

Oliver Murphy: Discovering Maths 4: EXERCISE 3A

1.

$$x^3 + 4x^2 + x - 6 = 0 \quad \text{Factors of } -6: \pm 1, \pm 2, \pm 3, \pm 6.$$

$$(1)^3 + 4(1)^2 + (1) - 6 = 0 \Rightarrow x = 1 \text{ is a root} \Rightarrow x - 1 \text{ is a factor.}$$

$$\begin{array}{r} x^2 + 5x + 6 \\ x-1 \overline{) x^3 + 4x^2 + x - 6} \\ \underline{x^3 - x^2} \phantom{- 6} \\ 5x^2 + x \phantom{- 6} \\ \underline{5x^2 - 5x} \phantom{- 6} \\ 6x - 6 \\ \underline{6x - 6} \\ 0 \end{array}$$

$$x^2 + 5x + 6 = (x + 2)(x + 3) = 0 \Rightarrow x = -2 \text{ or } x = -3$$

$$\text{Ans. } x = 1, x = -2 \text{ or } x = -3$$

2.

$$2x^3 - x^2 - 2x + 1 = 0 \quad \text{Factors of } +1: \pm 1..$$

$$2(1)^3 - (1)^2 - 2(1) + 1 = 0 \Rightarrow x = 1 \text{ is a root} \Rightarrow x - 1 \text{ is a factor.}$$

$$\begin{array}{r} 2x^2 + x - 1 \\ x-1 \overline{) 2x^3 - x^2 - 2x + 1} \\ \underline{2x^3 - 2x^2} \phantom{- 2x + 1} \\ x^2 - 2x \phantom{+ 1} \\ \underline{x^2 - x} \phantom{+ 1} \\ -x + 1 \\ \underline{-x + 1} \\ 0 \end{array}$$

$$2x^2 + x - 1 = (2x - 1)(x + 1) = 0 \Rightarrow x = \frac{1}{2} \text{ or } x = -1$$

$$\text{Ans. } x = 1, x = -1 \text{ or } x = \frac{1}{2}$$

**3.**

$$x^3 + 3x^2 - 4x - 12 = 0 \quad \text{Factors of } +12: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12.$$

$$(-2)^3 + 3(-2)^2 - 4(-2) - 12 = 0 \Rightarrow x = -2 \text{ is a root} \Rightarrow x + 2 \text{ is a factor.}$$

$$\begin{array}{r}
 x^2 + x - 6 \\
 x + 2 \overline{) x^3 + 3x^2 - 4x - 12} \\
 \underline{x^3 + 2x^2} \phantom{- 4x - 12} \\
 x^2 - 4x \phantom{- 12} \\
 \underline{x^2 + 2x} \phantom{- 12} \\
 -6x - 12 \\
 \underline{-6x - 12} \\
 0
 \end{array}$$

$$x^2 + x - 6 = (x + 3)(x - 2) = 0 \Rightarrow x = -3 \text{ or } x = 2$$

$$\text{Ans. } x = -2, x = -3 \text{ or } x = 2$$

**4.**

$$x^3 + x^2 - 5x + 3 = 0 \quad \text{Factors of } +3: \pm 1, \pm 3.$$

$$(1)^3 + (1)^2 - 5(1) + 3 = 0 \Rightarrow x = 1 \text{ is a root} \Rightarrow x - 1 \text{ is a factor.}$$

$$\begin{array}{r}
 x^2 + 2x - 3 \\
 x - 1 \overline{) x^3 + x^2 - 5x + 3} \\
 \underline{x^3 - x^2} \phantom{- 5x + 3} \\
 2x^2 - 5x \phantom{+ 3} \\
 \underline{2x^2 - 2x} \phantom{+ 3} \\
 -3x + 3 \\
 \underline{-3x + 3} \\
 0
 \end{array}$$

$$x^2 + 2x - 3 = (x + 3)(x - 1) = 0 \Rightarrow x = -3 \text{ or } x = 1$$

$$\text{Ans. } x = 1, x = -3 \text{ or } x = 1$$

5.

$$3x^3 - 11x^2 + x + 15 = 0 \text{ Factors of } +15: \pm 1, \pm 3, \pm 5, \pm 15.$$

$$3(-1)^3 - 11(-1)^2 + (-1) + 15 = 0 \Rightarrow x = -1 \text{ is a root} \Rightarrow x + 1 \text{ is a factor.}$$

$$\begin{array}{r} 3x^2 - 14x + 15 \\ x+1 \overline{) 3x^3 - 11x^2 + x + 15} \\ \underline{3x^3 + 3x^2} \phantom{+ x + 15} \\ -14x^2 + x \phantom{+ 15} \\ \underline{-14x^2 - 14x} \phantom{+ 15} \\ 15x + 15 \\ \underline{15x + 15} \\ 0 \end{array}$$

$$3x^2 - 14x + 15 = (3x - 5)(x - 3) = 0 \Rightarrow x = \frac{5}{3} \text{ or } x = 3$$

$$\text{Ans. } x = -1, x = 3 \text{ or } x = \frac{5}{3}$$

6.(i)

$$x^3 + 5x^2 - 4x - 20 = 0 \text{ Factors of } -20: \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20.$$

$$(-2)^3 + 5(-2)^2 - 4(-2) - 20 = 0 \Rightarrow x = -2 \text{ is a root} \Rightarrow x + 2 \text{ is a factor.}$$

$$\begin{array}{r} x^2 + 3x - 10 \\ x+2 \overline{) x^3 + 5x^2 - 4x - 20} \\ \underline{x^3 + 2x^2} \phantom{- 4x - 20} \\ 3x^2 - 4x \phantom{- 20} \\ \underline{3x^2 + 6x} \phantom{- 20} \\ -10x - 20 \\ \underline{-10x - 20} \\ 0 \end{array}$$

$$x^2 + 3x - 10 = (x + 5)(x - 2) = 0$$

$$\text{Ans. } x^3 + 5x^2 - 4x - 20 = (x + 2)(x + 5)(x - 2)$$

(ii)

$$x^3 + 5x^2 - 4x - 20 = (x + 2)(x + 5)(x - 2) = 0$$

$$\Rightarrow x = -2, x = -5, x = 2.$$

**7.**

$x - 2$  is a factor  $\Rightarrow x = 2$  is a root.

$$(2)^3 + 2(2)^2 - 5(2) + k = 0 \Rightarrow 6 + k = 0 \Rightarrow k = -6$$

$$\begin{array}{r} x^2 + 4x + 3 \\ x - 2 \overline{) x^3 + 2x^2 - 5x - 6} \\ \underline{x^3 - 2x^2} \phantom{- 6} \\ 4x^2 - 5x \phantom{- 6} \\ \underline{4x^2 - 8x} \phantom{- 6} \\ 3x - 6 \\ \underline{3x - 6} \\ 0 \end{array}$$

$$x^2 + 4x + 3 = (x + 1)(x + 3)$$

$$\therefore x^3 + 2x^2 - 5x - 6 = (x - 2)(x + 1)(x + 3)$$

**8.**

$x - 3$  is a factor  $\Rightarrow x = 3$  is a root.

$$3(3)^3 - 2(3)^2 + k(3) - 6 = 0 \Rightarrow 57 + 3k = 0 \Rightarrow k = -19$$

$$\begin{array}{r} 3x^2 + 7x + 2 \\ x - 3 \overline{) 3x^3 - 2x^2 - 19x - 6} \\ \underline{3x^3 - 9x^2} \phantom{- 6} \\ 7x^2 - 19x \phantom{- 6} \\ \underline{7x^2 - 21x} \phantom{- 6} \\ 2x - 6 \\ \underline{2x - 6} \\ 0 \end{array}$$

$$3x^2 + 7x + 2 = (3x + 1)(x + 2)$$

$$\therefore 3x^3 - 2x^2 - 19x - 6 = (x - 3)(3x + 1)(x + 2)$$

**9.**

$x + 2$  is a factor  $\Rightarrow x = -2$  is a root.

$$(-2)^3 + t(-2)^2 + 3(-2) - 10 = 0 \Rightarrow -24 + 4t = 0 \Rightarrow t = 6$$

$$\begin{array}{r} x^2 + 4x - 5 \\ x + 2 \overline{) x^3 + 6x^2 + 3x - 10} \\ \underline{x^3 + 2x^2} \phantom{- 10} \\ 4x^2 + 3x \phantom{- 10} \\ \underline{4x^2 + 8x} \phantom{- 10} \\ -5x - 10 \\ \underline{-5x - 10} \\ 0 \end{array}$$

$$x^2 + 4x - 5 = (x + 5)(x - 1) \Rightarrow x = -5 \text{ or } x = 1$$

Ans.  $x = -2$  or  $x = -5$  or  $x = 1$

**10.**

$x - 2$  is a factor  $\Rightarrow x = 2$  is a root.

$$(2)^3 + 7(2)^2 + a(2) + b = 0 \Rightarrow 36 + 2a + b = 0$$

$$\text{A: } 2a + b = -36$$

$x + 2$  is a factor  $\Rightarrow x = -2$  is a root.

$$(-2)^3 + 7(-2)^2 + a(-2) + b = 0 \Rightarrow 20 - 2a + b = 0$$

$$\text{B: } -2a + b = -20$$

$$\text{A + B: } 2b = -56 \Rightarrow b = -28$$

$$\text{Substitute in A: } 2a + (-28) = -36 \Rightarrow 2a = -8 \Rightarrow a = -4$$

$x - 2, x + 2$  are factors  $\Rightarrow (x - 2)(x + 2) = x^2 - 4$  is a factor

$$\begin{array}{r} x + 7 \\ x^2 - 4 \overline{) x^3 + 7x^2 - 4x - 28} \\ \underline{x^3 + 4x^2} \phantom{- 28} \\ 3x^2 - 4x - 28 \\ \underline{3x^2 - 12x} \phantom{- 28} \\ 8x - 28 \\ \underline{8x - 28} \\ 0 \end{array}$$

$$\therefore x^3 + 7x^2 - 4x - 28 = (x - 2)(x + 2)(x + 7)$$

**11.**

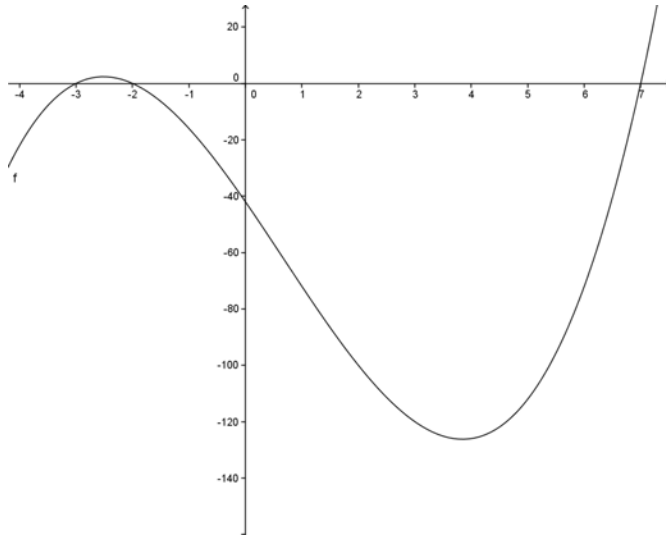
$$x^3 - 2x^2 - 29x - 42 = 0$$

$$(7)^3 - 2(7)^2 - 29(7) - 42 = 0 \Rightarrow x = 7 \text{ is a root} \Rightarrow x - 7 \text{ is a factor.}$$

$$\begin{array}{r} x^2 + 5x + 6 \\ x - 7 \overline{) x^3 - 2x^2 - 29x - 42} \\ \underline{x^3 - 7x^2} \phantom{- 42} \\ 5x^2 - 29x \phantom{- 42} \\ \underline{5x^2 - 35x} \phantom{- 42} \\ 6x - 42 \\ \underline{6x - 42} \\ 0 \end{array}$$

$$x^2 + 5x + 6 = (x + 2)(x + 3) = 0 \Rightarrow x = -2 \text{ or } x = -3$$

$$\text{Ans. } x = 7, x = -2 \text{ or } x = -3$$



**12.**

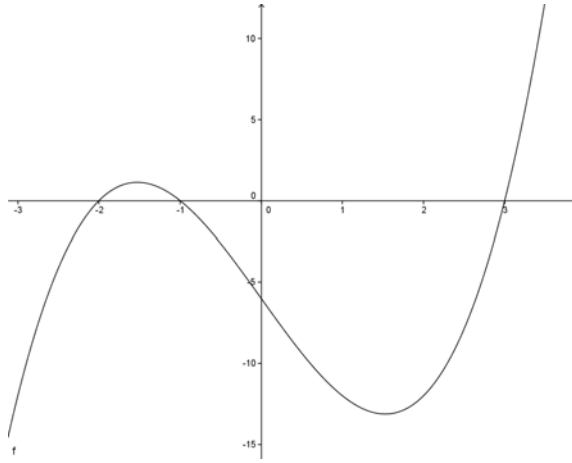
$$x^3 - 7x - 6 = 0$$

$$(-1)^3 - 7(-1) - 6 = 0 \Rightarrow x = -1 \text{ is a root} \Rightarrow x + 1 \text{ is a factor.}$$

$$\begin{array}{r} x^2 - x - 6 \\ x + 1 \overline{) x^3 - 7x - 6} \\ \underline{x^3 + x^2} \phantom{- 6} \\ -x^2 - 7x \phantom{- 6} \\ \underline{-x^2 - 1x} \phantom{- 6} \\ -6x - 6 \\ \underline{-6x - 6} \\ 0 \end{array}$$

$$x^2 - x - 6 = (x + 2)(x - 3) = 0 \Rightarrow x = -2 \text{ or } x = 3$$

$$\text{Ans. } x = -1, x = -2 \text{ or } x = 3$$



13.

$$x = \frac{1}{2} = 0 \Rightarrow x - \frac{1}{2} = 0 \Rightarrow 2x - 1 = 0$$

$$x^2 - 9x + 20$$

$$2x - 1 \overline{) 2x^3 - 19x^2 + 49x - 20}$$

$$\underline{2x^3 - x^2}$$

$$-18x^2 + 49x$$

$$\underline{-18x^2 + 9x}$$

$$40x - 20$$

$$\underline{40x - 20}$$

$$0$$

$$x^2 - 9x + 20 = (x - 4)(x - 5)$$

$$\therefore 2x^3 - 19x^2 + 49x - 20 = (2x - 1)(x - 4)(x - 5)$$

The roots are  $x = \frac{1}{2}, x = 4, x = 5$ .

14.(i)

$$-1, 2, 5 \text{ roots} \Rightarrow (x + 1)(x - 2)(x - 5) = 0$$

$$\Rightarrow (x + 1)(x^2 - 7x + 10) = 0$$

$$\Rightarrow x(x^2 - 7x + 10) + 1(x^2 - 7x + 10) = 0$$

$$\Rightarrow x^3 - 7x^2 + 10x + x^2 - 7x + 10 = 0$$

$$\Rightarrow x^3 - 6x^2 + 3x + 10 = 0$$

14.(i) (Alternative method)

$$p, q, r \text{ roots} \Rightarrow (x - p)(x - q)(x - r) = 0$$

$$\Rightarrow (x - p)(x^2 - qx - rx + qr) = 0$$

$$\Rightarrow x(x^2 - qx - rx + qr) - p(x^2 - qx - rx + qr) = 0$$

$$\Rightarrow x^3 - qx^2 - rx^2 + qrx - px^2 + pqx + prx - pqr = 0$$

$$\Rightarrow x^3 - (p + q + r)x^2 + (pq + qr + pr)x - pqr = 0$$

(ii)

$$3, \frac{1}{2}, -7 \text{ roots} \Rightarrow (x-3)\left(x-\frac{1}{2}\right)(x+7) = 0$$

$$\Rightarrow (x-3)(2x-1)(x+7) = 0$$

$$\Rightarrow (x-3)(2x^2+13x-7) = 0$$

$$\Rightarrow x(2x^2+13x-7) - 3(2x^2+13x-7) = 0$$

$$\Rightarrow 2x^3+13x^2-7x-6x^2-39x+21 = 0$$

$$\Rightarrow 2x^3+7x^2-46x+21 = 0$$

(iii)

$$1, -1, -\frac{1}{3} \text{ roots} \Rightarrow (x-1)(x+1)\left(x+\frac{1}{3}\right) = 0$$

$$\Rightarrow (x-1)(x+1)(3x+1) = 0$$

$$\Rightarrow (x-1)(3x^2+4x+1) = 0$$

$$\Rightarrow x(3x^2+4x+1) - 1(3x^2+4x+1) = 0$$

$$\Rightarrow 3x^3+4x^2+x-3x^2-4x-1 = 0$$

$$\Rightarrow 3x^3+x^2-3x-1 = 0$$

(iv)

$$0, 5, -2 \text{ roots} \Rightarrow x(x-5)(x+2) = 0$$

$$\Rightarrow x(x^2-3x-10) = 0 \Rightarrow x^3-3x^2-10x = 0$$

(v)

$$1, 1, 1 \text{ roots} \Rightarrow (x-1)(x-1)(x-1) = 0$$

$$\Rightarrow (x-1)(x^2-2x+1) = 0$$

$$\Rightarrow x(x^2-2x+1) - 1(x^2-2x+1) = 0$$

$$\Rightarrow x^3-2x^2+x-x^2+2x-1 = 0$$

$$\Rightarrow x^3-3x^2+3x-1 = 0$$

**15.(i)**

$$x^3 + ax^2 + bx + c \text{ has root } x = -3$$

$$\Rightarrow (-3)^3 + a(-3)^2 + b(-3) + c = 0 \Rightarrow -27 + 9a - 3b + c = 0$$

$$\text{A: } 9a - 3b + c = 27$$

$$x = -1 \text{ a root } \Rightarrow (-1)^3 + a(-1)^2 + b(-1) + c = 0$$

$$\text{B: } a - b + c = 1$$

$$x = 3 \text{ a root } \Rightarrow (3)^3 + a(3)^2 + b(3) + c = 0$$

$$\text{C: } 9a + 3b + c = -27$$

$$\text{C} - \text{A: } 6b = -54 \Rightarrow b = -9$$

$$\text{A} - \text{B: } 8a - 2b = 26 \Rightarrow 8a - 2(-9) = 26 \Rightarrow 8a = 26 - 18 = 8 \Rightarrow a = 1$$

$$\text{B: } (1) - (-9) + c = 1 \Rightarrow 10 + c = 1 \Rightarrow c = -9$$

$$\text{Ans: } a = 1, b = -9, c = -9.$$

**15.(i) Alternative method.**

$$-3, -1, 3 \text{ roots } \Rightarrow y = (x+3)(x+1)(x-3)$$

$$= (x+3)(x^2 - 2x - 3)$$

$$= x(x^2 - 2x - 3) + 3(x^2 - 2x - 3)$$

$$= x^3 - 2x^2 - 3x + 3x^2 - 6x - 9$$

$$= x^3 + x^2 - 9x - 9$$

$$= x^3 + ax^2 + bx + c$$

$$\text{Ans: } a = 1, b = -9, c = -9.$$

**(ii)**

$$-1, 4, 4 \text{ roots } \Rightarrow y = (x+1)(x-4)(x-4)$$

$$= (x+1)(x^2 - 8x + 16)$$

$$= x(x^2 - 8x + 16) + 1(x^2 - 8x + 16)$$

$$= x^3 - 8x^2 + 16x + x^2 - 8x + 16$$

$$= x^3 - 7x^2 + 8x + 16$$

$$= x^3 + ax^2 + bx + c$$

$$\text{Ans: } a = -7, b = 8, c = 16.$$

**(iii)**

$$-1, -1, 4 \text{ roots } \Rightarrow y = (x+1)(x+1)(x-4)$$

$$= (x+1)(x^2 - 3x - 4)$$

$$= x(x^2 - 3x - 4) + 1(x^2 - 3x - 4)$$

$$= x^3 - 3x^2 - 4x + x^2 - 3x - 4$$

$$= x^3 - 2x^2 - 7x - 4$$

$$= x^3 + ax^2 + bx + c$$

$$\text{Ans: } a = -2, b = -7, c = -4.$$

(iv)

$$\begin{aligned}
 -2, 1, 1 \text{ roots} &\Rightarrow y = (x+2)(x-1)(x-1) \\
 &= (x+2)(x^2-2x+1) \\
 &= x(x^2-2x+1) + 2(x^2-2x+1) \\
 &= x^3 - 2x^2 + x + 2x^2 - 4x + 2 \\
 &= x^3 + 0x^2 - 3x + 2 \\
 &= x^3 + ax^2 + bx + c
 \end{aligned}$$

Ans:  $a = 0, b = -3, c = 2$ .

16.

$$\begin{aligned}
 x^2 - (\alpha + \beta)x + \alpha\beta = 0 &\Rightarrow x^2 - (3 + \sqrt{5} + 3 - \sqrt{5})x + (3 + \sqrt{5})(3 - \sqrt{5}) = 0 \\
 \Rightarrow x^2 - 6x + 9 - 5 = 0 &\Rightarrow x^2 - 6x + 4 = 0 \\
 (x-4)(x^2 - 6x + 4) &= x(x^2 - 6x + 4) - 4(x^2 - 6x + 4) \\
 &= x^3 - 6x^2 + 4x - 4x^2 + 24x - 16 \\
 &= x^3 - 10x^2 + 28x - 16 = 0
 \end{aligned}$$

17.

$$\begin{aligned}
 x^2 - (\alpha + \beta)x + \alpha\beta = 0 &\Rightarrow x^2 - (1 - \sqrt{2} + 1 + \sqrt{2})x + (1 - \sqrt{2})(1 + \sqrt{2}) = 0 \\
 \Rightarrow x^2 - 2x + 1 - 2 = 0 &\Rightarrow x^2 - 2x - 1 = 0 \\
 x = \frac{1}{4} \text{ a root} &\Rightarrow x - \frac{1}{4} = 0 \Rightarrow 4x - 1 = 0 \\
 (4x-1)(x^2 - 2x - 1) &= 4x(x^2 - 2x - 1) - 1(x^2 - 2x - 1) \\
 &= 4x^3 - 8x^2 - 4x - x^2 + 2x + 1 \\
 &= 4x^3 - 9x^2 - 2x + 1 = 0
 \end{aligned}$$

18.

$$x^3 - 14x + 8 = 0$$

$$(-4)^3 - 14(-4) + 8 = -64 + 56 + 8 = 0 \Rightarrow x = -4 \text{ is a root} \Rightarrow x + 4 \text{ is a factor.}$$

$$\begin{array}{r} x^2 - 4x + 2 \\ x + 4 \overline{) x^3 - 14x + 8} \\ \underline{x^3 + 4x^2} \phantom{+ 8} \\ -4x^2 - 14x \phantom{+ 8} \\ \underline{-4x^2 - 16x} \phantom{+ 8} \\ 2x + 8 \\ \underline{2x + 8} \\ 0 \end{array}$$

$$\left\{ \begin{array}{l} ax^2 + bx + c = 0 \\ x^2 - 4x + 2 = 0 \end{array} \right\} \quad a = 1, b = -4, c = 2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm \sqrt{4} \sqrt{2}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

$$\text{Ans. } x = 2 + \sqrt{2} \text{ or } x = 2 - \sqrt{2}$$

**19.**

$$5x^3 + 7x^2 - kx + 3 = 0, x = \frac{3}{5} \text{ a root} \Rightarrow$$

$$5\left(\frac{3}{5}\right)^3 + 7\left(\frac{3}{5}\right)^2 - k\left(\frac{3}{5}\right) + 3 = 0 \Rightarrow \frac{33}{3} - \frac{3k}{5} = 0 \Rightarrow k = 11$$

$$\text{Now, } x = \frac{3}{5} \text{ a root} \Rightarrow x - \frac{3}{5} = 0 \Rightarrow 5x - 3 \text{ is a factor}$$

$$\begin{array}{r} x^2 + 2x - 1 \\ 5x - 3 \overline{) 5x^3 + 7x^2 - 11x + 3} \\ \underline{5x^3 - 3x^2} \phantom{+ 3} \\ 10x^2 - 11x \phantom{+ 3} \\ \underline{10x^2 - 6x} \phantom{+ 3} \\ -5x + 3 \\ \underline{-5x + 3} \\ 0 \end{array}$$

$$\left. \begin{array}{l} ax^2 + bx + c = 0 \\ x^2 + 2x - 1 = 0 \end{array} \right\} a = 1, b = 2, c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm \sqrt{4}\sqrt{2}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

**20.(i)**

$$x^3 - 4x = x(x^2 - 4) = x(x - 2)(x + 2) = 0$$

$$\Rightarrow x = 0, x = 2 \text{ or } x = -2$$

**(ii)**

$$x^3 - 4x^2 = x^2(x - 4) = 0$$

$$\Rightarrow x = 0, x = 0 \text{ or } x = 4$$

**(iii)**

$$4x^3 - x = x(4x^2 - 1) = x(2x - 1)(2x + 1) = 0$$

$$\Rightarrow x = 0, x = \frac{1}{2} \text{ or } x = -\frac{1}{2}$$

**(iv)**

$$x^3 - 2x^2 = x^2(x - 2) = 0$$

$$\Rightarrow x = 0, x = 0 \text{ or } x = 2$$

**(v)**

$$x^3 - 2x^2 + x = x(x^2 - 2x + 1) = x(x - 1)(x - 1) = 0$$

$$\Rightarrow x = 0, x = 1 \text{ or } x = 1.$$

**21.**

$$2x^3 - 3x^2 + 6x + 4 = 0, \quad x = -\frac{1}{2}$$

$$2\left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 + 6\left(-\frac{1}{2}\right) + 4 = -\frac{1}{4} - \frac{3}{4} - 3 + 4 = 0$$

Now,  $x = -\frac{1}{2}$  a root  $\Rightarrow x + \frac{1}{2} = 0 \Rightarrow 2x + 1$  is a factor

$$\begin{array}{r} x^2 - 2x + 4 \\ 2x+1 \overline{) 2x^3 - 3x^2 + 6x + 4} \\ \underline{2x^3 + x^2} \phantom{+ 6x + 4} \\ -4x^2 + 6x \phantom{+ 4} \\ \underline{-4x^2 - 2x} \phantom{+ 4} \\ 8x + 4 \\ \underline{8x + 4} \\ 0 \end{array}$$

$$\left\{ \begin{array}{l} ax^2 + bx + c = 0 \\ x^2 - 2x + 4 = 0 \end{array} \right\} \quad a = 1, \quad b = -2, \quad c = 4$$

$b^2 - 4ac = (-2)^2 - 4(1)(4) = -12 < 0$  thus the roots are unreal.

**22.(i)**

$$x^3 - 1 = (x - 1)(x^2 + x + 1)$$

**(ii)**

$$x^3 - 1 = (x - 1)(x^2 + x + 1) = 0$$

$\Rightarrow x - 1 = 0$  i.e.  $x = 1$

or  $x^2 + x + 1 = 0$

$$\left\{ \begin{array}{l} ax^2 + bx + c = 0 \\ x^2 + x + 1 = 0 \end{array} \right\} \quad a = 1, \quad b = 1, \quad c = 1$$

$b^2 - 4ac = (1)^2 - 4(1)(1) = -3 < 0$  thus the roots of the quadratic are unreal.

Thus,  $x^3 - 1 = 0$  has only one real root,  $x = 1$ .

23.

$$f(x) = x^3 + ax + b$$

$$f(2) = (2)^3 + a(2) + b = 0 \Rightarrow 2a + b = -8 \quad \text{A}$$

$$f(3) = (3)^3 + a(3) + b = -20 \Rightarrow 3a + b = -47 \quad \text{B}$$

$$\text{B: } 3a + b = -47$$

$$-\text{A: } \underline{-2a - b = 8}$$

$$a = -39$$

$$\text{Substitute in A: } 2(-39) + b = -8 \Rightarrow b = -8 + 78 = 70$$

Now,  $x = 2$  is a root  $\Rightarrow x - 2$  is a factor.

$$\begin{array}{r} x^2 - 2x - 35 \\ x - 2 \overline{) x^3 - 39x + 70} \\ \underline{x^3 - 2x^2} \phantom{+ 70} \\ -2x^2 - 39x \phantom{+ 70} \\ \underline{-2x^2 - 4x} \phantom{+ 70} \\ -35x + 70 \\ \underline{-35x + 70} \\ 0 \end{array}$$

$$x^2 - 2x - 35 = (x - 7)(x + 5) = 0$$

Ans. the roots are  $x = 2, x = 7$  and  $x = -5$

24.

$$f(x) = x^3 + ax^2 + bx + c = 0$$

$$f(3) = (3)^3 + a(3)^2 + b(3) + c = 0$$

$$\Rightarrow 9a + 3b + c = -27 \quad \text{A}$$

$$f(-2) = (-2)^3 + a(-2)^2 + b(-2) + c = 0$$

$$4a - 2b + c = 8 \quad \text{B}$$

$$f(1) = (1)^3 + a(1)^2 + b(1) + c = -36$$

$$a + b + c = -37 \quad \text{C}$$

$$\text{A} \quad 9a + 3b + c = -27$$

$$-\text{B} \quad \underline{-4a + 2b - c = -8}$$

$$5a + 5b = -35 \Rightarrow a + b = -7 \quad \text{D}$$

$$\text{B} \quad 4a - 2b + c = 8$$

$$-\text{C} \quad -a - b - c = 37$$

$$3a - 3b = 45 \Rightarrow a - b = 15 \quad \text{E}$$

$$\text{D+E} \quad 2a = 8 \Rightarrow a = 4$$

$$\text{Substitute in D:} \quad (4) + b = -7 \Rightarrow b = -11$$

$$\text{Substitute in C:} \quad (4) + (-11) + c = -37 \Rightarrow c = -30$$

$$3, -2 \text{ roots} \Rightarrow (x-3)(x+2) = x^2 - x - 6 \text{ is a factor}$$

$$\begin{array}{r} x^2 - x - 6 \quad \quad \quad x+5 \\ \hline x^3 + 4x^2 - 11x - 30 \\ \underline{x^3 - x^2 - 6x} \phantom{-30} \\ 5x^2 - 5x - 30 \\ \underline{5x^2 - 5x - 30} \\ 0 \end{array}$$

Thus,  $f(x) = 0$  has solutions  $x = -2$ ,  $x = 3$  and  $x = -5$