1.

$(x+a)^2+b\equiv x^2+8x+11$

$\Rightarrow x^2+2ax+a^2+b\equiv x^2+8x+11$

Now $x^2+2ax+a^2+b\equiv x^2+8x+11$

$\Rightarrow 2a=8\Rightarrow a=4$

and $a^2+b=11\Rightarrow 4^2+b=11\Rightarrow b=11-16=-5$

2.

$(x+p)^2-q\equiv x^2-4x-10$

$\Rightarrow x^2+2px+p^2-q\equiv x^2-4x-10$

Now $x^2+2px+p^2-q\equiv x^2-4x-10$

$\Rightarrow 2p=-4\Rightarrow p=-2$

and $p^2-q=-10\Rightarrow(-2)^2-q=-10\Rightarrow 4+10=14=q$

3.

$p(x+8)^2+k\equiv 3x^2+qx+200$

$\Rightarrow p\left(x^2+16x+64\right)+k\equiv 3x^2+qx+200$

$\Rightarrow px^2+16px+64p+k\equiv 3x^2+qx+200$

Thus $p=3$

$q=16p=16\times 3=48$

$64p+k=200\Rightarrow k=200-64\times 3=8$

4.

$q(x-p)^2+r\equiv 5x^2-20x+11$

$\Rightarrow q\left(x^2-2px+p^2\right)+r\equiv 5x^2-20x+11$

$\Rightarrow qx^2-2pqx+qp^2+r\equiv 5x^2-20x+11$

Thus $q=5$

$-2pq=-20\Rightarrow pq=10\Rightarrow 5p=10\Rightarrow p=2$

$qp^2+r=11\Rightarrow r=11-5\times 2^2=-9$

5.

$2x^2+7x+10\equiv p(x+q)^2+r$

$\Rightarrow 2x^2+7x+10\equiv p\left(x^2+2qx+q^2\right)+r$

$\Rightarrow 2x^2+7x+10\equiv px^2+2pqx+pq^2+r$

$\Rightarrow p=2$

$\Rightarrow 2pq=7\Rightarrow 4q=7\Rightarrow q=\frac{7}{4}$

$\Rightarrow pq^2+r=10\Rightarrow 2\left(\frac{7}{4}\right)^2+r=10\Rightarrow r=10-2\left(\frac{7}{4}\right)^2=\frac{31}{8}$

6.

$(x+a)^3 \equiv x^3 + px^2 + 3qx + 1$

$\Rightarrow x^3 + 3ax^2 + 3a^2x + a^3 \equiv x^3 + px^2 + 3qx + 1$

$\Rightarrow a^3 = 1 \Rightarrow a = \sqrt[3]{1} = 1$

$\Rightarrow 3q = 3a^2 \Rightarrow q = a^2 = 1^2 = 1$

$\Rightarrow p = 3a = 3 \times 1 = 3$

7.

$$\frac{a}{n+1} + \frac{b}{n+2} \equiv \frac{(n+2)a + (n+1)b}{(n+1)(n+2)} \equiv \frac{8n+11}{(n+1)(n+2)}$$

$\Rightarrow (n+2)a + (n+1)b \equiv 8n + 11$

$\Rightarrow an + 2a + bn + b \equiv 8n + 11$

$\Rightarrow (a+b)n + 2a + b \equiv 8n + 11$

A $\quad a + b = 8$

B $\quad 2a + b = 11$

-2A $\quad \underline{-2a - 2b = -16}$

Add $\quad -b = -5 \Rightarrow b = 5$

Substitute in A: $\quad a + (5) = 8 \Rightarrow a = 3$

8.

$$\frac{a}{n} + \frac{b}{n+2} \equiv \frac{(n+2)a + nb}{n(n+2)} \equiv \frac{6}{n(n+2)}$$

$\Rightarrow (n+2)a + nb \equiv 6$

$\Rightarrow an + 2a + nb \equiv 6$

$\Rightarrow (a+b)n + 2a \equiv (0)n + 6$

$2a = 6 \Rightarrow a = 3$

$a + b = 0 \Rightarrow 3 + b = 0 \Rightarrow b = -3$

9.

$p(x+a)^2 + b \equiv 5x^2 - 30x$

$\Rightarrow p(x^2 + 2ax + a^2) + b \equiv 5x^2 - 30x$

$\Rightarrow px^2 + 2apx + pa^2 + b \equiv 5x^2 - 30x$

Now $px^2 + 2apx + pa^2 + b \equiv 5x^2 - 30x + 0$

$\Rightarrow p = 5$

$\Rightarrow 2ap = -30 \Rightarrow ap = -15 \Rightarrow 5a = -15 \Rightarrow a = -3$

and $pa^2 + b = 0 \Rightarrow 5 \times (-3)^2 + b = 0 \Rightarrow b = -45$

10.

$$(x+a)^3+bx^2+cx \equiv x^3-8$$

$$\Rightarrow x^3+3ax^2+3a^2x+a^3+bx^2+cx \equiv x^3-8$$

$$\Rightarrow x^3+3ax^2+3a^2x+a^3+bx^2+cx \equiv x^3+(0)x^2+(0)x-8$$

$$\Rightarrow a^3=-8 \Rightarrow a=\sqrt[3]{-8}=-2$$

$$\Rightarrow 3a+b=0 \Rightarrow 3(-2)+b=0=b=6$$

$$\Rightarrow 3a^2+c=0 \Rightarrow c=0-3(-2)^2=12$$

11.

$$\begin{array}{r|l} & x+(a-p) \\ \hline x^2+px+q & x^3+ax^2+bx+c \\ & \underline{x^3+px^2+qx} \\ & (a-p)x^2+(b-q)x+c \\ & \underline{(a-p)x^2+p(a-p)x+q(a-p)} \\ & 0 \end{array}$$

Remainder $= 0$

$$\Rightarrow (b-q)x+c \equiv p(a-p)x+q(a-p)$$

$\Rightarrow$ (i) $\quad b-q=p(a-p)$

(ii) $\quad c=q(a-p)$

12.

$$\begin{array}{r|l} & 3x+(a+3) \\ \hline x^2-x-2 & 3x^3+ax^2-10x+b \\ & \underline{x^3-3x^2-6x} \\ & (a+3)x^2-4x+b \\ & \underline{(a+3)x^2-(a+3)x-2(a+3)} \\ & 0 \end{array}$$

Remainder $= 0$

$$\Rightarrow -4x+b \equiv -(a+3)x-2(a+3)$$

$$\Rightarrow a+3=4 \Rightarrow a=1$$

$$\Rightarrow b=-2(a+3)=-2(1+3)=-8$$

13.

x^2+px+q is a factor of x^3+ax^2+b

$$
\begin{array}{r|l}
 & x+a-p \\
\hline
x^2+px+q & \cancel{x^3}+ax^2+0x+b \\
 & \underline{\cancel{x^3}+px^2+qx} \\
 & \cancel{(a-p)x^2}-qx+b \\
 & \underline{\cancel{(a-p)x^2}+p(a-p)x+(a-p)q} \\
 & \qquad 0
\end{array}
$$

Remainder $=0 \Rightarrow$

(i) $b=q(a-p)$

(ii) $-q=p(a-p) \Rightarrow q=p(p-a)$

(iii) $qq+bp=qp(p-a)+q(a-p)p=q\cancel{p^2}-\cancel{qpa}+\cancel{qpa}-\cancel{qp^2}=0$