Oliver Murphy: Discovering Maths 4: EXERCISE 3C

RECALL: $\log_{\text{base}}(\text{bracket}) = \text{power}$

$\text{base}^{\text{power}} = \text{bracket}$

**1(i)** $\log_2(8) = x \Rightarrow 2^x = 8 \Rightarrow x = 3$

**(ii)** $\log_{10}(x) = 2 \Rightarrow 10^2 = x = 100$

**(iii)** $\log_3(81) = x \Rightarrow 3^x = 81 = 3^4 \Rightarrow x = 4$

**(iv)** $\log_5(x) = 3 \Rightarrow 5^3 = x = 125$

**(v)** $\log_2(x) = 10 \Rightarrow 2^{10} = x = 1024$

**(vi)** $\log_x(64) = 3 \Rightarrow x^3 = 64 = 4^3 \Rightarrow x = 4$

**(vii)** $\log_{10}(10) = x \Rightarrow 10^x = 10^1 \Rightarrow x = 1$

**(viii)** $\log_{10}(1) = x \Rightarrow 10^x = 1 = 10^0 \Rightarrow x = 0$

**(ix)** $\log_{25}(x) = \frac{1}{2} \Rightarrow 25^{\frac{1}{2}} = x = 5$

**2(i)** $\log_{10}(3x+1) = 2 \Rightarrow 3x+1 = 10^2 = 100 \Rightarrow x = 33$

**(ii)** $\log_2(x-1) = 3 \Rightarrow x-1 = 2^3 = 8 \Rightarrow x = 9$

**(iii)** $\log_3(5x+2) = 3 \Rightarrow 5x+2 = 3^3 = 27 \Rightarrow x = 5$

**(iv)** $\log_5(8x+1) = 2 \Rightarrow 8x+1 = 5^2 = 25 \Rightarrow x = 3$

**(v)** $\log_2(x+1) - \log_2(x-1) = 1$

$\log_2\left(\frac{x+1}{x-1}\right) = 1 \Rightarrow \frac{x+1}{x-1} = 2^1$

$\Rightarrow x+1 = 2(x-1) = 2x-2 \Rightarrow 2+1 = 2x-x \Rightarrow 3 = x$

**(vi)** $\log_3(10x+7) - \log_3(x+1) = 2$

$\log_3\left(\frac{10x+7}{x+1}\right) = 2 \Rightarrow \frac{10x+7}{x+1} = 3^2$

$\Rightarrow 10x+7 = 9(x+1) = 9x+9$

$\Rightarrow 10x - x = 9-7 \Rightarrow x = 2$

**(vii)** $3\log_2(3) = \log_2(x)$

$\log_2(3^3) = \log_2(x) \Rightarrow 27 = x$

**(viii)**

$\log_5(x+1) = \log_5(7x+1) - 1$

$\Rightarrow 1 = \log_5(7x+1) - \log_5(x+1)$

$\Rightarrow 1 = \log_5\left(\frac{7x+1}{x+1}\right)$

$\Rightarrow \frac{7x+1}{x+1} = 5^1$

$\Rightarrow 7x+1 = 5x+5 \Rightarrow 7x-5x = 5-1$

$\Rightarrow 2x = 4 \Rightarrow x = 2$

**(ix)**

$\log_{10}(9x+1)=2+\log_{10}(x-10)$

$\Rightarrow \log_{10}(9x+1)-\log_{10}(x-10)=2$

$\Rightarrow \log_{10}\left(\frac{9x+1}{x-10}\right)=2 \Rightarrow \frac{9x+1}{x-10}=10^2$

$\Rightarrow 9x+1=100x-1000 \Rightarrow 1000+1=100x-9x$

$1001=91x \Rightarrow x=1001\div 91=11$

**x**

$\log_2 3+\log_2(x+1)=\log_2(x+11)$

$\Rightarrow \log_2(3x+3)=\log_2(x+11)$

$\Rightarrow \log_2(3x+3)-\log_2(x+11)=0$

$\Rightarrow \log_2\left(\frac{3x+3}{x+11}\right)=0$

$\Rightarrow \frac{3x+3}{x+11}=2^0=1$

$\Rightarrow 3x+3=x+11 \Rightarrow 3x-x=11-3$

$\Rightarrow 2x=8 \Rightarrow x=4$

**3(i)**

Let $\log_a m=p$ and $\log_a n=q$

$\Rightarrow m=a^p$ and $n=a^q$

$\log_a\left(\frac{m}{n}\right)=\log_a\left(\frac{a^p}{a^q}\right)=\log_a\left(a^{p-q}\right)$

$=p-q=\log_a m-\log_a n$

**(ii)**

$\log_7 35-\log_7 5=\log_7\left(\frac{35}{5}\right)=\log_7 7=1$

**4(i)**

Let $\log_m x=p \Rightarrow x=m^p$

$\log_m\left(x^n\right)=\log_m\left(\left(m^p\right)^n\right)=\log_m\left(m^{pn}\right)=np=n\log_m x$

**(ii)**

$2^n=20 \Rightarrow \log_{10}\left(2^n\right)=\log_{10} 20$

$\Rightarrow n\log_{10} 2=\log_{10} 20$

$\Rightarrow n=\frac{\log_{10} 20}{\log_{10} 2}=4\cdot 32$ (2DP)

**5. (a)**

Let $\log_a b = p$

$\Rightarrow b = a^p$

$\log_{10} b = \log_{10}\left(a^p\right) = p\log_{10} a$

$\Rightarrow \dfrac{\log_{10} b}{\log_{10} a} = p = \log_a b$

**(b) (i)** $\log_{10} 3 = 0\cdot 48$ **(ii)** $\log_{10} 7 = 0\cdot 85$

**(iii)** $\log_3 7 = \dfrac{\log_{10} 7}{\log_{10} 3} = 1\cdot 77$ **(iv)** $\log_7 3 = \dfrac{\log_{10} 3}{\log_{10} 7} = 0\cdot 56$

**6. (a)**

$3^n > 1000000$

$\Rightarrow \log_{10}\left(3^n\right) > \log_{10}\left(1000000\right)$

$\Rightarrow n\log_{10}\left(3\right) > \log_{10}\left(1000000\right)$

$\Rightarrow n > \dfrac{\log_{10}\left(1000000\right)}{\log_{10}\left(3\right)} \Rightarrow n > 10\cdot 975$ i.e. $n = 11$

**(b)**

$2^n < 500000$

$\Rightarrow \log_{10}\left(2^n\right) < \log_{10}\left(500000\right)$

$\Rightarrow n\log_{10}\left(2\right) < \log_{10}\left(500000\right)$

$\Rightarrow n < \dfrac{\log_{10}\left(500000\right)}{\log_{10}\left(2\right)} \Rightarrow n < 18\cdot 932$ i.e. $n = 18$