1.

$$\left.\begin{cases} ax^2+bx+c=0 \\ x^2+6x+k=0 \end{cases}\right\} \quad a=1,\ b=6,\ c=k$$

Equal roots $\Rightarrow b^2-4ac=0 \Rightarrow (6)^2-4(1)(k)=0$

$\Rightarrow 36-4k=0 \Rightarrow k=9$

2.

$$\left.\begin{cases} ax^2+bx+c=0 \\ x^2-12x+9t=0 \end{cases}\right\} \quad a=1,\ b=-12,\ c=9t$$

Equal roots $\Rightarrow b^2-4ac=0 \Rightarrow (-12)^2-4(1)(9t)=0$

$\Rightarrow 144-36t=0 \Rightarrow 144 \div 36 = t = 4$

3.

$$\left.\begin{cases} ax^2+bx+c=0 \\ x^2+8x+p=0 \end{cases}\right\} \quad a=1,\ b=8,\ c=p$$

Unreal roots $\Rightarrow b^2-4ac<0 \Rightarrow (8)^2-4(1)(p)<0$

$\Rightarrow 64-4p<0 \Rightarrow 64<4p \Rightarrow 64 \div 4 < p$

$\Rightarrow 16<p \Rightarrow p>16$

4.

$$\left.\begin{cases} ax^2+bx+c=0 \\ x^2+kx+(k+3)=0 \end{cases}\right\} \quad a=1,\ b=k,\ c=(k+3)$$

Equal roots $\Rightarrow b^2-4ac=0 \Rightarrow (k)^2-4(1)(k+3)=0$

$\Rightarrow k^2-4k-12=0 \Rightarrow (k-6)(k+2)=0 \Rightarrow k=6$ or $k=-2$

$k=6 \Rightarrow x^2+6x+9=0 \Rightarrow (x+3)^2=0 \Rightarrow x=-3$

$k=-2 \Rightarrow x^2-2x+1=0 \Rightarrow (x-1)^2=0 \Rightarrow x=1$

5.

$$\left.\begin{cases} ax^2+bx+c=0 \\ x^2-3kx+(k^2-6)=0 \end{cases}\right\} \quad a=1,\ b=-3k,\ c=(k^2-6)$$

$$\begin{aligned} b^2-4ac &= (-3k)^2-4(1)(k^2-6) \\ &= 9k^2-4k^2+24 \\ &= 5k^2+24 \geq 0 \text{ for all } k \in R \end{aligned}$$

i.e. $b^2-4ac \geq 0$ so the quadratic has real roots for all $k \in R$.

6.

$$\left.\begin{cases} ax^2+bx+c=0 \\ x^2+2kx+(k+2)=0 \end{cases}\right\} \quad a=1,\ b=2k,\ c=(k+2)$$

Equal roots $\Rightarrow b^2-4ac=0 \Rightarrow (2k)^2-4(1)(k+2)=0$

$\Rightarrow 4k^2-4k-8=0 \Rightarrow k^2-k-2=0$

$\Rightarrow (k-2)(k+1)=0 \Rightarrow k=2$ or $k=-1$

$k=2 \Rightarrow x^2+4x+4=0 \Rightarrow (x+2)^2=0 \Rightarrow x=-2$

$k=-1 \Rightarrow x^2-2x+1=0 \Rightarrow (x-1)^2=0 \Rightarrow x=1$

7.

$$\left.\begin{cases} ax^2+bx+c=0 \\ ax^2+(2a-1)x-2=0 \end{cases}\right\} \quad a=a,\ b=(2a-1),\ c=-2$$

$b^2-4ac=(2a-1)^2-4(a)(-2)=4a^2-4a+1+8a$

$\qquad =4a^2+4a+1=(2a+1)^2 \geq 0$ for all $a \in R$

8.

$x^2+3x+3=q(x^2+5)=qx^2+5q$

$\Rightarrow x^2-qx^2+3x+3-5q=0$

$$\left.\begin{cases} ax^2+bx+c=0 \\ (1-q)x^2+3x+(3-5q)=0 \end{cases}\right\} \quad a=(1-q),\ b=3,\ c=(3-5q)$$

Equal roots $\Rightarrow b^2-4ac=0 \Rightarrow (3)^2-4(1-q)(3-5q)=0$

$\Rightarrow 9-4(3-8q+5q^2)=0 \Rightarrow 9-12+32q-20q^2=0$

$\Rightarrow 20q^2-32q+3=0 \Rightarrow (2q-3)(10q-1)=0$

$\Rightarrow q=\frac{3}{2}$ or $q=\frac{1}{10}$

9.

$$y=\frac{x^2+2x+2}{x^2+5x+5}\Rightarrow y\left(x^2+5x+5\right)=x^2+2x+2$$

$$\Rightarrow yx^2+5yx+5y-x^2-2x-2=0$$

$$\left\{\begin{matrix} ax^2+bx+c=0 \\ (y-1)x^2+(5y-2)x+(5y-2)=0 \end{matrix}\right\}$$

$a=(y-1),\ b=(5y-2),\ c=(5y-2)$

Real roots $\Rightarrow b^2-4ac\geq 0\Rightarrow (5y-2)^2-4(y-1)(5y-2)\geq 0$

$$\Rightarrow 25y^2-20y+4-4\left(5y^2-7y+2\right)\geq 0$$

$$\Rightarrow 25y^2-20y^2-20y+28y+4-8\geq 0$$

$$\Rightarrow 5y^2+8y-4\geq 0$$

Now to find the roots of this quadratic.

$5y^2+8y-4=0\Rightarrow (5y-2)(y+2)=0\Rightarrow y=\frac{2}{5}$ or $y=-2$.

The inequality is true <u>outside</u> these roots,

thus $y\leq -2$ or $y\geq \frac{2}{5}$.

10.

$$y(x-3)=(x-3)(x-3)+\frac{1}{(x-3)}(x-3)$$

$$\Rightarrow yx-3y=x^2-6x+9+1\Rightarrow 0=x^2-6x-yx+10+3y$$

$$\left\{\begin{matrix} ax^2+bx+c=0 \\ x^2-(6+y)x+(3y+10)=0 \end{matrix}\right\}$$

$a=1,\ b=-(2+y),\ c=(3y+10)$

Real roots $\Rightarrow b^2-4ac\geq 0\Rightarrow \left(-(6+y)\right)^2-4(1)(3y+10)\geq 0$

$$\Rightarrow 36+12y+y^2-12y-40\geq 0\Rightarrow y^2-4\geq 0$$

Now to find the roots of this quadratic.

$y^2-4=0\Rightarrow (y+2)(y-2)=0\Rightarrow y=-2$ or $y=2$.

The inequality is true <u>outside</u> these roots,

thus $y\leq -2$ or $y\geq 2$.

11.

$$y=\frac{x^2-6}{2x-5}\Rightarrow y(2x-5)=x^2-6$$

$$\Rightarrow 2yx-5y=x^2-6\Rightarrow 0=x^2-2yx+5y-6$$

$$\left\{\begin{matrix} ax^2+bx+c=0 \\ x^2-2yx+(5y-6)=0 \end{matrix}\right\}$$

$a=1,\ b=-2y,\ c=(5y-6)$

Real roots $\Rightarrow b^2-4ac\geq 0\Rightarrow(-2y)^2-4(1)(5y-6)\geq 0$

$\Rightarrow 4y^2-20y+24\geq 0\Rightarrow y^2-5y+6\geq 0$

Now to find the roots of this quadratic.

$y^2-5y+6=0\Rightarrow(y-2)(y-3)=0\Rightarrow y=2$ or $y=3$.

The inequality is true <u>outside</u> these roots,

thus $y=\dfrac{x^2-6}{2x-5}\leq 2$ or $y=\dfrac{x^2-6}{2x-5}\geq 3$.

12.

$$(x-1)(x+7)=k(x+2)$$

$$\Rightarrow x^2+6x-7=kx+2k\Rightarrow x^2+(6-k)x-7-2k=0$$

$$\left\{\begin{matrix} ax^2+bx+c=0 \\ x^2+(6-k)x-7-2k=0 \end{matrix}\right\}\quad a=1,\ b=(6-k),\ c=(-7-2k)$$

Equal roots $\Rightarrow b^2-4ac=0\Rightarrow(6-k)^2-4(1)(-7-2k)=0$

$\Rightarrow 36-12k+k^2-28+8k=0\Rightarrow k^2-4k+8=0$

$$\left\{\begin{matrix} ax^2+bx+c=0 \\ k^2-4k+8=0 \end{matrix}\right\}\quad a=1,\ b=-4,\ c=8$$

$\Rightarrow b^2-4ac=(-4)^2-4(1)(8)=16-32=-16<0$

Unreal roots, thus $(x-1)(x+7)=k(x+2)$

cannot have equal roots.