

LC HIGHER MATHS: TEST ON CHAPTER ONE

1(a) 5 marks (b) 5 marks (c) 5 marks

2(a) 10 marks (b) 15 marks

3. 10 marks

QUESTION ONE

Factorise (a) $8a^3 - 125b^3$ (b) $2x^2 - 32$

(c) Rationalise the denominator: $\frac{(1-\sqrt{2})}{(7-\sqrt{5})}$

ANSWER

$$\begin{aligned} \text{(a) } 8a^3 - 125b^3 &= (2a)^3 - (5b)^3 \\ &= (2a - 5b) \left((2a)^2 + (2a)(5b) + (5b)^2 \right) \\ &= (2a - 5b) (4a^2 + 10ab + 25b^2) \end{aligned}$$

$$\text{(b) } 2x^2 - 32 = 2(x^2 - 16) = 2(x - 4)(x + 4)$$

$$\begin{aligned} \text{(c) } \frac{(1-\sqrt{2})(7+\sqrt{5})}{(7-\sqrt{5})(7+\sqrt{5})} &= \frac{1(7+\sqrt{5}) - \sqrt{2}(7+\sqrt{5})}{7^2 - (\sqrt{5})^2} \\ &= \frac{7 + \sqrt{5} - 7\sqrt{2} - \sqrt{2}\sqrt{5}}{49 - 5} \\ &= \frac{7 + \sqrt{5} - 7\sqrt{2} - \sqrt{2}\sqrt{5}}{44} \end{aligned}$$

QUESTION TWO

(a) Write the solutions to the quadratic equation

$$9x^2 - 12x - 1 = 0$$

in the form $\frac{a \pm \sqrt{b}}{c}$ where $a, b \in \mathbb{N}$.

ANSWER

$$\left. \begin{array}{l} ax^2 + bx + c = 0 \\ 9x^2 - 12x - 1 = 0 \end{array} \right\} a = 9, b = -12, c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(9)(-1)}}{2(9)}$$

$$x = \frac{12 \pm \sqrt{180}}{18} = \frac{12 \pm \sqrt{36} \sqrt{5}}{18} = \frac{12 \pm 6\sqrt{5}}{18} = \frac{2 \pm \sqrt{5}}{3}$$

LC HIGHER MATHS: TEST ON CHAPTER ONE

(b) Solve the equation $x - 2 = \sqrt{x+18}$

ANSWER

$$(x-2)^2 = (\sqrt{x+18})^2$$

$$x^2 - 4x + 4 = x + 18$$

$$x^2 - 4x + 4 - x - 18 = 0$$

$$x^2 - 5x - 14 = x^2 + 2x - 7x - 14 = 0$$

$$x(x+2) - 7(x+2) = 0$$

$$(x-7)(x+2) = 0 \Rightarrow x = 7 \text{ or } x = -2$$

$$\text{CHECK: } x - 2 = \sqrt{x+18}$$

$$(7) - 2 = \sqrt{(7)+18} \Rightarrow 5 = 5 \text{ TRUE}$$

$$(-2) - 2 = \sqrt{(-2)+18} \Rightarrow -4 = 4 \text{ FALSE}$$

$$\text{ANSWER: } x = 7$$

QUESTION THREE

Show that $p^3 + q^3 - (p+q)^3 = -3pq(p+q)$

ANSWER

$$\begin{aligned}(p+q)^3 &= (p+q)(p+q)^2 \\ &= (p+q)(p^2 + 2pq + q^2) \\ &= p(p^2 + 2pq + q^2) + q(p^2 + 2pq + q^2) \\ &= p^3 + 2p^2q + pq^2 + qp^2 + 2pq^2 + q^3 \\ &= p^3 + 3p^2q + 3pq^2 + q^3\end{aligned}$$

$$\begin{aligned}p^3 + q^3 - (p+q)^3 &= p^3 + q^3 - (p^3 + 3p^2q + 3pq^2 + q^3) \\ &= \cancel{p^3} + \cancel{q^3} - \cancel{p^3} - 3p^2q - 3pq^2 - \cancel{q^3} \\ &= -3p^2q - 3pq^2 = -3pq(p+q)\end{aligned}$$

COMMON PROBLEMS:

- Not knowing bookwork accurately.
- Not using correct conjugates to rationalise the denominator.
- **Not squaring e.g. $(x-2)^2$ properly.**

LC HIGHER MATHS: TEST ON CHAPTER ONE

SQUARE THE FOLLOWING

1. $(5x+2)^2$
2. $(3x+7)^2$
3. $(4x-9)^2$
4. $(6a-5b)^2$

ANSWERS

1. $(5x+2)^2 = (5x)^2 + 2(5x)(2) + (2)^2$
 $= 25x^2 + 20x + 4$
2. $(3x+7)^2 = (3x)^2 + 2(3x)(7) + (7)^2$
 $= 9x^2 + 42x + 49$
3. $(4x-9)^2 = (4x)^2 - 2(4x)(9) + (9)^2$
 $= 16x^2 - 72x + 81$
4. $(6a-5b)^2 = (6a)^2 - 2(6a)(5b) + (5b)^2$
 $= 36a^2 - 60ab + 25b^2$

FACTORISE

1. $121x^2 - 64y^2$
2. $32a^2 - 50b^2$
3. $64x^3 - 27y^3$
4. $216a^3 + 343b^3$
5. $3x^3 + 24$
6. $2y^3 - 54$

ANSWERS

1. $121x^2 - 64y^2 = (11x-8y)(11x+8y)$
2. $32a^2 - 50b^2 = 2(16a^2 - 25b^2)$
 $= 2(4a-5b)(4a+5b)$
3. $64x^3 - 27y^3 = (4x)^3 - (3y)^3$
 $= (4x-3y)((4x)^2 + (4x)(3y) + (3y)^2)$
 $= (4x-3y)(16x^2 + 12xy + 9y^2)$
4. $216a^3 + 343b^3 = (6a)^3 + (7b)^3$
 $= (6a+7b)((6a)^2 - (6a)(7b) + (7b)^2)$
 $= (6a+7b)(36a^2 - 42ab + 49b^2)$
5. $3x^3 + 24 = 3(x^3 + 8) = 3(x+2)(x^2 - 2x + 4)$
6. $2y^3 - 54 = 2(y^3 - 27) = 2(y-3)(y^2 + 3y + 9)$