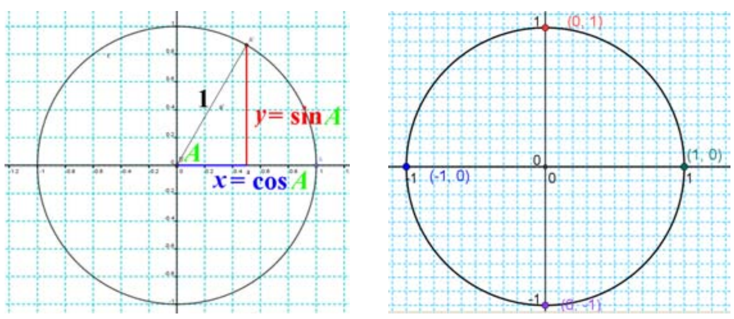
Now, $\tan(45^\circ) = 1$.

2nd and 4th quadrants for negative tan $\Rightarrow$

$\tan(\theta) = -1 = \tan(180^\circ + 45^\circ) = \tan(360^\circ - 45^\circ)$

$\Rightarrow \theta = 225^\circ$ or $\theta = 315^\circ$

**5. (a)**




We wish to solve

$\sin\theta = 0,\ 0 \le \theta \le 2\pi$ i.e. $\sin\theta = 0,\ 0^\circ \le \theta \le 360^\circ$.

Now, the *y*-coordinate of a point on the unit circle is equal to the sine of the angle.

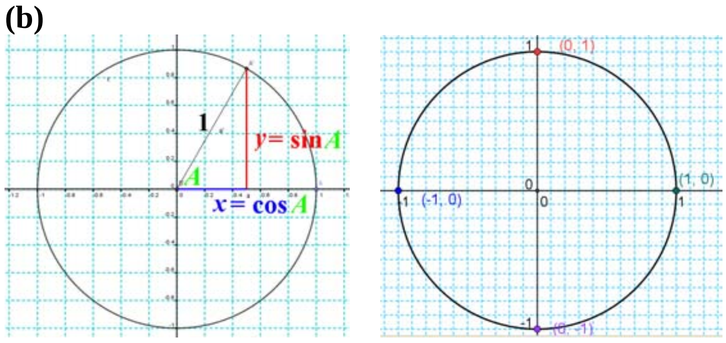
Here, the *y*-coordinate is zero at (1, 0) and (−1, 0).

For (1, 0) the angles are $0^\circ$ and $360^\circ$.

For (−1, 0) the angle is $180^\circ$.

In radians, $\theta = 0,\ \pi$ or $2\pi$.

**(b)**




$\cos\alpha + 1 = 0 \Rightarrow \cos\alpha = -1$.

From the unit circle, the *x*-coordinate is −1 at the angle $180^\circ$.

Thus, in radians, $0 \le \alpha \le 2\pi \Rightarrow \alpha = \pi$.

**6. (i)**

$5\sin A = 2 \Rightarrow \sin A = \frac{2}{5} \Rightarrow A = \sin^{-1}\left(\frac{2}{5}\right) \Rightarrow A = 23\cdot 6^\circ$

Positive sin $\Rightarrow$ 1st or 2nd quadrant (Alfie stole two chickens.)

$\Rightarrow A = 180^\circ - 23\cdot 6^\circ = 156\cdot 4^\circ$.

Answer: $A = 23\cdot 6^\circ$ or $A = 156\cdot 4^\circ$.

**(ii)**

$7\cos A + 3 = 0 \Rightarrow \cos A = -\frac{3}{7}$.

Let's be positive: $\cos A = \frac{3}{7} \Rightarrow A = \cos^{-1}\left(\frac{3}{7}\right) = 64\cdot 6^\circ$.

Negative cos $\Rightarrow$ 2nd or 3rd quadrant (Alfie stole two chickens.)

$\Rightarrow A = 180^\circ - 64\cdot 6^\circ = 115\cdot 4^\circ$ or $A = 180^\circ + 64\cdot 6^\circ = 244\cdot 6^\circ$.

**(iii)**

$2\tan A + 7 = 0 \Rightarrow \tan A = -\frac{7}{2}$.

Let's be positive: $\tan A = \frac{7}{2} \Rightarrow A = \tan^{-1}\left(\frac{7}{2}\right) = 74\cdot 1^\circ$.

Negative tan $\Rightarrow$ 2nd or 4th quadrant (Alfie stole two chickens.)

$\Rightarrow A = 180^\circ - 74\cdot 1^\circ = 105\cdot 9^\circ$ or $A = 360^\circ - 74\cdot 1^\circ = 285\cdot 9^\circ$.

**(iv)**

$3\sec A = 5 \Rightarrow \sec A = \frac{5}{3} = \frac{1}{\cos A}$

$\Rightarrow \frac{3}{5} = \cos A \Rightarrow A = \cos^{-1}\left(\frac{3}{5}\right) \Rightarrow A = 53\cdot 1^\circ$

Positive cos $\Rightarrow$ 1st or 4th quadrant (Alfie stole two chickens.)

$\Rightarrow A = 360^\circ - 53\cdot 1^\circ = 306\cdot 9^\circ$.

Answer: $A = 53\cdot 1^\circ$ or $A = 306\cdot 9^\circ$.

**(v)**

$2\text{cosec } A - 11 = 0 \Rightarrow \text{cosec } A = \frac{11}{2} = \frac{1}{\sin A}$

$\Rightarrow \sin A = \frac{2}{11} \Rightarrow A = \sin^{-1}\left(\frac{2}{11}\right) = 10\cdot 5^\circ$

Positive sin $\Rightarrow$ 1st or 2nd quadrant (Alfie stole two chickens.)

$\Rightarrow A = 180^\circ - 10\cdot 5^\circ = 169\cdot 5^\circ$.

Answer: $A = 10\cdot 5^\circ$ or $A = 169\cdot 5^\circ$.

**(vi)**

$17\cot A+23=1\Rightarrow 17\cot A=1-23=-22\Rightarrow \cot A=-\dfrac{22}{17}$

$\Rightarrow \dfrac{1}{\tan A}=-\dfrac{22}{17}\Rightarrow \tan A=-\dfrac{17}{22}$

Let's be positive: $\tan A=\dfrac{17}{22}\Rightarrow A=\tan^{-1}\left(\dfrac{17}{22}\right)=37\cdot 7^\circ$.

Negative tan $\Rightarrow$ 2nd or 4th quadrant (Alfie stole two chickens.)

$\Rightarrow A=180^\circ-37\cdot 7^\circ=142\cdot 3^\circ$ or $A=360^\circ-37\cdot 7^\circ=322\cdot 3^\circ$.

**7. (i)**

$\alpha=30^\circ\Rightarrow \cos 2\alpha=\cos 60^\circ=\dfrac{1}{2}$

$\cos\alpha+\cos\alpha=\dfrac{\sqrt{3}}{2}+\dfrac{\sqrt{3}}{2}=\sqrt{3}\neq\cos 2\alpha$

**(ii)**

$\sin 3\alpha=\sin 90^\circ=1$

$3\sin\alpha=3\sin 30^\circ=1\cdot 5\neq\sin 3\alpha$

**(iii)**

$\tan\left(\dfrac{1}{2}\alpha\right)=\tan 15^\circ=2-\sqrt{3}$ (calculator)

$\dfrac{1}{2}\tan\alpha=\dfrac{1}{2}\tan 30^\circ=\dfrac{\sqrt{3}}{6}\neq\tan\left(\dfrac{1}{2}\alpha\right)$

**8. (i)**

$\sin(x+y)=\sin(10^\circ+23^\circ)=\sin 33^\circ=0\cdot 5446$

$\sin x+\sin y=\sin 10^\circ+\sin 23^\circ=0\cdot 5644\neq\sin(x+y)$

**(ii)**

$\cos(y-x)=\cos(23^\circ-10^\circ)=\cos 13^\circ=0\cdot 9744$

$\cos y-\cos x=\cos 23^\circ-\cos 10^\circ=-0\cdot 0643\neq\cos(y-x)$

**(iii)**

$\tan(x+y)=\tan(10^\circ+23^\circ)=\tan 33^\circ=0\cdot 6494$

$\tan x+\tan y=\tan 10^\circ+\tan 23^\circ=0\cdot 6008\neq\tan(x+y)$

**10.**


Let's be positive: $\sin(60^\circ) = \dfrac{\sqrt{3}}{2}$ (tables)

3rd and 4th quadrants for negative $\sin \Rightarrow$

$\sin(\theta) = -\dfrac{\sqrt{3}}{2} = \sin(180^\circ + 60^\circ) = \sin(360^\circ - 60^\circ)$

$\Rightarrow \theta = 240^\circ$ or $\theta = 300^\circ$

Now, $\cos 240^\circ = \dfrac{1}{2}$ but $\cos 300^\circ = \dfrac{1}{2}$ (calculator).

Thus, $\theta = 240^\circ = 240^\circ \times \dfrac{\pi}{180^\circ} = \dfrac{4\pi}{3}$ radians.