

1.(i)

$$x^2 - 2x - 3 = 0 \Rightarrow x^2 - 3x + x - 3 = 0$$

$$\Rightarrow x(x-3) + 1(x-3) = 0$$

$$\Rightarrow (x-3)(x+1) = 0 \Rightarrow x = 3 \text{ or } x = -1$$

(ii)

$$x^2 - 2x + 1 = 0 \Rightarrow x^2 - x - x + 1 = 0$$

$$\Rightarrow x(x-1) - 1(x-1) = 0$$

$$\Rightarrow (x-1)(x-1) = 0 \Rightarrow x = 1$$

(iii)

$$x^2 - 2x = 0 \Rightarrow x(x-2) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 2$$

(iv)

$$4x^2 - 3x - 1 = 0 \Rightarrow 4x^2 - 4x + x - 1 = 0$$

$$\Rightarrow 4x(x-1) + 1(x-1) = 0$$

$$\Rightarrow (x-1)(4x+1) = 0$$

$$x-1 = 0 \Rightarrow x = 1$$

$$4x+1 = 0 \Rightarrow x = -\frac{1}{4}$$

(v)

$$4x^2 - 4x + 1 = 0 \Rightarrow 4x^2 - 2x - 2x + 1 = 0$$

$$\Rightarrow 2x(2x-1) - 1(2x-1) = 0$$

$$\Rightarrow (2x-1)(2x-1) = 0$$

$$2x-1 = 0 \Rightarrow x = \frac{1}{2}$$

(vi)

$$4x^2 - 5x + 1 = 0 \Rightarrow 4x^2 - 4x - x + 1 = 0$$

$$\Rightarrow 4x(x-1) - 1(x-1) = 0$$

$$\Rightarrow (x-1)(4x-1) = 0$$

$$x-1 = 0 \Rightarrow x = 1$$

$$4x-1 = 0 \Rightarrow x = \frac{1}{4}$$

(vii)

$$x^2 - 6x - 7 = 0 \Rightarrow x^2 - 7x + x - 7 = 0$$

$$\Rightarrow x(x-7) + 1(x-7) = 0$$

$$\Rightarrow (x-7)(x+1) = 0 \Rightarrow x = 7 \text{ or } x = -1$$

(viii)

$$x^2 - 6x + 5 = 0 \Rightarrow x^2 - 5x - x + 5 = 0$$

$$\Rightarrow x(x-5) - 1(x-5) = 0$$

$$\Rightarrow (x-5)(x-1) = 0 \Rightarrow x = 5 \text{ or } x = 1$$

(ix)

$$4x^2 - 9 = 0 \Rightarrow (2x - 3)(2x + 3) = 0$$

$$2x - 3 = 0 \Rightarrow x = \frac{3}{2}$$

$$2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$$

(x)

$$49x^2 - 121 = 0 \Rightarrow (7x - 11)(7x + 11) = 0$$

$$7x - 11 = 0 \Rightarrow x = \frac{11}{7}$$

$$7x + 11 = 0 \Rightarrow x = -\frac{11}{7}$$

2. (i)

$$(2x - 1)(x + 3) = 0$$

$$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$$

$$x + 3 = 0 \Rightarrow x = -3$$

(ii)

$$(2x - 1)(x + 3) = 15$$

$$\Rightarrow 2x^2 + 6x - x - 3 - 15 = 0$$

$$\Rightarrow 2x^2 + 5x - 18 = 0 \Rightarrow 2x^2 + 9x - 4x - 18 = 0$$

$$\Rightarrow x(2x + 9) - 2(2x + 9) = 0 \Rightarrow (x - 2)(2x + 9) = 0$$

$$x - 2 = 0 \Rightarrow x = 2$$

$$2x + 9 = 0 \Rightarrow x = -\frac{9}{2}$$

(iii)

$$(2x - 1)(x + 3) = 4x$$

$$\Rightarrow 2x^2 + 6x - x - 3 - 4x = 0$$

$$\Rightarrow 2x^2 + x - 3 = 0 \Rightarrow 2x^2 - 2x + 3x - 3 = 0$$

$$\Rightarrow 2x(x - 1) + 3(x - 1) = 0 \Rightarrow (2x + 3)(x - 1) = 0$$

$$2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$$

$$x - 1 = 0 \Rightarrow x = 1$$

(iv)

$$(2x - 1)(x + 3) = 5x$$

$$\Rightarrow 2x^2 + 6x - x - 3 - 5x = 0$$

$$\Rightarrow 2x^2 - 3 = 0 \Rightarrow x^2 = \frac{3}{2} \Rightarrow x = \pm\sqrt{\frac{3}{2}}$$

3.

$$\begin{cases} ax^2 + bx + c = 0 \\ x^2 - 6x - 2 = 0 \end{cases} \quad a = 1, b = -6, c = -2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{44}}{2} = \frac{6 \pm \sqrt{4}\sqrt{11}}{2} = \frac{6 \pm 2\sqrt{11}}{2} = 3 \pm \sqrt{11}$$

$$x = t + 3 = 3 \pm \sqrt{11} \Rightarrow t = \pm \sqrt{11}$$

4.

$$x^2 - 8x + 12 = 0 \Rightarrow x^2 - 2x - 6x + 12 = 0$$

$$\Rightarrow x(x-2) - 6(x-2) = 0 \Rightarrow (x-6)(x-2) = 0$$

$$x - 6 = 0 \Rightarrow x = 6$$

$$x - 2 = 0 \Rightarrow x = 2$$

$$t^2 + t = 6 \Rightarrow t^2 + t - 6 = 0$$

$$t^2 + 3t - 2t - 6 = 0 \Rightarrow t(t+3) - 2(t+3)$$

$$\Rightarrow (t-2)(t+3) = 0 \Rightarrow t = 2 \text{ or } t = -3$$

$$t^2 + t = 2 \Rightarrow t^2 + t - 2 = 0$$

$$t^2 + 2t - t - 6 = 0 \Rightarrow t(t+2) - 1(t+2)$$

$$\Rightarrow (t-1)(t+2) = 0 \Rightarrow t = 1 \text{ or } t = -2$$

5.

$$x^2 - 15x + 56 = 0 \Rightarrow x^2 - 7x - 8x + 56 = 0$$

$$\Rightarrow x(x-7) - 8(x-7) = 0 \Rightarrow (x-8)(x-7) = 0$$

$$x-8=0 \Rightarrow x=8$$

$$x-7=0 \Rightarrow x=7$$

$$y + \frac{10}{y} = 8 \Rightarrow y^2 + 10 = 8y \Rightarrow y^2 - 8y + 10 = 0$$

$$\left\{ \begin{array}{l} ax^2 + bx + c = 0 \\ y^2 - 8y + 10 = 0 \end{array} \right\} \quad a=1, b=-8, c=10$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(10)}}{2(1)}$$

$$y = \frac{8 \pm \sqrt{24}}{2} = \frac{8 \pm \sqrt{4}\sqrt{6}}{2} = \frac{8 \pm 2\sqrt{6}}{2} = 4 \pm \sqrt{6}$$

$$y + \frac{10}{y} = 7 \Rightarrow y^2 + 10 = 7y \Rightarrow y^2 - 7y + 10 = 0$$

$$\Rightarrow y^2 - 5y - 2y + 10 = 0 \Rightarrow y(y-5) - 2(y-5) = 0$$

$$\Rightarrow (y-2)(y-5) = 0 \Rightarrow y=2 \text{ or } y=5$$

6.

$$x^2 - 8x + 15 = 0 \Rightarrow x^2 - 3x - 5x + 15 = 0$$

$$\Rightarrow x(x-3) - 5(x-3) = 0 \Rightarrow (x-5)(x-3) = 0$$

Ans. 1: $x = 5$ or $x = 3$

$$\sqrt{p} + \sqrt{p+3} = 5 \Rightarrow \sqrt{p+3} = 5 - \sqrt{p}$$

$$(\sqrt{p+3})^2 = (5 - \sqrt{p})^2$$

$$p+3 = (5)^2 - 2(5)(\sqrt{p}) + (\sqrt{p})^2$$

$$\cancel{p} + 3 = 25 - 10\sqrt{p} + \cancel{p}$$

$$\Rightarrow 10\sqrt{p} \Rightarrow 25 - 3 = 22$$

$$\Rightarrow 100p = 22^2 = 484 = p \Rightarrow \frac{484}{100} = \frac{121}{25}$$

CHECK: $\sqrt{p} + \sqrt{p+3} = 5$

$$\sqrt{\left(\frac{121}{25}\right)} + \sqrt{\left(\frac{121}{25}\right) + 3}$$

$$= \frac{11}{5} + \sqrt{\frac{121+75}{25}} = \frac{11}{5} + \sqrt{\frac{196}{25}} = 5 \text{ TRUE}$$

$$\sqrt{p} + \sqrt{p+3} = 3 \Rightarrow \sqrt{p+3} = 3 - \sqrt{p}$$

$$(\sqrt{p+3})^2 = (3 - \sqrt{p})^2$$

$$p+3 = (3)^2 - 2(3)(\sqrt{p}) + (\sqrt{p})^2$$

$$\cancel{p} + 3 = 9 - 6\sqrt{p} + \cancel{p}$$

$$\Rightarrow 6\sqrt{p} \Rightarrow 9 - 3 = 6 \Rightarrow \sqrt{p} \Rightarrow 1$$

$$\Rightarrow p = 1^2 = 1$$

CHECK: $\sqrt{p} + \sqrt{p+3} = 3$

$$\sqrt{(1)} + \sqrt{(1)+3} = 1 + \sqrt{4} = 3 \text{ TRUE}$$

Ans. 2: $p = \frac{121}{25}$ or $p = 1$

7.

$$x + y = 7 \Rightarrow y = 7 - x$$

$$xy = 10 \Rightarrow x(7 - x) = 10 \Rightarrow 7x - x^2 = 10$$

$$\Rightarrow 0 = x^2 - 7x + 10 = x^2 - 2x - 5x + 10$$

$$\Rightarrow x(x - 2) - 5(x - 2) = 0 \Rightarrow (x - 5)(x - 2) = 0$$

$$x = 5 \Rightarrow y = 7 - (5) = 2$$

$$x = 2 \Rightarrow y = 7 - (2) = 5$$

Ans. (5,2) or (2,5)

8.

$$2x + y = 7 \Rightarrow y = 7 - 2x$$

$$xy = 3 \Rightarrow x(7 - 2x) = 3 \Rightarrow 7x - 2x^2 = 3$$

$$\Rightarrow 0 = 2x^2 - 7x + 3 = 2x^2 - 1x - 6x + 3$$

$$\Rightarrow x(2x - 1) - 3(2x - 1) = 0 \Rightarrow (x - 3)(2x - 1) = 0$$

$$x - 3 \Rightarrow x = 3$$

$$2x - 1 \Rightarrow x = \frac{1}{2}$$

$$x = 3 \Rightarrow y = 7 - 2(3) = 1$$

$$x = \frac{1}{2} \Rightarrow y = 7 - 2\left(\frac{1}{2}\right) = 6$$

Ans. (3,1) or $\left(\frac{1}{2}, 6\right)$

9.

$$x - y = 1 \Rightarrow x = 1 + y$$

$$x^2 + y^2 = 5 \Rightarrow (1 + y)^2 + y^2 = 5 \Rightarrow 1 + 2y + y^2 + y^2 = 5$$

$$\Rightarrow 2y^2 + 2y - 4 = 0 \Rightarrow y^2 + y - 2 = 0 \Rightarrow y^2 - y + 2y - 2 = 0$$

$$\Rightarrow y(y - 1) + 2(y - 1) = 0 \Rightarrow (y + 2)(y - 1) = 0$$

$$\Rightarrow y = -2 \text{ or } y = 1$$

$$y = -2 \Rightarrow x = 1 + (-2) = -1$$

$$y = 1 \Rightarrow x = 1 + (1) = 1$$

Ans. (1,1) or (-1,-2)

10.

$$x + y = 5 \Rightarrow y = 5 - x$$

$$\begin{aligned} x^2 + y^2 = 13 &\Rightarrow x^2 + (5 - x)^2 = 13 \Rightarrow x^2 + 25 - 10x + x^2 = 13 \\ &\Rightarrow 2x^2 - 10x + 12 = 0 \Rightarrow x^2 - 5x + 6 = 0 \Rightarrow x^2 - 3x - 2x + 6 = 0 \\ &\Rightarrow x(x - 3) - 2(x - 3) = 0 \Rightarrow (x - 2)(x - 3) = 0 \\ &\Rightarrow x = 2 \text{ or } x = 3 \end{aligned}$$

$$x = 2 \Rightarrow y = 5 - (2) = 3$$

$$x = 3 \Rightarrow y = 5 - (3) = 2$$

Ans. (2,3) or (3,2)

11.

$$2a - b = 1 \Rightarrow 2a - 1 = b$$

$$\begin{aligned} ab = 6 &\Rightarrow a(2a - 1) = 6 \Rightarrow 2a^2 - a = 6 \\ &\Rightarrow 2a^2 - a - 6 = 0 \Rightarrow 2a^2 - 4a + 3a - 6 = 0 \\ &\Rightarrow 2a(a - 2) + 3(a - 2) = 0 \Rightarrow (2a + 3)(a - 2) = 0 \end{aligned}$$

$$2a + 3 \Rightarrow a = -\frac{3}{2}$$

$$a - 2 \Rightarrow a = 2$$

$$a = -\frac{3}{2} \Rightarrow b = 2\left(-\frac{3}{2}\right) - 1 = -4$$

$$a = 2 \Rightarrow b = 2(2) - 1 = 3$$

Ans. $\left(-\frac{3}{2}, -4\right)$ or (2,3)

12.

$$x + 2y = 5 \Rightarrow x = 5 - 2y$$

$$\begin{aligned} x^2 + y^2 = 5 &\Rightarrow (5 - 2y)^2 + y^2 = 5 \Rightarrow 25 - 20y + 4y^2 + y^2 = 5 \\ &\Rightarrow 5y^2 - 20y + 20 = 0 \Rightarrow y^2 - 4y + 4 = 0 \Rightarrow y^2 - 2y - 2y + 4 = 0 \\ &\Rightarrow y(y - 2) - 2(y - 2) = 0 \Rightarrow (y - 2)(y - 2) = 0 \\ &\Rightarrow y = 2 \end{aligned}$$

$$y = 2 \Rightarrow x = 5 - 2(2) = 1$$

Ans. (1,2)

13.

$$x - y = 0 \Rightarrow x = y$$

$$\begin{aligned} x^2 + y^2 - 3x + y - 12 = 0 &\Rightarrow x^2 + x^2 - 3x + x - 12 = 0 \\ &\Rightarrow 2x^2 - 2x - 12 = 0 \Rightarrow x^2 - x - 6 = 0 \Rightarrow x^2 - 3x + 2x - 6 = 0 \\ &\Rightarrow x(x - 3) + 2(x - 3) = 0 \Rightarrow (x + 2)(x - 3) = 0 \\ &\Rightarrow x = -2 \text{ or } x = 3 \end{aligned}$$

$$x = -2 \Rightarrow y = -2$$

$$x = 3 \Rightarrow y = 3$$

Ans. (-2, -2) or (3,3)

14.

$$x = 0$$

$$\begin{aligned} x^2 + y^2 - 3x - 2y - 15 = 0 &\Rightarrow y^2 - 2y - 15 = 0 \Rightarrow \\ &\Rightarrow y^2 - 5y + 3y - 15 = 0 \Rightarrow y(y - 5) + 3(y - 5) = 0 \\ &\Rightarrow (y + 3)(y - 5) = 0 \Rightarrow y = -3 \text{ or } y = 5 \end{aligned}$$

Ans. (0, -3) or (0,5)

15.

$$2x - 3y = 13 \Rightarrow 2x = 13 + 3y \Rightarrow x = \frac{13 + 3y}{2} = 6.5 + 1.5y$$

$$x^2 + y^2 = 13 \Rightarrow (6.5 + 1.5y)^2 + y^2 = 13$$

$$\Rightarrow 6 \cdot 5^2 + 2 \times 6 \cdot 5 \times 1.5y + (1.5y)^2 + y^2 = 13$$

$$\Rightarrow 42 \cdot 25 + 19 \cdot 5y + 2 \cdot 25y^2 + y^2 = 13 \Rightarrow 169 + 78y + 9y^2 + 4y^2 = 52$$

$$\Rightarrow 13y^2 + 78y + 117 = 0 \Rightarrow y^2 + 6y + 9 = 0 \Rightarrow (y + 3)^2 = 0 \Rightarrow y = -3$$

$$x = 6.5 + 1.5(-3) = 2$$

Ans. (2, -3)

16.

$$5x - 2y = 3 \Rightarrow 5x = 3 + 2y \Rightarrow x = \frac{3 + 2y}{5}$$

$$x^2 + y^2 - 2xy = 0 \Rightarrow \left(\frac{3 + 2y}{5}\right)^2 + y^2 - 2\left(\frac{3 + 2y}{5}\right)y = 0$$

$$\Rightarrow \frac{1}{25}(3 + 2y)^2 + y^2 - \frac{2}{5}(3 + 2y)y = 0$$

$$\Rightarrow 25 \times \frac{1}{25}(3 + 2y)^2 + 25 \times y^2 - 25 \times \frac{2}{5}(3 + 2y)y = 0$$

$$\Rightarrow (3 + 2y)^2 + 25y^2 - 10(3 + 2y)y = 0$$

$$\Rightarrow 9 + 12y + 4y^2 + 25y^2 - 30y - 20y^2 = 0$$

$$\Rightarrow 9y^2 - 18y + 9 = 0 \Rightarrow y^2 - 2y + 1 = (y - 1)^2 = 0$$

$$\Rightarrow y = 1$$

$$y = 1 \Rightarrow x = \frac{3 + 2(1)}{5} = 1$$

Ans. (1, 1)

17.

$$2 \times \frac{1}{2}(x+y) = 2 \times \frac{5}{2} \Rightarrow x+y=5 \Rightarrow y=5-x$$

$$xy+6=3x+2y \Rightarrow x(5-x)+6=3x+2(5-x)$$

$$\Rightarrow 5x-x^2+6=3x+10-2x$$

$$\Rightarrow 0 = -5x+x^2-6+3x+10-2x$$

$$\Rightarrow 0 = x^2-4x+4 = (x-2)^2 \Rightarrow x=2$$

$$y=5-x=5-2=3$$

Ans. (2,3)

18.

$$x+y=7 \Rightarrow y=7-x$$

$$x^2+y^2=26 \frac{1}{2} \Rightarrow x^2+(7-x)^2=26 \frac{1}{2}$$

$$\Rightarrow x^2+49-14x+x^2=26 \frac{1}{2} \Rightarrow 2x^2-14x+22 \frac{1}{2}=0$$

$$\Rightarrow 4x^2-28x+45=0 \Rightarrow 4x^2-18x-10x+45=0$$

$$\Rightarrow 2x(2x-9)-5(2x-9)=0 \Rightarrow (2x-5)(2x-9)=0$$

$$2x-5=0 \Rightarrow x=\frac{5}{2}$$

$$2x-9=0 \Rightarrow x=\frac{9}{2}$$

$$x=\frac{5}{2} \Rightarrow y=7-\left(\frac{5}{2}\right)=\frac{9}{2}$$

$$x=\frac{9}{2} \Rightarrow y=7-\left(\frac{9}{2}\right)=\frac{5}{2}$$

Ans. The two numbers are $\frac{5}{2}$ and $\frac{9}{2}$

19. (i)

$$\text{Perimeter} = 60 \Rightarrow x+y+26=60 \Rightarrow x+y=34$$

$$\text{Pythagoras}' \Rightarrow x^2+y^2=26^2 \Rightarrow x^2+y^2=676$$

(ii)

$$x + y = 34 \Rightarrow y = 34 - x$$

$$x^2 + y^2 = 676 \Rightarrow x^2 + (34 - x)^2 = 676$$

$$\Rightarrow x^2 + 1156 - 68x + x^2 = 676 \Rightarrow 2x^2 - 68x + 480 = 0$$

$$\Rightarrow x^2 - 34x + 240 = 0 \Rightarrow x^2 - 10x - 24x + 240 = 0$$

$$x(x - 10) - 24(x - 10) = 0 \Rightarrow (x - 24)(x - 10) = 0$$

$$\Rightarrow x = 24 \text{ or } x = 10$$

$$\Rightarrow y = 34 - 24 = 10 \text{ or } y = 34 - 10 = 24$$

$$x > y \Rightarrow x = 24 \text{ and } y = 10$$

20.

$$x + y = 10 \Rightarrow y = 10 - x$$

$$x^2 + y^2 = 58 \Rightarrow x^2 + (10 - x)^2 = 58$$

$$\Rightarrow x^2 + 100 - 20x + x^2 = 58 \Rightarrow 2x^2 - 20x + 42 = 0$$

$$\Rightarrow x^2 - 10x + 21 = 0 \Rightarrow x^2 - 3x - 7x + 21 = 0$$

$$\Rightarrow x(x - 3) - 7(x - 3) = 0 \Rightarrow (x - 7)(x - 3) = 0$$

$$\Rightarrow x = 3 \text{ or } x = 7$$

$$x < y \Rightarrow x = 3 \text{ and } y = 10 - 3 = 7$$

21.

$$0 = x^4 - 10x^2 + 9 = x^4 - x^2 - 9x^2 + 9$$

$$= x^2(x^2 - 1) - 9(x^2 - 1) = (x^2 - 9)(x^2 - 1)$$

$$x^2 - 1 = 0 \Rightarrow x = \pm 1$$

$$x^2 - 9 = 0 \Rightarrow x = \pm 3$$