

Oliver Murphy: Discovering Maths 4: EXERCISE 2B

1.

$$\text{A: } x + y + z = 6$$

$$\text{B: } 3x - 2y + 3z = 8$$

$$\text{C: } 2x + y - 3z = -5$$

$$2 \times \text{A: } 2x + 2y + 2z = 12$$

$$\text{B: } \underline{3x - 2y + 3z = 8}$$

$$\text{ADD: } 5x + 5z = 20 \Rightarrow x + z = 4 \quad \text{D}$$

$$\text{B: } 3x - 2y + 3z = 8$$

$$2 \times \text{C: } \underline{4x + 2y - 6z = -10}$$

$$\text{ADD: } 7x - 3z = -2 \quad \text{E}$$

$$\text{E: } 7x - 3z = -2$$

$$3 \times \text{D: } \underline{3x + 3z = 12}$$

$$\text{ADD: } 10x = 10 \Rightarrow x = 1$$

Substitute $x = 1$ in D:

$$(1) + z = 4 \Rightarrow z = 3$$

Substitute $x = 1$ and $z = 3$ in A:

$$(1) + y + (3) = 6 \Rightarrow y + 4 = 6 \Rightarrow y = 2$$

Ans: $x = 1$, $y = 2$, $z = 3$

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2.

A: $2x + y + 4z = 23$

B: $5x + y + 2z = 19$

C: $3x - 2y + z = 13$

B: $5x + y + 2z = 19$

-A: $\underline{-2x - y - 4z = -23}$

ADD: $3x - 2z = -4$ D

$2 \times B$: $10x + 2y + 4z = 38$

C: $\underline{3x - 2y + z = 13}$

ADD: $13x + 5z = 51$ E

$5 \times D$: $15x - 10z = -20$

$2 \times E$: $\underline{26x + 10z = 102}$

ADD: $41x = 82 \Rightarrow x = 2$

Substitute $x = 2$ in E:

$$13(2) + 5z = 51 \Rightarrow 5z = 51 - 26 = 25 \Rightarrow z = 5$$

Substitute $x = 2$ and $z = 5$ in A:

$$2(2) + y + 4(5) = 23 \Rightarrow y + 24 = 23 \Rightarrow y = -1$$

Ans: $x = 2, y = -1, z = 5$

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3.

A: $x + y + z = 1$

B: $2x + 3y + z = 4$

C: $4x + 9y + z = 16$

B: $2x + 3y + z = 4$

-A: $\underline{-x - y - z = -1}$

ADD: $x + 2y = 3$ D

C: $4x + 9y + z = 16$

-B: $\underline{-2x - 3y - z = -4}$

ADD: $2x + 6y = 12 \Rightarrow x + 3y = 6$ E

E: $x + 3y = 6$

-D: $\underline{-x - 2y = -3}$

ADD: $y = 3$

Substitute $y = 3$ in D:

$$x + 2(3) = 3 \Rightarrow x = -3$$

Substitute $x = -3$ and $y = 3$ in A:

$$(-3) + (3) + z = 1 \Rightarrow z = 1$$

Ans: $x = -3, y = 3, z = 1$

4.

A: $a - b + c = 4$

B: $a + b - c = 2$

C: $a + b + c = 8$

A: $a - b + c = 4$

B: $\underline{a + b - c = 2}$

ADD: $2a = 6 \Rightarrow a = 3$

B: $a + b - c = 2$

C: $\underline{a + b + c = 8}$

ADD: $2a + 2b = 10$

Substitute $a = 3 \Rightarrow 2(3) + 2b = 10$

$$\Rightarrow 2b = 10 - 6 = 4 \Rightarrow b = 2$$

Substitute $a = 3$ and $b = 2$ in C:

$$(3) + (2) + c = 8 \Rightarrow c = 3$$

Ans: $a = 3, b = 2, c = 3$

5.

A: $2x + y + z = -7$

B: $x + 2y + z = -8$

C: $x + y + 2z = -9$

A: $2x + y + z = -7$

-B: $\underline{-x - 2y - z = 8}$

ADD: $x - y = 1$ D

$2 \times$ A: $4x + 2y + 2z = -14$

-C: $\underline{-x - y - 2z = 9}$

ADD: $3x + y = -5$ E

D: $x - y = 1$

E: $\underline{3x + y = -5}$

ADD: $4x = -4 \Rightarrow x = -1$

Substitute $x = -1$ in E:

$$3(-1) + y = -5 \Rightarrow y = -5 + 3 \Rightarrow y = -2$$

Substitute $x = -1$ and $y = -2$ in A:

$$2(-1) + (-2) + z = -7 \Rightarrow z - 4 = -7 \Rightarrow z = -3$$

Ans: $x = -1, y = -2, z = -3$

6.

A: $x + y = z$

B: $3x + 2y - 4z = -1$

C: $x - 3y + 3z = 2$

Substitute A: $x + y = z$ in B:

B: $3x + 2y - 4(x + y) = -1$

$\Rightarrow 3x + 2y - 4x - 4y = -1$

$\Rightarrow -x - 2y = -1 \quad \Rightarrow x + 2y = 1 \quad \text{D}$

Substitute A: $x + y = z$ in C:

C: $x - 3y + 3(x + y) = 2$

$\Rightarrow x - 3y + 3x + 3y = 2$

$\Rightarrow 4x = 2 \quad \Rightarrow x = \frac{2}{4} = \frac{1}{2}$

Substitute $x = \frac{1}{2}$ in D:

$$\left(\frac{1}{2}\right) + 2y = 1$$

$\Rightarrow 2y = 1 - \frac{1}{2} = \frac{1}{2} \quad \Rightarrow y = \frac{1}{4}$

Substitute $x = \frac{1}{2}, y = \frac{1}{4}$ in A:

$$z = \left(\frac{1}{2}\right) + \left(\frac{1}{4}\right) = \frac{3}{4}$$

Ans: $x = \frac{1}{2}, y = \frac{1}{4}, z = \frac{3}{4}$

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7.

A: $x + y = z$

B: $x + z = 11$

C: $y + z = 7$

Substitute A: $x + y = z$ in B:

B: $x + (x + y) = 11$

$\Rightarrow 2x + y = 11$ D

Substitute A: $x + y = z$ in C:

C: $y + (x + y) = 7$

$\Rightarrow x + 2y = 7$ E

$2 \times E:$ $2x + 4y = 14$

$-D:$ $\underline{-2x - y = -11}$

ADD: $3y = 3 \Rightarrow y = 1$

Substitute $y = 1$ in E:

$x + 2(1) = 7 \Rightarrow x = 5$

Substitute $x = 5, y = 1$ in A:

$z = (5) + (1) = 6$

Ans: $x = 5, y = 1, z = 6$

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8.

$$\text{A:} \quad 5x - 4y + z = 3$$

$$\text{B:} \quad 3x + y - 2z = 31$$

$$\text{C:} \quad x + 4y = 21$$

$$2 \times \text{A:} \quad 10x - 8y + 2z = 6$$

$$\text{B:} \quad \underline{3x + y - 2z = 31}$$

$$\text{ADD:} \quad 13x - 7y = 37 \quad \text{D}$$

$$4 \times \text{D:} \quad 52x - 28y = 148$$

$$7 \times \text{C:} \quad \underline{7x + 28y = 147}$$

$$\text{ADD:} \quad 59x = 295 \quad \Rightarrow x = 5$$

Substitute $x = 5$ in D:

$$13(5) - 7y = 37$$

$$\Rightarrow 65 - 37 = 7y = 28 \quad \Rightarrow y = 4$$

Substitute $x = 5$ and $y = 4$ in A:

$$5(5) - 4(4) + z = 3$$

$$\Rightarrow 25 - 16 + z = 3 \Rightarrow 9 + z = 3 \Rightarrow z = -6$$

Ans: $x = 5$, $y = 4$, $z = -6$

9.

A: $x + y + z = 5$

B: $2x - y + 3z = 3$

C: $3x + 2y - 5z = 21$

A: $x + y + z = 5$

B: $2x - y + 3z = 3$

ADD: $3x + 4z = 8$ D

$2 \times B$: $4x - 2y + 6z = 6$

C: $3x + 2y - 5z = 21$

ADD: $7x + z = 27$ E

$4 \times E$: $28x + 4z = 108$

$-D$: $-3x - 4z = -8$

ADD: $25x = 100 \Rightarrow x = 4$

Substitute $x = 4$ in E:

$$7(4) + z = 27 \Rightarrow z = -1$$

Substitute $x = 4$ and $z = -1$ in A:

$$(4) + y + (-1) = 5 \Rightarrow y = 2$$

Ans 1: $x = 4, y = 2, z = -1$

$$a^2 = x = 4 \Rightarrow a = \pm 2$$

$$2b = y = 2 \Rightarrow b = 1$$

$$c + 1 = z = -1 \Rightarrow c = -2$$

Ans 2: $a = \pm 2, b = 1, c = -2$

10.

A: $x + y + z = 6$

B: $x - 2y - z = -2$

C: $3x - 5y + z = 0$

A: $x + y + z = 6$

B: $x - 2y - z = -2$

ADD: $2x - y = 4$ D

B: $x - 2y - z = -2$

C: $3x - 5y + z = 0$

ADD: $4x - 7y = -2$ E

$7 \times D$: $14x - 7y = 28$

$-E$: $-4x + 7y = 2$

ADD: $10x = 30 \Rightarrow x = 3$

Substitute $x = 3$ in D:

$$2(3) - y = 4 \Rightarrow 6 - 4 = y = 2$$

Substitute $x = 3$ and $y = 2$ in A:

$$(3) + (2) + z = 6 \Rightarrow z = 6 - 5 = 1$$

Ans 1: $x = 3, y = 2, z = 1$

$$x = 2a + 1 = 3 \Rightarrow 2a = 2 \Rightarrow a = 1$$

$$y = b - 1 = 2 \Rightarrow b = 3$$

$$z = (c - 3)^2 = 1$$

$$\Rightarrow c^2 - 6c + 9 = 1 \Rightarrow c^2 - 6c + 8 = 0$$

$$\Rightarrow c^2 - 2c - 4c + 8 = 0 \Rightarrow c(c - 2) - 4(c - 2) = 0$$

$$\Rightarrow (c - 4)(c - 2) = 0 \Rightarrow c = 4 \text{ or } c = 2$$

Ans 2: $a = 1, b = 3, c = 4 \text{ or } 2$

11 (a)

$$\frac{1}{x} = \frac{2}{13} \Rightarrow x = \frac{13}{2}$$

$$\frac{1}{x+y} = \frac{2}{7} \Rightarrow x+y = \frac{7}{2} \Rightarrow y = \frac{7}{2} - \frac{13}{2} = -\frac{6}{2} = -3$$

(b)

$$\frac{1}{x} = \frac{2}{13} \Rightarrow x = \frac{13}{2}$$

$$\frac{1}{x} + \frac{1}{y} = \frac{2}{7} \Rightarrow \frac{1}{y} = \frac{2}{7} - \frac{2}{13} = \frac{12}{91}$$

$$\Rightarrow y = \frac{91}{12}$$

12.

A: $x + 3y - 8 = 0 \Rightarrow x + 3y = 8$

B: $y - 2z - 5 = 0 \Rightarrow y - 2z = 5$

C: $z - x = 0 \Rightarrow z = x$

Substitute in $z = x$ in B:

$$y - 2(x) = 5 \Rightarrow -2x + y = 5 \quad \text{D}$$

$2 \times \text{A:}$ $2x + 6y = 16$

D: $\frac{-2x + y = 5}{}$

ADD: $7y = 21 \Rightarrow y = 3$

Substitute in $y = 3$ in A:

$$x + 3(3) = 8 \Rightarrow x = -1$$

C: $z = x = -1$

Ans: $x = -1, y = 3, z = -1$

13.

A: $a + b + c = 192$

B: $2a + 5b = 284$

C: $3a + b + 3c = 480$

$3 \times \text{A:}$ $3a + 3b + 3c = 576$

$-\text{C:}$ $\frac{-3a - b - 3c = -480}{}$

ADD: $2b = 96 \Rightarrow b = 48$

Substitute in $b = 48$ in B:

$$2a + 5(48) = 284 \Rightarrow 2a = 284 - 240 = 44$$

$$\Rightarrow a = 22$$

Substitute in $a = 22$ and $b = 48$ in A:

$$(22) + (48) + c = 192 \Rightarrow c = 192 - 48 - 22 = 122$$

Ans: $a = 22, b = 48, c = 122$

14.

$$\frac{1}{x} = \frac{2}{3} \Rightarrow x = \frac{3}{2}$$

$$\frac{1}{x+y} = \frac{2}{5} \Rightarrow x+y = \frac{5}{2} \Rightarrow y = \frac{5}{2} - \frac{3}{2} = \frac{2}{2} = 1$$

$$\frac{1}{x+y+z} = 1 \Rightarrow x+y+z = 1 \Rightarrow z = 1 - \frac{3}{2} - 1 = -\frac{3}{2}$$

Ans. $x = \frac{3}{2}$, $y = 1$, $z = -\frac{3}{2}$

15.

$$\frac{1}{x} = \boxed{1\frac{1}{4}} = \frac{5}{4} \Rightarrow x = \frac{4}{5}$$

$$\frac{1}{x} + \frac{1}{y} = 2\frac{1}{3} \Rightarrow \frac{1}{y} = 2\frac{1}{3} - \boxed{1\frac{1}{4}} = 1\frac{1}{12} = \frac{13}{12} \Rightarrow y = \frac{12}{13}$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 3\frac{1}{2} \Rightarrow \frac{1}{z} = 3\frac{1}{2} - 1\frac{1}{4} - 1\frac{1}{12} = \frac{7}{6} \Rightarrow z = \frac{6}{7}$$

Ans. $x = \frac{4}{5}$, $y = \frac{12}{13}$, $z = \frac{6}{7}$

16.

$$\text{A: } x + y - 2z = 316$$

$$\text{B: } 2x + y - 3z = 654$$

$$\text{C: } 3x + 4y - 6z = 279$$

$$2 \times \text{B: } 4x + 2y - 6z = 1308$$

$$-3 \times \text{A: } \underline{-3x - 3y + 6z = -948}$$

$$\text{ADD: } x - y = 360 \quad \text{D}$$

$$\text{C: } 3x + 4y - 6z = 279$$

$$-3 \times \text{A: } \underline{-3x - 3y + 6z = -948}$$

$$\text{ADD: } y = -669$$

Substitute $y = -669$ in D:

$$x - (-669) = 360 \Rightarrow x = 360 - 669 = -309$$

Substitute $x = -309$ and $y = -669$ in A:

$$(-309) + (-669) - 2z = 316$$

$$\Rightarrow -309 - 669 - 316 = 2z = -1294 \Rightarrow z = -647$$

Ans 1: $x = -309$, $y = -669$, $z = -647$

$$x = 3a = -309 \Rightarrow a = -103$$

$$y = b = -669$$

$$z = c - 1 = -647 \Rightarrow c = -646$$

Ans 2: $a = -103$, $b = -669$, $c = -646$

17.

$$\text{A:} \quad x + y = 3$$

$$\text{B:} \quad 3x - 2y = 14$$

$$2 \times \text{A:} \quad 2x + 2y = 6$$

$$\text{B:} \quad \underline{3x - 2y = 14}$$

$$\text{ADD:} \quad 5x = 20 \Rightarrow x = 4$$

Substitute in A:

$$(4) + y = 3 \Rightarrow 3 - 4 \Rightarrow y = -1$$

Ans 1: $x = 4, y = -1$

$$x = \frac{1}{a} = 4 \Rightarrow a = \frac{1}{4}$$

$$y = \frac{1}{b} = -1 \Rightarrow b = \frac{1}{-1} = -1$$

18.

$$\text{A:} \quad 2x - 5y + 6z = 7$$

$$\text{B:} \quad x - y - z = 3$$

$$\text{C:} \quad 3x + 2y + z = 4$$

$$\text{A:} \quad 2x - 5y + 6z = 7$$

$$6 \times \text{B:} \quad \underline{6x - 6y - 6z = 18}$$

$$\text{ADD:} \quad 8x - 11y = 25 \quad \text{D}$$

$$\text{B:} \quad x - y - z = 3$$

$$\text{C:} \quad \underline{3x + 2y + z = 4}$$

$$\text{ADD:} \quad 4x + y = 7 \quad \text{E}$$

$$\text{D:} \quad 8x - 11y = 25$$

$$11 \times \text{E:} \quad \underline{44x + 11y = 77}$$

$$\text{ADD:} \quad 52x = 102 \Rightarrow x = \frac{102}{52} = \frac{51}{26}$$

Substitute $x = \frac{51}{26}$ in E:

$$4\left(\frac{51}{26}\right) + y = 7 \Rightarrow y = 7 - \frac{204}{26} = -\frac{11}{13}$$

Substitute $x = \frac{51}{26}$ and $y = -\frac{11}{13}$ in C:

$$3\left(\frac{51}{26}\right) + 2\left(-\frac{11}{13}\right) + z = 4$$

$$\Rightarrow z = 4 - \frac{109}{26} = -\frac{5}{26}$$

$$\text{Ans 1: } x = \frac{51}{26}, y = -\frac{11}{13}, z = -\frac{5}{26}$$

$$x = \frac{1}{a} = \frac{51}{26} \Rightarrow a = \frac{26}{51}$$

$$y = \frac{1}{b} = -\frac{11}{13} \Rightarrow b = -\frac{13}{11}$$

$$z = \frac{1}{c} = -\frac{5}{26} \Rightarrow c = -\frac{26}{5}$$

$$\text{Ans 2: } a = \frac{26}{51}, b = -\frac{13}{11}, c = -\frac{26}{5}$$

19.

$$x = \frac{1}{a}, y = \frac{1}{b}, z = \frac{1}{c} \Rightarrow$$

$$\text{A:} \quad 2x + y + z = 1$$

$$\text{B:} \quad x - 2y + 2z = 1$$

$$\text{C:} \quad 4x - y + 3z = 1$$

$$2 \times \text{A:} \quad 4x + 2y + 2z = 2$$

$$\text{B:} \quad \underline{x - 2y + 2z = 1}$$

$$\text{ADD:} \quad 5x + 4z = 3 \quad \text{D}$$

$$\text{A:} \quad 2x + y + z = 1$$

$$\text{C:} \quad \underline{4x - y + 3z = 1}$$

$$\text{ADD:} \quad 6x + 4z = 2 \Rightarrow 3x + 2z = 1 \quad \text{E}$$

$$2 \times \text{E:} \quad 6x + 4z = 2$$

$$-\text{D:} \quad \underline{-5x - 4z = -3}$$

$$\text{ADD:} \quad x = -1$$

Substitute $x = -1$ in E:

$$3(-1) + 2z = 1 \Rightarrow 2z = 4 \Rightarrow z = 2$$

Substitute $x = -1$ and $z = 2$ in A:

$$2(-1) + y + (2) = 1 \Rightarrow y = 1$$

Ans 1: $x = -1, y = 1, z = 2$

$$x = \frac{1}{a} = -1 \Rightarrow a = -1$$

$$y = \frac{1}{b} = 1 \Rightarrow b = 1$$

$$z = \frac{1}{c} = 2 \Rightarrow c = \frac{1}{2}$$

Ans 2: $a = -1, b = 1, c = \frac{1}{2}$

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20.

$$\begin{aligned} \text{A:} \quad & 40\left(\frac{2x}{5}\right) + 40\left(\frac{y}{8}\right) + 40 \times z = 40\left(\frac{5}{2}\right) \\ & \Rightarrow 16x + 5y + 40z = 100 \end{aligned}$$

$$\begin{aligned} \text{B:} \quad & 6\left(\frac{x+1}{3}\right) - 6\left(\frac{y}{2}\right) - 6(4z) = 0 \\ & \Rightarrow 2x + 2 - 3y - 24z = 0 \\ & \Rightarrow 2x - 3y - 24z = -2 \end{aligned}$$

$$\text{C:} \quad \frac{x+y+z}{2} = 1 \Rightarrow x+y+z = 2$$

$$3 \times \text{A:} \quad 48x + 15y + 120z = 300$$

$$5 \times \text{B:} \quad \underline{10x - 15y - 120z = -10}$$

$$\text{ADD:} \quad 58x = 290 \Rightarrow x = 5$$

$$\text{B:} \quad 2x - 3y - 24z = -2$$

$$3 \times \text{C:} \quad \underline{3x + 3y + 3z = 6}$$

$$\text{ADD:} \quad 5x - 21z = 4 \qquad \qquad \qquad \text{D}$$

Substitute $x = 5$ in D:

$$5(5) - 21z = 4 \Rightarrow 25 - 4 = 21z \Rightarrow z = 1$$

Substitute $x = 5$ and $z = 1$ in C:

$$(5) + y + (1) = 2 \Rightarrow y + 6 = 2 \Rightarrow y = -4$$

Ans: $x = 5$, $y = -4$, $z = 1$