1.(i)

$x^2-(\text{sum})x+\text{product}=0$

$x^2-(3+2)x+3\times 2=0$

$\Rightarrow x^2-5x+6=0$

(ii)

$x^2-(\text{sum})x+\text{product}=0$

$x^2-(5+7)x+5\times 7=0$

$\Rightarrow x^2-12x+35=0$

(iii)

$x^2-(\text{sum})x+\text{product}=0$

$x^2-(5+(-2))x+5\times(-2)=0$

$\Rightarrow x^2-3x-10=0$

(iv)

$x^2-(\text{sum})x+\text{product}=0$

$x^2-(6+0)x+6\times 0=0$

$\Rightarrow x^2-6x=0$

(v)

$x^2-(\text{sum})x+\text{product}=0$

$x^2-\left(\frac{5}{2}+(-3)\right)x+\frac{5}{2}\times(-3)=0$

$\Rightarrow x^2+\frac{1}{2}x-\frac{15}{2}=0\Rightarrow 2x^2+x-15=0$

(vi)

$x^2-(\text{sum})x+\text{product}=0$

$x^2-\left(\frac{1}{2}+\frac{1}{5}\right)x+\frac{1}{2}\times\frac{1}{5}=0$

$\Rightarrow x^2-\frac{7}{10}x+\frac{1}{10}=0\Rightarrow 10x^2-7x+1=0$

(vii)

$x^2-(\text{sum})x+\text{product}=0$

$x^2-\left(-\frac{3}{4}+\frac{3}{4}\right)x+\left(-\frac{3}{4}\right)\times\frac{3}{4}=0$

$\Rightarrow x^2-\frac{9}{16}=0\Rightarrow 16x^2-9=0$

(viii)

$x^2-(\text{sum})x+\text{product}=0$

$x^2-\left(4+\sqrt{3}+4-\sqrt{3}\right)x+\left(4+\sqrt{3}\right)\times\left(4-\sqrt{3}\right)=0$

$\Rightarrow x^2-8x+4^2-3=0\Rightarrow x^2-8x+13=0$

(ix)

$$x^2-(\text{sum})x+\text{product=0}$$

$$x^2-\left(\left(-1-\sqrt{2}\right)+\left(-1+\sqrt{2}\right)\right)x+\left(-1-\sqrt{2}\right)\times\left(-1+\sqrt{2}\right)=0$$

$$\Rightarrow x^2+2x+1^2-2=0\Rightarrow x^2+2x-1=0$$

(x)

$$x^2-(\text{sum})x+\text{product=0}$$

$$x^2-\left(\left(\frac{1-\sqrt{3}}{2}\right)+\left(\frac{1+\sqrt{3}}{2}\right)\right)x+\left(\frac{1-\sqrt{3}}{2}\right)\times\left(\frac{1+\sqrt{3}}{2}\right)=0$$

$$\Rightarrow x^2-\frac{2}{2}x+\frac{1^2-3}{4}=0\Rightarrow x^2-x-\frac{2}{4}=0$$

$$\Rightarrow x^2-x-\frac{1}{2}=0\Rightarrow 2x^2-2x-1=0$$

2.(i)

$$x^2-(\text{sum})x+\text{product=0}$$

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$x^2-2x+7=0\Rightarrow \alpha+\beta=2$$

(ii)

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$x^2-2x+7=0\Rightarrow \alpha\beta=7$$

(iii)

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$$

$$=2^2-2\times7=-10$$

(iv)

$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}=\frac{2}{7}$$

(v)

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\beta\alpha}=\frac{-10}{7}=-\frac{10}{7}$$

(vi)

$$\alpha^3+\beta^3=(a+\beta)\left(\alpha^2-\alpha\beta+\beta^2\right)$$

$$=(2)(-10-7)=-34$$

(vii)

$$\alpha^2\beta^2=(\alpha\beta)^2=(7)^2=49$$

(viii)

$$\alpha^2\beta+\beta^2\alpha=\alpha\beta(\alpha+\beta)=7(2)=14$$

3.(i)

$$x^2-(\text{sum})x+\text{product=0}$$

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$x^2-7x+3=0\Rightarrow \alpha+\beta=7$$

(ii)

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2-7x+3=0 \Rightarrow \alpha\beta=3$

(iii)

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=7^2-2\times 3=49-6=43$

(iv)

$\alpha^2\beta^2=(\alpha\beta)^2=(3)^2=9$

4.(i)

$x^2-(\text{sum})x+\text{product}=0$

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2-5x+2=0 \Rightarrow \alpha+\beta=5$

(ii)

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2-5x+2=0 \Rightarrow \alpha\beta=2$

(iii)

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}=\frac{5}{2}$

(iv)

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=5^2-2\times 2=25-4=21$

(v)

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\beta\alpha}=\frac{21}{2}$

5(i)

$x^2-(\text{sum})x+\text{product}=0$

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2+6x+11=0 \Rightarrow \alpha+\beta=-6$ and $\alpha\beta=11$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(-6)^2-2\times 11=36-22=14$

(ii)

$\alpha^3+\beta^3=(a+\beta)(\alpha^2-\alpha\beta+\beta^2)$

$=(-6)(14-11)=-18$

(iii)

$\alpha^3\beta+\beta^3\alpha=\alpha\beta(\alpha^2+\beta^2)=11(14)=154$

(iv)

$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\beta\alpha}=-\frac{18}{11}$

6.

$$x^2-(\text{sum})x+\text{product=0}$$

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$2x^2+3x+7=0\Rightarrow x^2+\frac{3}{2}x+\frac{7}{2}=0$$

$$\Rightarrow \alpha+\beta=-\frac{3}{2}\text{ and }\alpha\beta=\frac{7}{2}$$

$$\alpha^3+\beta^3=(a+\beta)(\alpha^2-\alpha\beta+\beta^2)$$

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\left(-\frac{3}{2}\right)^2-2\times\frac{7}{2}=-\frac{19}{4}$$

$$\Rightarrow \alpha^3+\beta^3=(a+\beta)(\alpha^2-\alpha\beta+\beta^2)=\left(-\frac{3}{2}\right)\left(-\frac{19}{4}-\frac{7}{2}\right)=\frac{99}{8}$$

7.(i)

$$x^2-(\text{sum})x+\text{product=0}$$

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$4x^2-8x+5=0\Rightarrow x^2-2x+\frac{5}{4}=0$$

$$\Rightarrow \alpha+\beta=2\text{ and }\alpha\beta=\frac{5}{4}$$

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=2^2-2\times\frac{5}{4}=\frac{3}{2}$$

(ii)

$$\alpha^4+\beta^4=(\alpha^2+\beta^2)^2-2(\alpha\beta)^2=\left(\frac{3}{2}\right)^2-2\left(\frac{5}{4}\right)^2=-\frac{7}{8}$$

8.

$$x^2-(\text{sum})x+\text{product=0}$$

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$x^2-px+q=0$$

$$\Rightarrow \alpha+\beta=p\text{ and }\alpha\beta=q$$

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(p)^2-2\times q=p^2-2q$$

$$\Rightarrow \alpha^3+\beta^3=(a+\beta)(\alpha^2-\alpha\beta+\beta^2)=(p)(p^2-2q-q)=p^3-3pq$$

9.

$$x^2-(\text{sum})x+\text{product}=0$$

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$x^2-2px+q=0 \Rightarrow \alpha+\beta=2p \text{ and } \alpha\beta=q$$

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(2p)^2-2\times q=4p^2-2q$$

$$\alpha^4+\beta^4=\left(\alpha^2+\beta^2\right)^2-2(\alpha\beta)^2=\left(4p^2-2q\right)^2-2(q)^2$$

$$=16p^4-16p^2q+4q^2-2q^2=16p^4-16p^2q+2q^2$$

10.

$$x^2-(\text{sum})x+\text{product}=0$$

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$ax^2+bx+c=0 \Rightarrow x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$\Rightarrow \alpha+\beta=-\frac{b}{a} \text{ and } \alpha\beta=\frac{c}{a}$$

$$\frac{\beta}{\alpha}+\frac{\alpha}{\beta}=\frac{\beta^2+\alpha^2}{\beta\alpha}$$

$$\beta^2+\alpha^2=(\beta+\alpha)^2-2\alpha\beta=\left(-\frac{b}{a}\right)^2-2\times\frac{c}{a}=\frac{b^2}{a^2}-\frac{2c}{a}=\frac{b^2-2ac}{a^2}$$

$$\frac{\beta}{\alpha}+\frac{\alpha}{\beta}=\frac{\beta^2+\alpha^2}{\beta\alpha}=\frac{\frac{b^2-2ac}{a^2}}{\frac{c}{a}}=\frac{b^2-2ac}{a^2}\times\frac{a}{c}=\frac{b^2-2ac}{ac}$$

11.

$$x^2-(\text{sum})x+\text{product}=0$$

$$x^2-(\alpha+\beta)x+\alpha\beta=0 \Rightarrow x^2-(\alpha+3\alpha)x+\alpha(3\alpha)$$

$$x^2-4\alpha x+3\alpha^2=0$$

$$x^2+px+q=0$$

$$\Rightarrow 4\alpha=-p \text{ and } 3\alpha^2=q$$

$$\Rightarrow 3p^2=3(4\alpha)^2=3\times16\alpha^2=48\alpha^2$$

$$16q=16\left(3\alpha^2\right)=48\alpha^2 \text{ thus } 3p^2=16q$$

12.

$x^2-(\text{sum})x+\text{product}=0$

$x^2-(\alpha+\beta)x+\alpha\beta=0 \Rightarrow x^2-(\alpha+\alpha+2)x+\alpha(\alpha+2)$

$x^2-(2\alpha+2)x+\alpha^2+2\alpha=0$

$x^2-2kx+t=0$

$\Rightarrow \alpha^2+2\alpha=t$ and $2\alpha+2=2k \Rightarrow \alpha+1=k$

$\Rightarrow (k-1)(k+1)=k^2-1=(\alpha+1)^2-1=\alpha^2+2\alpha+1-1=\alpha^2+2\alpha=t$

13.(i)

$x^2-(\text{sum})x+\text{product}=0$

$x^2-(\alpha+\beta)x+\alpha\beta=0 \Rightarrow x^2-(\alpha+2\alpha)x+\alpha(2\alpha)$

$x^2-3\alpha x+2\alpha^2=0$

$x^2+\frac{b}{a}x+\frac{c}{a}=0$

$\left(\frac{b}{a}\right)^2=(-3\alpha)^2 \Rightarrow \frac{b^2}{a^2}=9\alpha^2 \Rightarrow 2b^2=18a^2\alpha^2$

$\frac{c}{a}=2\alpha^2 \Rightarrow c=2a\alpha^2 \Rightarrow 9ac=18a^2\alpha^2$

$\Rightarrow 2b^2=9ac$

(ii)

$\left.\begin{cases} ax^2+bx+c=0 \\ ax^2-15x+25=0 \end{cases}\right\}$ $a=a,\ b=-15,\ c=25$

One root is double the other $\Rightarrow 2b^2=9ac$

$\Rightarrow 2(-15)^2=9a(25) \Rightarrow 450=225a$

$\Rightarrow a=450\div 225=2$

14.(i)

$x^2-(\text{sum})x+\text{product}=0$

$x^2-(\alpha+\beta)x+\alpha\beta=0 \Rightarrow x^2-(\alpha+\alpha)x+\alpha(\alpha)$

$x^2-2\alpha x+\alpha^2=0$

$x^2-bx+c=0$

$b^2=(2\alpha)^2 \Rightarrow b^2=4\alpha^2$

$4c=4\alpha^2 \Rightarrow b^2=4c$

(ii)

$\left.\begin{cases} x^2+bx+c=0 \\ x^2-14x+k=0 \end{cases}\right\}$ $b=-14,\ c=k$

Equal roots $\Rightarrow b^2=4a$

$\Rightarrow (-14)^2=4k \Rightarrow 196=4k$

$\Rightarrow k=196\div 4=49$

15.(i)

$x^2-(\text{sum})x+\text{product}=0$

$x^2-(\alpha+\beta)x+\alpha\beta=0 \Rightarrow x^2-(\alpha+\alpha+1)x+\alpha(\alpha+1)$

$x^2-(2\alpha+1)x+\alpha^2+\alpha=0$

$x^2+\dfrac{b}{a}x+\dfrac{c}{a}=0$

$\Rightarrow\left(\dfrac{b}{a}\right)^2=\left(-(2\alpha+1)\right)^2 \Rightarrow \dfrac{b^2}{a^2}=4\alpha^2+4\alpha+1$

$\Rightarrow b^2=4a^2\alpha^2+4a^2\alpha+a^2 \Rightarrow b^2-a^2=4a^2\alpha^2+4a^2\alpha$

$\dfrac{c}{a}=\alpha^2+\alpha \Rightarrow c=a\alpha^2+a\alpha \Rightarrow 4ac=4a^2\alpha^2+4a^2\alpha$

$\Rightarrow b^2-a^2=4ac$

(ii)

$\left.\begin{cases} ax^2+bx+c=0 \\ 4x^2-16x+k=0 \end{cases}\right\}$ $a=4,\ b=-16,\ c=k$

Roots differ by 1 $\Rightarrow b^2-a^2=4ac$

$\Rightarrow(-16)^2-4^2=4(4)k \Rightarrow 240=16k$

$\Rightarrow k=240\div 16=15$

16.(i)

$x^2-(\text{sum})x+\text{product}=0$

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2-5x-7=0$

$\Rightarrow \alpha+\beta=5$ and $\alpha\beta=-7$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(5)^2-2\times(-7)=39$

(ii)

$\alpha^2\beta^2=(\alpha\beta)^2=(-7)^2=49$

The quadratic with roots α^2 and β^2 is given by

$x^2-(\alpha^2+\beta^2)x+\alpha^2\beta^2=0 \Rightarrow x^2-39x+49=0$

17.(i)

$x^2-(\text{sum})x+\text{product}=0$

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2+3x+7=0$

$\Rightarrow \alpha+\beta=-3$ and $\alpha\beta=7$

$\dfrac{1}{\alpha}+\dfrac{1}{\beta}=\dfrac{\beta+\alpha}{\alpha\beta}=-\dfrac{3}{7},\quad \dfrac{1}{\alpha\beta}=\dfrac{1}{7}$

(ii)

The quadratic with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is given by:

$$x^2-\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)x+\frac{1}{\alpha\beta}=0$$

$$\Rightarrow x^2-\left(-\frac{3}{7}\right)x+\frac{1}{7}=0\Rightarrow 7x^2+3x+1=0$$

18.(i)

$$x^2-(\text{sum})x+\text{product=0}$$

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$2x^2-x-8=0\Rightarrow x^2-\frac{1}{2}x-4=0$$

$$\Rightarrow \alpha+\beta=\frac{1}{2} \text{ and } \alpha\beta=-4$$

$$\alpha^3+\beta^3=(a+\beta)(\alpha^2-\alpha\beta+\beta^2)$$

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\left(\frac{1}{2}\right)^2-2\times(-4)=8\cdot 25$$

$$\Rightarrow \alpha^3+\beta^3=(a+\beta)(\alpha^2-\alpha\beta+\beta^2)=\left(\frac{1}{2}\right)(8\cdot 25-(-4))=6\cdot 125$$

$$\alpha^3\beta^3=(\alpha\beta)^3=(-4)^3=-64$$

(ii)

The quadratic with roots α^3 and β^3 is given by:

$$x^2-(\alpha^3+\beta^3)x+\alpha^3\beta^3=0$$

$$\Rightarrow x^2-6\cdot 125x+(-64)=0\Rightarrow 8x^2-49x-512=0$$

19.

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$2x^2-9x+8=0\Rightarrow x^2-\frac{9}{2}x+4=0$$

$$\Rightarrow \alpha+\beta=\frac{9}{2} \text{ and } \alpha\beta=4$$

The quadratic with roots 10α and 10β is given by:

$$x^2-(10\alpha+10\beta)x+(10\alpha)(10\beta)=0$$

$$\Rightarrow x^2-10(\alpha+\beta)x+100\alpha\beta=0\Rightarrow x^2-10\left(\frac{9}{2}\right)x+100(4)=0$$

$$\Rightarrow x^2-45x+400=0$$

20.(i)

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$x^2-7x+2=0\Rightarrow \alpha+\beta=7 \text{ and } \alpha\beta=2$$

$$(\alpha-\beta)^2=\alpha^2-2\alpha\beta+\beta^2=(\alpha+\beta)^2-2\alpha\beta-2\alpha\beta$$

$$=(\alpha+\beta)^2-4\alpha\beta=7^2-4\times 2=41$$

$$\alpha-\beta=\pm\sqrt{(\alpha-\beta)^2}=\pm\sqrt{41}$$

But $\alpha>\beta\Rightarrow\alpha-\beta>0\Rightarrow\alpha-\beta=\sqrt{41}$

(ii)

$$x^2-(\text{sum})x+\text{product}=0$$

$$x^2-\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)x+\frac{\alpha}{\beta}\frac{\beta}{\alpha}=0$$

$$x^2-\left(\frac{\alpha^2+\beta^2}{\beta\alpha}\right)x+1=0$$

$$x^2-\left(\frac{(\alpha+\beta)^2-\alpha\beta}{\beta\alpha}\right)x+1=0$$

$$x^2-\left(\frac{7^2-2}{2}\right)x+1=0$$

$$x^2-\frac{47}{2}x+1=0\Rightarrow 2x^2-47x+2=0$$

21.

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$3x^2-6x+2=0\Rightarrow x^2-2x+\frac{2}{3}=0$$

$$\Rightarrow\alpha+\beta=2 \text{ and } \alpha\beta=\frac{2}{3}$$

The quadratic with roots $2\alpha-\beta$ and $2\beta-\alpha$ is given by:

$$x^2-(2\alpha-\beta+2\beta-\alpha)x+(2\alpha-\beta)(2\beta-\alpha)=0$$

$$\Rightarrow x^2-(\alpha+\beta)x+4\alpha\beta-2\alpha^2-2\beta^2+\beta\alpha=0$$

Now

$$4\alpha\beta-2\alpha^2-2\beta^2+\beta\alpha=5\alpha\beta-2(\alpha^2+\beta^2)$$

$$=5\alpha\beta-2\left[(\alpha+\beta)^2-2\alpha\beta\right]$$

$$=5\alpha\beta-2(\alpha+\beta)^2+4\alpha\beta$$

$$=9\alpha\beta-2(\alpha+\beta)^2$$

$$\Rightarrow x^2-(\alpha+\beta)x+9\alpha\beta-2(\alpha+\beta)^2=0$$

$$\Rightarrow x^2-2x+9\left(\frac{2}{3}\right)-2(2)^2=0$$

$$\Rightarrow x^2-2x-2=0$$

22.(i)

$x^2-(\text{sum})x+\text{product}=0$

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2+px+q=0 \Rightarrow \alpha+\beta=-p$ and $\alpha\beta=q$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(-p)^2-2\times q=p^2-2q$

(ii)

$\alpha^4+\beta^4=(\alpha^2+\beta^2)^2-2(\alpha\beta)^2=(p^2-2q)^2-2(q)^2$

$=p^4-4p^2q+4q^2-2q^2$

$=p^4-4p^2q+2q^2$

(iii)

$x^2-(\alpha^4+\beta^4)x+\alpha^4\beta^4=0$

$\Rightarrow x^2-(p^4-4p^2q+2q^2)x+q^4=0$

23.

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$ax^2+bx+c=0 \Rightarrow x^2+\frac{b}{a}x+\frac{c}{a}=0$

$\Rightarrow \alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$

The quadratic with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is given by:

$x^2-\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)x+\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)=0$

$\Rightarrow x^2-\left(\frac{\beta+\alpha}{\alpha\beta}\right)x+\frac{1}{\alpha\beta}=0$

$\Rightarrow \alpha\beta x^2-(\beta+\alpha)x+1=0$

$\Rightarrow \left(\frac{c}{a}\right)x^2-\left(-\frac{b}{a}\right)x+1=0 \Rightarrow cx^2+bx+a=0$

24.(i)

$x^2-(\text{sum})x+\text{product}=0$

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$2x^2-5x-1=0 \Rightarrow x^2-\frac{5}{2}x-\frac{1}{2}=0$

$\Rightarrow \alpha+\beta=\frac{5}{2}$ and $\alpha\beta=-\frac{1}{2}$

$\alpha^3+\beta^3=(a+\beta)(\alpha^2-\alpha\beta+\beta^2)$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\left(\frac{5}{2}\right)^2-2\times\left(-\frac{1}{2}\right)=\frac{29}{4}$

$\Rightarrow \alpha^3+\beta^3=(a+\beta)(\alpha^2-\alpha\beta+\beta^2)=\left(\frac{5}{2}\right)\left(\frac{29}{4}-\left(-\frac{1}{2}\right)\right)=\frac{155}{8}$

(ii)

The quadratic with roots $1+\frac{1}{\alpha}$ and $1+\frac{1}{\beta}$ is given by:

$x^2-\left(1+\frac{1}{\alpha}+1+\frac{1}{\beta}\right)x+\left(1+\frac{1}{\alpha}\right)\left(1+\frac{1}{\beta}\right)=0$

$\Rightarrow x^2-\left(2+\frac{1}{\alpha}+\frac{1}{\beta}\right)x+1+\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\alpha\beta}=0$

$\Rightarrow x^2-\left(2+\frac{\frac{5}{2}}{\left(-\frac{1}{2}\right)}\right)x+1+\frac{\frac{5}{2}}{\left(-\frac{1}{2}\right)}+\frac{1}{\left(-\frac{1}{2}\right)}=0$

$\Rightarrow x^2-(2-5)x+1-5-2=0 \Rightarrow x^2+3x-6=0$

25.

Suppose $x^2-2x-3b=0$ has roots α, β,

and $x^2-7x+2b=0$ has roots α, γ.

A: $\alpha+\beta=2$

B: $\alpha\beta=-3b$

C: $\alpha+\gamma=7$

D: $\alpha\gamma=2b$

Now D $\Rightarrow \gamma=\dfrac{2b}{\alpha}$, substitute in C

$\Rightarrow \alpha+\dfrac{2b}{\alpha}=7 \Rightarrow \alpha^2+2b=7\alpha$ E

A $\Rightarrow \beta=2-\alpha$, substitute in B

$\alpha(2-\alpha)=-3b \Rightarrow 2\alpha-\alpha^2=-3b$

$\Rightarrow \alpha^2-2\alpha=3b \Rightarrow \dfrac{\alpha^2-2\alpha}{3}=b$ F

Substitute F in E: $\alpha^2+2\left(\dfrac{\alpha^2-2\alpha}{3}\right)=7\alpha$

$\Rightarrow 3\alpha^2+2\alpha^2-4\alpha=21\alpha \Rightarrow 5\alpha^2-25\alpha=0$

$\Rightarrow \alpha(7-5\alpha)=0 \Rightarrow \alpha=0$ or $\alpha=5$

Substitute in F: $b=\dfrac{5^2-2\times5}{3}=\dfrac{15}{3}=5$

or $b=\dfrac{0^2-2\times0}{3}=0$ but $b\neq0 \Rightarrow \alpha=5$ and $b=5$

26.

A: $2x^2 - 5x - k = 0$

B: $10x^2 + 11x + k = 0$

If A and B have a common root, then there is

a value of x for which $\text{A} = 0$ and $\text{B} = 0$ and hence $\text{A} + \text{B} = 0$.

$\text{A} + \text{B}$: $12x^2 + 6x = 0 \Rightarrow 2x^2 + x = 0 \Rightarrow x(2x+1) = 0$

$\Rightarrow x = 0$ or $x = -\frac{1}{2}$

Substituting $x = 0$ in A $\Rightarrow -k = 0 \Rightarrow k = 0$.

Substituting $x = 0$ in B $\Rightarrow k = 0$.

Substituting $x = -\frac{1}{2}$ in A

$\Rightarrow 2\left(-\frac{1}{2}\right)^2 - 5\left(-\frac{1}{2}\right) - k = 0 \Rightarrow 3 - k = 0 \Rightarrow k = 3.$

Substituting $x = -\frac{1}{2}$ in B

$\Rightarrow 10\left(-\frac{1}{2}\right)^2 + 11\left(-\frac{1}{2}\right) + k = 0 \Rightarrow -3 + k = 0 \Rightarrow k = 3.$

Thus $k \neq 0 \Rightarrow x = -\frac{1}{2}$ and $k = 3$.