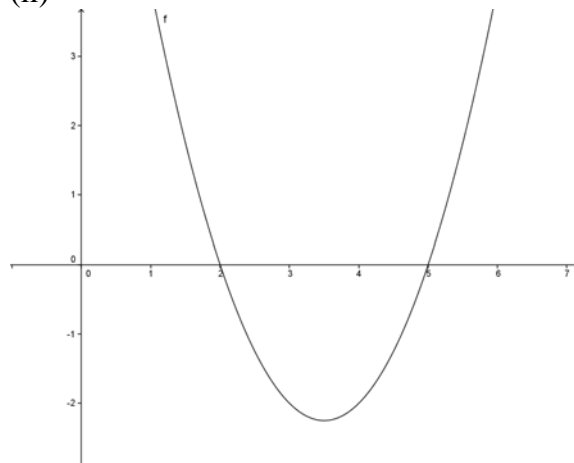


1.(i)

$$x^2 - 7x + 10 = 0 \Rightarrow (x - 2)(x - 5) = 0$$

$$\Rightarrow x = 2 \text{ or } x = 5$$

(ii)



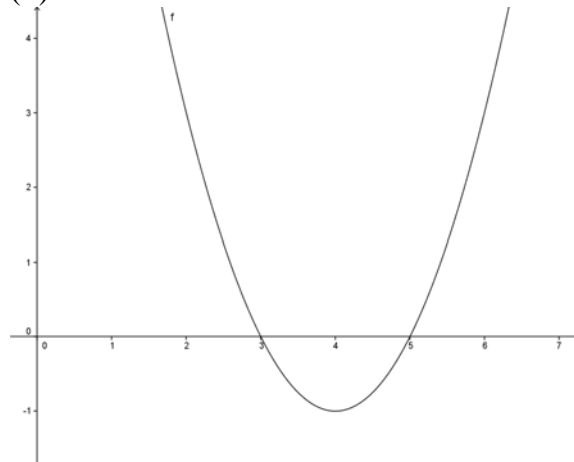
$$x^2 - 7x + 10 < 0 \Rightarrow 2 < x < 5$$

2.(i)

$$x^2 - 8x + 15 = 0 \Rightarrow (x - 3)(x - 5) = 0$$

$$\Rightarrow x = 3 \text{ or } x = 5$$

(ii)

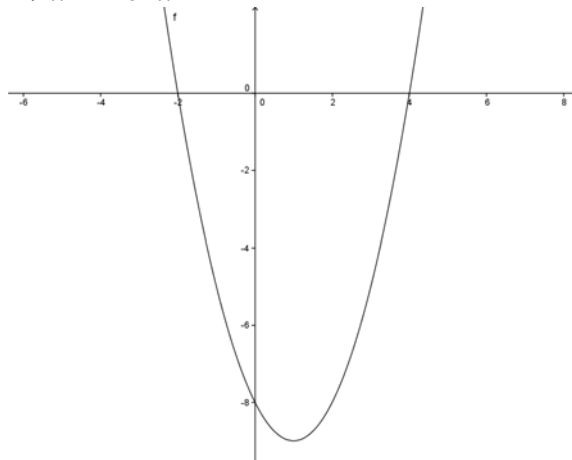


$$x^2 - 8x + 15 > 0 \Rightarrow x < 3 \text{ or } x > 5$$

3.(i)

$$x^2 - 2x - 8 = 0 \Rightarrow (x - 4)(x + 2) = 0$$

$$\Rightarrow x = 4 \text{ or } x = -2$$

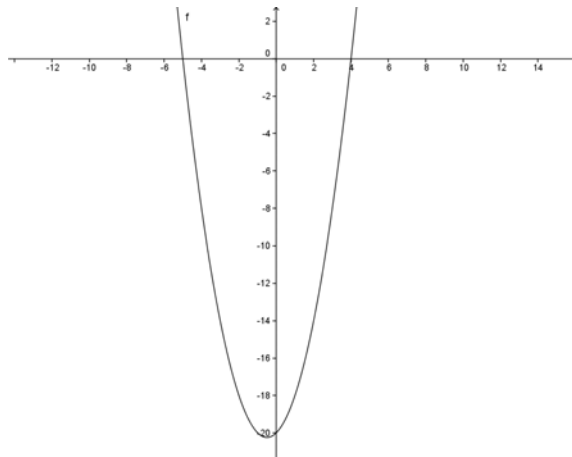


$$x^2 - 2x - 8 \leq 0 \Rightarrow -2 \leq x \leq 4$$

(ii)

$$x^2 + x - 20 = 0 \Rightarrow (x + 5)(x - 4) = 0$$

$$\Rightarrow x = -5 \text{ or } x = 4$$

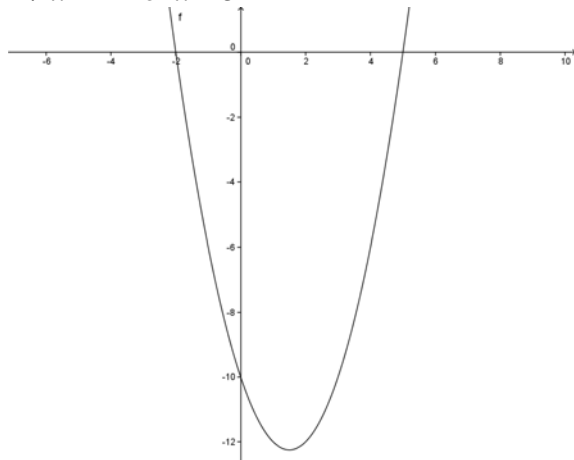


$$x^2 + x - 20 \geq 0 \Rightarrow x \leq -5 \text{ or } x \geq 4$$

(iii)

$$x^2 - 3x - 10 = 0 \Rightarrow (x + 2)(x - 5) = 0$$

$$\Rightarrow x = -2 \text{ or } x = 5$$

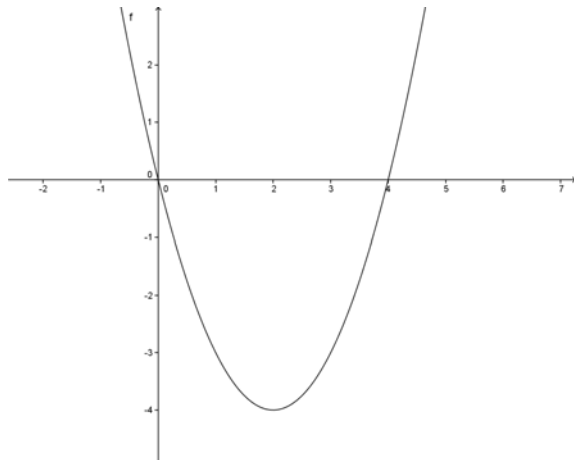


$$x^2 - 3x - 10 < 0 \Rightarrow -2 < x < 5$$

(iv)

$$x^2 - 4x = 0 \Rightarrow x(x - 4) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 4$$

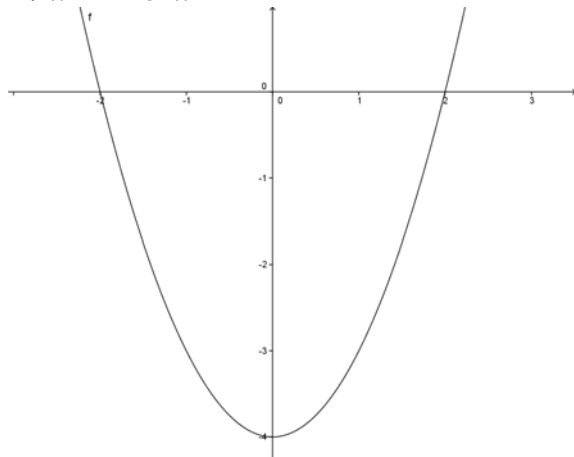


$$x^2 - 4x > 0 \Rightarrow x < 0 \text{ or } x > 4$$

(v)

$$x^2 - 4 = 0 \Rightarrow (x + 2)(x - 2) = 0$$

$$\Rightarrow x = -2 \text{ or } x = 2$$

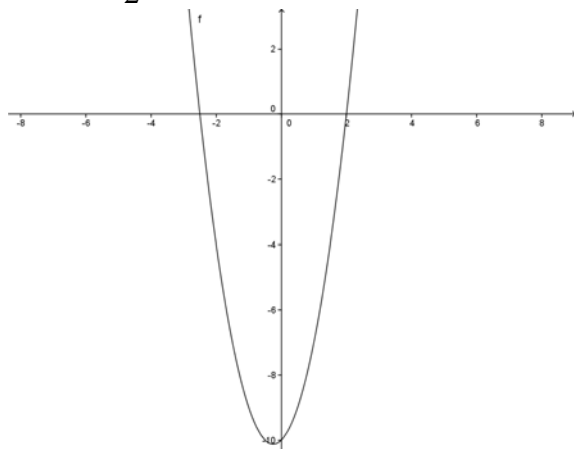


$$x^2 - 4 \leq 0 \Rightarrow -2 \leq x \leq 2$$

4.(i)

$$2x^2 + x - 10 = 0 \Rightarrow (2x + 5)(x - 2) = 0$$

$$\Rightarrow x = -\frac{5}{2} \text{ or } x = 2$$



$$2x^2 + x - 10 < 0 \Rightarrow -\frac{5}{2} < x < 2$$

(ii)

$$6x^2 = x + 1 \Rightarrow 6x^2 - x - 1 = 0$$

$$\Rightarrow (3x + 1)(2x - 1) = 0$$

$$\Rightarrow x = -\frac{1}{3} \text{ or } x = \frac{1}{2}$$

$$6x^2 \geq x + 1 \Rightarrow x \leq -\frac{1}{3} \text{ or } x \geq \frac{1}{2}$$

(iii)

$$2x^2 + 4x + 7 = x^2 + 12 \Rightarrow x^2 + 4x - 5 = 0$$

$$\Rightarrow (x+5)(x-1) = 0 \Rightarrow x = -5 \text{ or } x = 1$$

$$2x^2 + 4x + 7 < x^2 + 12 \Rightarrow -5 < x < 1$$

(iv)

$$(x-3)(1-x) \geq -15$$

$$\Rightarrow x(1-x) - 3(1-x) \geq -15$$

$$\Rightarrow x - x^2 - 3 + 3x + 15 \geq 0$$

$$\Rightarrow -x^2 + 4x + 12 \geq 0$$

$$\Rightarrow 0 \geq x^2 - 4x - 12$$

$$x^2 - 4x - 12 = (x+2)(x-6) = 0 \Rightarrow x = -2 \text{ or } x = 6$$

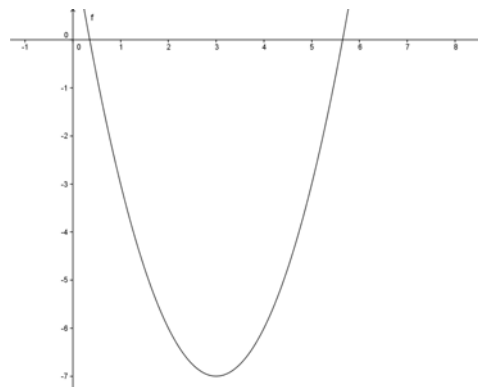
$$x^2 - 4x - 12 \leq 0 \Rightarrow -2 \leq x \leq 6$$

5.

$$\begin{cases} ax^2 + bx + c = 0 \\ x^2 - 6x + 2 = 0 \end{cases} \quad a = 1, b = -6, c = 2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{28}}{2} = \frac{6 \pm \sqrt{4}\sqrt{7}}{2} = \frac{6 \pm 2\sqrt{7}}{2} = 3 \pm \sqrt{7}$$



$$x^2 - 6x + 2 \leq 0 \Rightarrow 3 - \sqrt{7} \leq x \leq 3 + \sqrt{7}$$

**6. (i)**

$$\left\{ \begin{array}{l} ax^2 + bx + c = 0 \\ x^2 - 13x - 13 = 0 \end{array} \right\} \quad a = 1, b = -13, c = -13$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(1)(-13)}}{2(1)}$$

$$x = \frac{13 \pm \sqrt{221}}{2} \Rightarrow x = \frac{13 - \sqrt{221}}{2} = -0.9$$

$$\text{or } x = \frac{13 + \sqrt{221}}{2} = 13.9$$

**(ii)**

$$n^2 - 13n - 13 > 0 \Rightarrow n > 13.9 \text{ i.e. } n = 14$$

(Note:  $n$  cannot be negative so we do not consider the case where  $n < -0.9$ )

**7. (i)**

$$\left\{ \begin{array}{l} ax^2 + bx + c = 0 \\ 2x^2 - 11x - 22 = 0 \end{array} \right\} \quad a = 2, b = -11, c = -22$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(2)(-22)}}{2(2)}$$

$$x = \frac{11 \pm \sqrt{297}}{4} \Rightarrow x = \frac{11 - \sqrt{297}}{4} = -1.6$$

$$\text{or } x = \frac{11 + \sqrt{297}}{4} = 7.1$$

**(ii)**

$$2n^2 < 11(n + 2) \Rightarrow 2n^2 - 11n - 22 < 0$$

$$\Rightarrow -1.6 < n < 7.1$$

$\Rightarrow$  the greatest value of  $n$  is 7

8.

$$\begin{aligned} (x-4)^2 \frac{x+3}{x-4} &> 3(x-4)^2 \\ \Rightarrow (x-4)(x+3) &> 3(x^2 - 8x + 16) \\ \Rightarrow x^2 - x - 12 &> 3x^2 - 24x + 48 \\ \Rightarrow 0 &> 3x^2 - x^2 - 24x + x + 48 + 12 \\ \Rightarrow 0 &> 2x^2 - 23x + 60 \\ \Rightarrow 2x^2 - 23x + 60 &< 0 \\ 2x^2 - 23x + 60 = 0 &\Rightarrow (2x-15)(x-4) = 0 \\ \Rightarrow x = 4 \text{ or } x = 7.5 \\ \Rightarrow 2x^2 - 23x + 60 < 0 &\Rightarrow 4 < x < 7.5 \end{aligned}$$

9.

$$\begin{aligned} (x-2)^2 \frac{x+1}{x-4} &< 2(x-2)^2 \\ \Rightarrow (x-2)(x+1) &< 2(x^2 - 4x + 4) \\ \Rightarrow x^2 - x - 2 &> 2x^2 - 8x + 8 \\ \Rightarrow 0 &< 2x^2 - x^2 - 8x + x + 8 + 2 \\ \Rightarrow 0 &< x^2 - 7x + 10 \\ \Rightarrow x^2 - 7x + 10 &> 0 \\ x^2 - 7x + 10 = 0 &\Rightarrow (x-2)(x-5) = 0 \\ \Rightarrow x = 2 \text{ or } x = 5 \\ \Rightarrow x^2 - 7x + 10 > 0 &\Rightarrow x < 2 \text{ or } x > 5 \end{aligned}$$

10.

$$\begin{aligned} (x+1)^2 \frac{x-3}{x+1} &> 2(x+1)^2 \\ \Rightarrow (x+1)(x-3) &> 2(x^2 + 2x + 1) \\ \Rightarrow x^2 - 2x - 3 &> 2x^2 + 4x + 2 \\ \Rightarrow 0 &> 2x^2 - x^2 + 4x + 2x + 2 + 3 \\ \Rightarrow 0 &> x^2 + 6x + 5 \\ \Rightarrow x^2 + 6x + 5 &< 0 \\ x^2 + 6x + 5 = 0 &\Rightarrow (x+1)(x+5) = 0 \\ \Rightarrow x = -5 \text{ or } x = -1 \\ \Rightarrow x^2 + 6x + 5 < 0 &\Rightarrow -5 < x < -1 \end{aligned}$$

11.

$$\begin{aligned} (x+1)^2 \frac{x-4}{x+1} &< 2(x+1)^2 \\ \Rightarrow (x+1)(x-4) &< 2(x^2+2x+1) \\ \Rightarrow x^2-3x-4 &> 2x^2+4x+2 \\ \Rightarrow 0 &< 2x^2-x^2+4x+3x+2+4 \\ \Rightarrow 0 &< x^2+7x+6 \\ \Rightarrow x^2+7x+6 &> 0 \\ x^2+7x+6=0 &\Rightarrow (x+1)(x+6)=0 \\ \Rightarrow x &=-1 \text{ or } x=-6 \\ \Rightarrow x^2+7x+6 &> 0 \Rightarrow x < -6 \text{ or } x > -1 \end{aligned}$$

12.

$$\begin{aligned} (x-1)^2 \frac{x+1}{x-1} &< 3(x-1)^2 \\ \Rightarrow (x-1)(x+1) &< 3(x^2-2x+1) \\ \Rightarrow x^2-1 &> 3x^2-6x+3 \\ \Rightarrow 0 &< 3x^2-x^2-6x+3+1 \\ \Rightarrow 0 &< 2x^2-6x+4 \\ \Rightarrow 2x^2-6x+4 &> 0 \Rightarrow x^2-3x+2 > 0 \\ x^2-3x+2=0 &\Rightarrow (x-1)(x-2)=0 \\ \Rightarrow x &=1 \text{ or } x=2 \\ \Rightarrow x^2-3x+2 &> 0 \Rightarrow x < 1 \text{ or } x > 2 \end{aligned}$$

13.

$$\begin{aligned} (x-3)^2 \frac{2x-1}{x-3} &\geq 3(x-3)^2 \\ \Rightarrow (x-3)(2x-1) &\geq 3(x^2-6x+9) \\ \Rightarrow 2x^2-7x+3 &\geq 3x^2-18x+27 \\ \Rightarrow 0 &\geq 3x^2-2x^2-18x+7x+27-3 \\ \Rightarrow 0 &\geq x^2-11x+24 \\ \Rightarrow x^2-11x+24 &\leq 0 \\ x^2-11x+24=0 &\Rightarrow (x-3)(x-8)=0 \\ \Rightarrow x &=3 \text{ or } x=8 \\ \Rightarrow x^2-11x+24 &\leq 0 \Rightarrow 3 < x < 8 \end{aligned}$$

14.

$$(x+5)^2 \frac{2x+1}{x+5} < 0 \times (x+5)^2$$

$$\Rightarrow (x+5)(2x+1) < 0$$

$$\Rightarrow 2x^2 + 11x + 5 < 0$$

$$2x^2 + 11x + 5 = 0 \Rightarrow (2x+1)(x+5) = 0$$

$$\Rightarrow x = -5 \text{ or } x = -\frac{1}{2}$$

$$\Rightarrow x^2 + 7x + 6 < 0 \Rightarrow -5 < x < -\frac{1}{2}$$

15.

$$(3x+5)^2 \frac{x-1}{3x+5} \geq 0 \times (3x+5)^2$$

$$\Rightarrow (3x+5)(x-1) \geq 0$$

$$\Rightarrow 3x^2 + 2x - 5 \geq 0$$

$$3x^2 + 2x - 5 = 0 \Rightarrow (3x+5)(x-1) = 0$$

$$\Rightarrow x = -\frac{5}{3} \text{ or } x = 1$$

$$3x^2 + 2x - 5 \geq 0 \Rightarrow x \leq -\frac{5}{3} \text{ or } x \geq 1$$

16.

$$(x-3)^2 \frac{x+2}{x-3} > 1(x-3)^2$$

$$\Rightarrow (x-3)(x+2) > 1(x^2 - 6x + 9)$$

$$\Rightarrow x^2 - x - 6 > x^2 - 6x + 9$$

$$\Rightarrow x^2 - x^2 - x + 6x - 6 - 9 > 0$$

$$\Rightarrow 5x - 15 > 0 \Rightarrow x - 3 > 0$$

$$\Rightarrow x > 3$$

17.

$$f(x) > 5 \Rightarrow \frac{100}{x+1} > 5$$

$$\Rightarrow (x+1)^2 \frac{100}{x+1} > 5(x+1)^2$$

$$\Rightarrow (x+1)100 > 5(x^2 + 2x + 1)$$

$$\Rightarrow 100x + 100 > 5x^2 + 10x + 5$$

$$\Rightarrow 0 > 5x^2 + 10x - 100x + 5 - 100$$

$$\Rightarrow 0 > 5x^2 - 90x - 95 \Rightarrow 0 > x^2 - 18x - 19$$

$$\Rightarrow x^2 - 18x - 19 < 0$$

$$x^2 - 18x - 19 = 0 \Rightarrow (x+1)(x-19) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 19$$

$$x^2 - 18x - 19 < 0 \Rightarrow -1 < x < 19$$