Oliver Murphy: Discovering Maths 4: EXERCISE 1G

1 (a)

$$q = p + p\sqrt{p - p^2}$$
$$= (0\cdot1) + (0\cdot1)\sqrt{(0\cdot1) - (0\cdot1)^2}$$

$$= 0\cdot1 + 0\cdot1 \times \sqrt{0\cdot09} = 0\cdot1 + 0\cdot1 \times 0\cdot3$$
$$= 0\cdot1 + 0\cdot03 = 0\cdot13$$

(b)

$$\left(\frac{1}{x} - \frac{1}{x+h}\right) \div h = \left(\frac{x+h-x}{x(x+h)}\right) \div h$$
$$= \left(\frac{h}{x(x+h)}\right) \div h = \frac{1}{x(x+h)}$$

(c)

$$\frac{6t+1}{2t-1} = x \Rightarrow 6t+1 = x(2t-1)$$
$$\Rightarrow 6t+1 = 2xt - x \Rightarrow 1 + x = 2xt - 6t$$
$$\Rightarrow 1 + x = t(2x-6) \Rightarrow \frac{1+x}{2x-6} = t$$

$$t = \frac{1+x}{2x-6} = \frac{1+(3\cdot5)}{2(3\cdot5)-6} = \frac{4\cdot5}{1} = 4\cdot5$$

t is not defined if $x = 3$ as this would lead to division by zero.

2 (a) (i) $$\frac{1}{10+5\pi} = \frac{1}{25\cdot708} = 0\cdot039$$

(i) $1000 + 50\pi^5 = 16300\cdot98 = 16000$ (2 s.f.)

(b)

$$b + c\left(d^3 - e\right) = a \Rightarrow c\left(d^3 - e\right) = a - b$$
$$\Rightarrow d^3 - e = \frac{a-b}{c} \Rightarrow d^3 = \frac{a-b}{c} + e = \frac{a-b+ec}{c}$$
$$d = \sqrt[3]{\frac{a-b+ec}{c}}$$

$$d = \sqrt[3]{\frac{8\cdot9 - 0\cdot4 + 5\times1\cdot675}{5}} = \sqrt[3]{\frac{16\cdot875}{1\cdot675}}$$
$$= \sqrt[3]{3\cdot375} = 1\cdot5$$

(c)

$$\frac{\left(7-\sqrt{23}\right)}{\left(5+\sqrt{23}\right)}\frac{\left(5-\sqrt{23}\right)}{\left(5-\sqrt{23}\right)}=\frac{7\left(5-\sqrt{23}\right)-\sqrt{23}\left(5-\sqrt{23}\right)}{5^2-\left(\sqrt{23}\right)^2}$$

$$=\frac{35-7\sqrt{23}-5\sqrt{23}+23}{25-23}$$

$$=\frac{58-12\sqrt{23}}{2}=29-6\sqrt{23}$$

3. (a)

$$\sqrt{x+6}=x-6\Rightarrow\left(\sqrt{x+6}\right)^2=(x-6)^2$$

$$\Rightarrow x+6=x^2-12x+36\Rightarrow 0=x^2-12x+36-x-6$$

$$\Rightarrow x^2-13x+30=0\Rightarrow x^2-3x-10x+30$$

$$\Rightarrow x(x-3)-10(x-3)=0=(x-3)(x-10)$$

$$\Rightarrow x=3 \text{ or } x=10$$

CHECK: $\sqrt{x+6}=x-6$

$$\sqrt{(3)+6}=(3)-6\Rightarrow\sqrt{9}=-3\Rightarrow 3=-3 \text{ FALSE}$$

$$\sqrt{(10)+6}=(10)-6\Rightarrow\sqrt{16}=4\Rightarrow 4=4 \text{ TRUE}$$

ANS. $x=10$

(b)

$$x=2+\sqrt{5}$$

$$\frac{x^2+1}{x}=\frac{\left(2+\sqrt{5}\right)^2+1}{\left(2+\sqrt{5}\right)}$$

$$=\frac{2^2+2(2)\left(\sqrt{5}\right)+\left(\sqrt{5}\right)^2+1}{\left(2+\sqrt{5}\right)}$$

$$=\frac{4+4\sqrt{5}+5+1}{\left(2+\sqrt{5}\right)}=\frac{\left(10+4\sqrt{5}\right)}{\left(2+\sqrt{5}\right)}\frac{\left(2-\sqrt{5}\right)}{\left(2-\sqrt{5}\right)}$$

$$=\frac{10\left(2-\sqrt{5}\right)+4\sqrt{5}\left(2-\sqrt{5}\right)}{2^2-\left(\sqrt{5}\right)^2}$$

$$=\frac{20-10\sqrt{5}+8\sqrt{5}-4\sqrt{5}\sqrt{5}}{4-5}$$

$$=\frac{20-2\sqrt{5}-20}{-1}=\frac{-2\sqrt{5}}{-1}=2\sqrt{5}=k\sqrt{5}$$

ANS. $k=2$

(c)

$$x=\frac{2t}{1+t^2},\ y=\frac{1-t^2}{1+t^2}$$

$$1-y=1-\frac{1-t^2}{1+t^2}=\frac{1+t^2-\left(1-t^2\right)}{1+t^2}$$
$$=\frac{1+t^2-1+t^2}{1+t^2}=\frac{2t^2}{1+t^2}$$

$$\frac{x}{1-y}=\left(\frac{2t}{1+t^2}\right)\div\left(\frac{2t^2}{1+t^2}\right)$$
$$=\left(\frac{\cancel{2}t}{\cancel{1+t^2}}\right)\times\left(\frac{\cancel{1+t^2}}{\cancel{2}t^2}\right)=\frac{t}{t^2}=\frac{1}{t}$$

4. (a)

$$\frac{8x^3+27}{4x^2-9}=\frac{(2x)^3+3^3}{(2x)^2-3^2}=\frac{[2x+3][(2x)^2-(2x)(3)+3^2]}{(2x-3)(2x+3)}$$
$$=\frac{\cancel{[2x+3]}[4x^2-6x+9]}{(2x-3)\cancel{(2x+3)}}=\frac{4x^2-6x+9}{2x-3}$$

(b)

$$1+\sqrt{2x-1}=x-1\Rightarrow\sqrt{2x-1}=x-1-1$$
$$\Rightarrow\left(\sqrt{2x-1}\right)^2=\left(x-2\right)^2$$
$$\Rightarrow 2x-1=x^2-4x+4$$
$$\Rightarrow 0=x^2-4x+4-2x+1=x^2-6x+5$$
$$\Rightarrow 0=x^2-x-5x+5=x(x-1)-5(x-1)$$
$$\Rightarrow 0=(x-1)(x-5)\Rightarrow x=1 \text{ or } x=5$$

CHECK: $1+\sqrt{2x-1}=x-1$

$1+\sqrt{2(1)-1}=(1)-1\Rightarrow 1+\sqrt{1}=0$ FALSE

$1+\sqrt{2(5)-1}=(5)-1\Rightarrow 1+\sqrt{9}=4$ TRUE

ANS. $x=5$

(c)

TO PROVE: $(\text{hypotenuse})^2=(\text{opposite})^2+(\text{adjacent})^2$

$$(\text{hypotenuse})^2=\left(m^2+n^2\right)^2=\left(m^2\right)^2+2\left(m^2\right)\left(n^2\right)+\left(n^2\right)^2$$
$$=m^4+2m^2n^2+n^4$$

$$\begin{aligned}&(\text{opposite})^2+(\text{adjacent})^2\\&=(2mn)^2+\left(m^2-n^2\right)^2\\&=(2mn)(2mn)+\left(m^2\right)^2-2\left(m^2\right)\left(n^2\right)+\left(n^2\right)^2\\&=4m^2n^2+m^4-2m^2n^2+n^4=m^4+2m^2n^2+n^4\\&=(\text{hypotenuse})^2\end{aligned}$$

Thus, by Pythagoras' theorem, the triangle is right-angled.

$$m=5,\ n=2$$
$$m^2+n^2=5^2+2^2=29$$
$$2mn=2(5)(2)=20$$
$$m^2-n^2=5^2-2^2=21$$

5. (a)

$$\frac{\left(2-\sqrt{3}\right)}{\left(2+\sqrt{3}\right)}\frac{\left(2-\sqrt{3}\right)}{\left(2-\sqrt{3}\right)}=\frac{2^2-2(2)\left(\sqrt{3}\right)+\left(\sqrt{3}\right)^2}{2^2-\left(\sqrt{3}\right)^2}=\frac{4-4\sqrt{3}+3}{4-3}=\frac{7-4\sqrt{3}}{1}=7-4\sqrt{3}$$

(b)

$$\frac{8}{6x-15}-\frac{5}{4x-10}=\frac{2\times 8}{2\times(6x-15)}-\frac{3\times 5}{3\times(4x-10)}=\frac{16-15}{12x-30}=\frac{1}{6(2x-5)}$$

(c)

$$\frac{\sqrt{5}}{\sqrt{6}}\frac{\sqrt{6}}{\sqrt{6}}+\frac{\sqrt{6}}{\sqrt{5}}\frac{\sqrt{5}}{\sqrt{5}}+\frac{\sqrt{120}}{\sqrt{49}}=\frac{\sqrt{30}}{6}+\frac{\sqrt{30}}{5}+\frac{\sqrt{4}\sqrt{30}}{7}=\left(\frac{1}{6}+\frac{1}{5}+\frac{2}{7}\right)\sqrt{30}=\frac{137}{210}\sqrt{30}$$

6. (a)

$$\frac{x^2-x-6}{x^2-4}=\frac{x^2-3x+2x-6}{x^2-2^2}=\frac{x(x-3)+2(x-3)}{(x-2)(x+2)}=\frac{(x-3)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}=\frac{x-3}{x-2}$$

(b) (i) $a^2+2ab+b^2=(a+b)^2$

(ii) $a^2+b^2-c^2+2ab=(a+b)^2-c^2$
$=(a+b-c)(a+b+c)$

(c)

$\sqrt{x+4}=\sqrt{x-1}+1$

$\left(\sqrt{x+4}\right)^2=\left(\sqrt{x-1}+1\right)^2$

$x+4=\left(\sqrt{x-1}\right)^2+2\left(\sqrt{x-1}\right)(1)+1^2$

$\cancel{x}+4=\cancel{x}-\cancel{1}+2\sqrt{x-1}+\cancel{1}$

$2=\sqrt{x-1}\Rightarrow 4=x-1\Rightarrow 4+1=x=5$

CHECK: $\sqrt{x+4}=\sqrt{x-1}+1$

$\sqrt{(5)+4}=\sqrt{(5)-1}+1\Rightarrow\sqrt{9}=\sqrt{4}+1$ TRUE

ANS. $x=5$

7. (a)

$$\frac{1}{x+2}-\frac{1}{2x-1}=\frac{(2x-1)-(x+2)}{(x+2)(2x-1)}=\frac{2x-1-x-2}{(x+2)(2x-1)}$$

$$=\frac{x-3}{(x+2)(2x-1)}$$

$$\frac{1}{2x^2+3x-2}=\frac{1}{2x^2+4x-1x-2}$$

$$=\frac{1}{2x(x+2)-1(x+2)}=\frac{1}{(x+2)(2x-1)}$$

Thus, $\frac{1}{x+2}-\frac{1}{2x-1}=\frac{1}{2x^2+3x-2}$

$$\Rightarrow\frac{x-3}{\cancel{(x+2)(2x-1)}}=\frac{1}{\cancel{(x+2)(2x-1)}}$$

$\Rightarrow x-3=1\Rightarrow x=1+3=4$

(b)

$x^2+x-6=x^2+3x-2x-6$

$=x(x+3)-2(x+3)=(x+3)(x-2)$

Thus, $\frac{4}{x-2}+\frac{5}{x+3}-\frac{20}{x^2+x-6}$

$$=\frac{4}{x-2}+\frac{5}{x+3}-\frac{20}{(x+3)(x-2)}$$

$$=\frac{4(x+3)+5(x-2)-20}{(x-2)(x+3)}$$

$$=\frac{4x+12+5x-10-20}{(x-2)(x+3)}$$

$$=\frac{9x+18}{(x-2)(x+3)}=\frac{9\,\cancel{(x+2)}}{\cancel{(x-2)}\,(x+3)}$$

$$=\frac{9}{x+3}$$

(c)

$$8x^3-27y^3+4x^2-9y^2$$

$$=(2x)^3-(3y)^3+(2x)^2-(3y)^2$$

$$=[2x-3y]\left[(2x)^2+(2x)(3y)+(3y)^2\right]+(2x-3y)(2x+3y)$$

$$=[2x-3y]\left[4x^2+6xy+9y^2\right]+(2x-3y)(2x+3y)$$

$$=[2x-3y]\left[4x^2+6xy+9y^2+2x+3y\right]$$

8 (a)

$$\frac{3(x-4)}{x-3}-\frac{x}{3-x}=\frac{3(x-4)}{x-3}+\frac{x}{x-3}$$

$$=\frac{3x-12+x}{x-3}=\frac{4x-12}{x-3}$$

$$=\frac{4\,\cancel{(x-3)}}{\cancel{x-3}}=4$$

(b) (i) $\frac{21}{\sqrt{7}}\frac{\sqrt{7}}{\sqrt{7}}=\frac{21\sqrt{7}}{7}=3\sqrt{7}$

(ii)

$$\frac{\left(1+\sqrt{7}\right)}{\left(\sqrt{7}-2\right)}\frac{\left(\sqrt{7}+2\right)}{\left(\sqrt{7}+2\right)}=\frac{1\left(\sqrt{7}+2\right)+\sqrt{7}\left(\sqrt{7}+2\right)}{\left(\sqrt{7}\right)^2-(2)^2}$$

$$=\frac{\sqrt{7}+2+7+2\sqrt{7}}{7-4}=\frac{9+3\sqrt{7}}{3}$$

$$=3+\sqrt{7}$$

(c)

$$\left(x^3+\sqrt{8}+\frac{4}{x^3}\right)\left(x^3-\sqrt{8}+\frac{4}{x^3}\right)$$

$$=x^3\left(x^3-\sqrt{8}+\frac{4}{x^3}\right)+\sqrt{8}\left(x^3-\sqrt{8}+\frac{4}{x^3}\right)+\frac{4}{x^3}\left(x^3-\sqrt{8}+\frac{4}{x^3}\right)$$

$$=x^6-\cancel{\sqrt{8}x^3}+\cancel{4}+\cancel{\sqrt{8}x^3}-\cancel{8}+\cancel{\frac{4\sqrt{8}}{x^3}}+\cancel{4}-\cancel{\frac{4\sqrt{8}}{x^3}}+\frac{16}{x^6}$$

$$=x^6+\frac{16}{x^6}$$