

Oliver Murphy: Discovering Maths 4: EXERCISE 3D

1.

$$2^x + 2^{2-x} = 5$$

$$2^x + \frac{2^2}{2^x} = 5$$

$$m + \frac{4}{m} = 5 \quad \text{Multiply both sides by } m$$

$$m^2 + 4 = 5m \Rightarrow m^2 - 5m + 4 = 0$$

$$(m-1)(m-4) = 0 \Rightarrow m = 1 \text{ or } m = 4$$

$$\Rightarrow 2^x = 1 \Rightarrow x = 0 \text{ or } 2^x = 4 \Rightarrow x = 2$$

2.

$$3^x + 3^{1-x} = 4$$

$$3^x + \frac{3^1}{3^x} = 4$$

$$m + \frac{3}{m} = 4 \quad \text{Multiply both sides by } m$$

$$m^2 + 3 = 4m \Rightarrow m^2 - 4m + 3 = 0$$

$$(m-1)(m-3) = 0 \Rightarrow m = 1 \text{ or } m = 3$$

$$\Rightarrow 3^x = 1 \Rightarrow x = 0 \text{ or } 3^x = 3 \Rightarrow x = 1$$

3.

$$2^x + 2^{-x} = \frac{17}{4}$$

$$2^x + \frac{1}{2^x} = \frac{17}{4}$$

$$m + \frac{1}{m} = \frac{17}{4} \quad \text{Multiply both sides by } 4m$$

$$4m^2 + 4 = 17m \Rightarrow 4m^2 - 17m + 4 = 0$$

$$(4m-1)(m-4) = 0 \Rightarrow m = \frac{1}{4} \text{ or } m = 4$$

$$\Rightarrow 2^x = \frac{1}{4} = 2^{-2} \Rightarrow x = -2 \text{ or } 2^x = 4 = 2^2 \Rightarrow x = 2$$

4.

$$3^{2x} + 3^x = 90$$

$$3^x \cdot 3^x + 3^x = 90$$

$$m^2 + m = 90 \Rightarrow m^2 + m - 90 = 0$$

$$(m-9)(m+10) = 0 \Rightarrow m = 9 \text{ or } m = -10$$

$$\Rightarrow 3^x = 9 \Rightarrow x = 2$$

or $3^x = -10$ no solution.

5.

$$5^{2x} + 2(5^x) = 35$$

$$\Rightarrow 5^x \cdot 5^x + 2(5^x) = 35$$

$$\Rightarrow m^2 + 2m = 35 \Rightarrow m^2 + 2m - 35 = 0$$

$$\Rightarrow (m+7)(m-5) = 0 \Rightarrow m = -7 \text{ or } m = 5$$

$$5^x = -7 \text{ no solution}$$

$$5^x = 5 \Rightarrow x = 1$$

6.

$$2^x + 2^{3-x} = 7$$

$$2^x + \frac{2^3}{2^x} = 7$$

$$m + \frac{8}{m} = 7 \quad \text{Multiply both sides by } m$$

$$m^2 + 8 = 7m \Rightarrow m^2 - 7m + 8 = 0$$

$$(m-8)(m+1) = 0 \Rightarrow m = 8 \text{ or } m = -1$$

$$\Rightarrow 2^x = 8 \Rightarrow x = 3$$

$$\text{or } 2^x = -1 \Rightarrow \text{no solution.}$$

7.

$$3^x - 3^{1-x} + 2 = 0$$

$$3^x - \frac{3^1}{3^x} + 2 = 0$$

$$m - \frac{3}{m} + 2 = 0 \quad \text{Multiply both sides by } m$$

$$m^2 + 2m - 3 = 0$$

$$\Rightarrow (m-1)(m+3) = 0 \Rightarrow m = 1 \text{ or } m = -3$$

$$\Rightarrow 3^x = 1 \Rightarrow x = 0 \text{ or } 3^x = -3 \Rightarrow \text{no solution.}$$

8.

$$3^{x+1} + 3^{1-x} = 10$$

$$3(3^x) + \frac{3^1}{3^x} = 10$$

$$3m + \frac{3}{m} = 10 \quad \text{Multiply both sides by } m$$

$$3m^2 + 3 = 10m \Rightarrow 3m^2 - 10m + 3 = 0$$

$$(3m-1)(m-3) = 0 \Rightarrow m = \frac{1}{3} \text{ or } m = 3$$

$$\Rightarrow 3^x = \frac{1}{3} \Rightarrow x = -1 \text{ or } 3^x = 3 \Rightarrow x = 1$$

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9.

$$2^{x-3} - 2^{6-x} = 2$$

$$\frac{2^x}{2^3} - \frac{2^6}{2^x} = 2$$

$$\frac{m}{8} - \frac{64}{m} = 2 \quad \text{Multiply both sides by } m$$

$$\frac{m^2}{8} - 64 = 2m \Rightarrow m^2 - 16m - 512 = 0$$

$$(m - 32)(m + 16) = 0 \Rightarrow m = 32 \text{ or } m = -16$$

$$\Rightarrow 2^x = 32 \Rightarrow x = 5 \text{ or } 2^x = -16 \Rightarrow \text{no solution.}$$

10.