

1.(i) $36^{\frac{1}{2}} = \sqrt{36} = 6$ **(ii)** $125^{\frac{1}{3}} = \sqrt[3]{125} = 5$
(iii) $16^{\frac{1}{4}} = \sqrt[4]{16} = 4$ **(iv)** $1000^{\frac{1}{3}} = \sqrt[3]{1000} = 10$
(v) $1000^{\frac{2}{3}} = \left(\sqrt[3]{1000}\right)^2 = 10^2 = 100$
(vi) $2^{-5} = \frac{1}{2^5} = \frac{1}{32}$ **(vii)** $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$
(viii) $8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$
(ix) $4^{\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$
(x) $4^{-1} = \frac{1}{4^1} = \frac{1}{4}$

2.(i) $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$ **(ii)** $\left(\frac{1}{4}\right)^{\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$
(iii) $32^{-\frac{3}{5}} = \frac{1}{32^{\frac{3}{5}}} = \frac{1}{\left(\sqrt[5]{32}\right)^3} = \frac{1}{2^3} = \frac{1}{8}$
(iv) $16^{\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}} = \frac{1}{\sqrt[4]{16}} = \frac{1}{2}$
(v) $27^{-\frac{2}{3}} = \frac{1}{27^{\frac{2}{3}}} = \frac{1}{\left(\sqrt[3]{27}\right)^2} = \frac{1}{3^2} = \frac{1}{9}$

3.(i) $2^5 = 32, 5^2 = 25 \Rightarrow 2^5 > 5^2$
(ii) $4^{\frac{1}{2}} = \sqrt{4} = 2, \left(\frac{1}{2}\right)^4 = \frac{1}{16} \Rightarrow 4^{\frac{1}{2}} > \left(\frac{1}{2}\right)^4$
(iii) $2^{-\frac{1}{2}} = \frac{1}{2^{\frac{1}{2}}} = \frac{1}{\sqrt{2}} = 1.414$
 $\left(-\frac{1}{2}\right)^2 = \frac{1}{4} = 0.25 \Rightarrow 2^{-\frac{1}{2}} > \left(-\frac{1}{2}\right)^2$
(iv) $\left(\frac{1}{2}\right)^7 = \frac{1}{2}\left(\frac{1}{2}\right)^6 \Rightarrow \left(\frac{1}{2}\right)^6 > \left(\frac{1}{2}\right)^7$
(v) $7^2 \times 7^3 = 7^5$
 $\left(7^2\right)^3 = 7^{2 \times 3} = 7^6 \Rightarrow \left(7^2\right)^3 > 7^2 \times 7^3$

4.(i) $2^k = 8 = 2^3 \Rightarrow k = 3$
(ii) $4^k = 64 = 4^3 \Rightarrow k = 3$
(iii) $8^k = 64 = 8^2 \Rightarrow k = 2$
(iv) $2^k = 128 = 2^7 \Rightarrow k = 7$

$$(v) 4^k = 2 = 4^{\frac{1}{2}} \Rightarrow k = \frac{1}{2}$$

$$(vi) 25^k = 5 = 25^{\frac{1}{2}} \Rightarrow k = \frac{1}{2}$$

$$(vii) 8^k = 4 \Rightarrow (2^3)^k = 2^2 \Rightarrow 2^{3k} = 2^2 \Rightarrow k = \frac{2}{3}$$

$$(viii) 1000^k = 100$$

$$(10^3)^k = 10^2 \Rightarrow 10^{3k} = 10^2$$

$$3k = 2 \Rightarrow k = \frac{2}{3}$$

$$(ix) 32^k = 16$$

$$(2^5)^k = 2^4 \Rightarrow 2^{5k} = 2^4$$

$$5k = 4 \Rightarrow k = \frac{4}{5}$$

$$(x) 8^k = \frac{1}{2}$$

$$(2^3)^k = 2^{-1} \Rightarrow 2^{3k} = 2^{-1}$$

$$3k = -1 \Rightarrow k = -\frac{1}{3}$$

$$5.(i) a^7 \div a^2 = a^{7-2} = a^5$$

$$(ii) a^7 \times a^2 = a^{7+2} = a^9$$

$$(iii) (a^7)^2 = a^{7 \times 2} = a^{14}$$

$$(iv) \sqrt{a} = a^{\frac{1}{2}}$$

$$(v) \sqrt[3]{a} = a^{\frac{1}{3}}$$

$$(vi) \sqrt{a^7} = (a^7)^{\frac{1}{2}} = a^{7 \times \frac{1}{2}} = a^{\frac{7}{2}}$$

$$(vii) \frac{1}{a^3} = a^{-3}$$

$$(viii) \frac{1}{\sqrt{a}} = \frac{1}{a^{\frac{1}{2}}} = a^{-\frac{1}{2}}$$

$$(ix) (\sqrt{a})^3 = \left(a^{\frac{1}{2}}\right)^3 = a^{\frac{1}{2} \times 3} = a^{\frac{3}{2}}$$

$$(x) \frac{1}{a\sqrt{a}} = \frac{1}{a^1 \times a^{\frac{1}{2}}} = \frac{1}{a^{1+\frac{1}{2}}} = \frac{1}{a^{\frac{3}{2}}} = a^{-\frac{3}{2}}$$

$$6. \sqrt[3]{\sqrt{2}} = \sqrt[3]{(\sqrt{2})} = \left(2^{\frac{1}{2}}\right)^{\frac{1}{3}} = 2^{\frac{1}{2} \times \frac{1}{3}} = 2^{\frac{1}{6}}$$

$$7.(i) 2^{3k} \cdot 2^k = 16 \Rightarrow 2^{4k} = 2^4 \Rightarrow k = 1$$

Oliver Murphy: Discovering Maths 4: EXERCISE 3B

(ii) $2^{2k+1} = 8^2 = (2^3)^2 = 2^6 \Rightarrow 2k+1 = 6 \Rightarrow k = 2.5$

(iii) $16^4 \cdot 8 = 2^k = (2^4)^4 \cdot 2^3 = 2^{16} \cdot 2^3 = 2^{19} \Rightarrow k = 19$

(iv) $5^{2k+1} = 125 = 5^3 \Rightarrow 2k+1 = 3 \Rightarrow k = 1$

(v) $3^{k+1} = 9^{k-1} = (3^2)^{k-1} = 3^{2k-2}$

$$\Rightarrow k+1 = 2k-2 \Rightarrow 2+1 = 2k-k$$

$$\Rightarrow 3 = k$$

(vi) $10^k \cdot 10^{2k} = 1000000$

$$\Rightarrow 10^{3k} = 10^6 \Rightarrow k = 2$$

(vii) $2^{k^2} \cdot 2^k = 64 \Rightarrow 2^{k^2+k} = 2^6$

$$\Rightarrow k^2 + k - 6 = 0 \Rightarrow (k+3)(k-2) = 0$$

$$\Rightarrow k = -3 \text{ or } k = 2$$

(viii) $10^{k^2} \div 10^{2k} = 1000 \Rightarrow 10^{k^2-2k} = 10^3$

$$\Rightarrow k^2 - 2k - 3 = 0 \Rightarrow (k-3)(k+1) = 0$$

$$\Rightarrow k = 3 \text{ or } k = -1$$

(ix) $5^k = \frac{1}{125} = 5^{-3} \Rightarrow k = -3$

(x) $2^k = \frac{1}{4\sqrt{2}} = \frac{1}{2^{\frac{1}{2}}} = 2^{-\frac{1}{2}} \Rightarrow k = -2\frac{1}{2}$

8(i) $(2^9)^2 = 2^{18}$ (ii) $2^9 \times 2^2 = 2^{11}$

(iii) $(2^5 \cdot 2^2)^2 = (2^7)^2 = 2^{14}$

(iv) $(\sqrt{2})^{100} = \left(2^{\frac{1}{2}}\right)^{100} = 2^{50}$

9(i) $2^3 \times 5^3 = (2 \times 5)^3 = 10^3$ TRUE

(ii) $(x^6)^7 = x^{42} \neq x^{13} \therefore$ FALSE

(iii) $(3\sqrt{3})^3 = \left(3^{\frac{1}{2}}\right)^3 = 3^{\frac{3}{2}} = 3^{1.5}$ TRUE

(iv) $(2\sqrt{2})^4 = \left(2^{\frac{1}{2}}\right)^4 = 2^{\frac{1}{2} \times 4} = 2^2 = 4$ TRUE

(v) $2^7 \times 3^7 = (2 \times 3)^7 = 6^7 \neq 5^7 \therefore$ FALSE