

Oliver Murphy: Discovering Maths 4: EXERCISE 1E

1.

$$\sqrt{x+3} = x-3 \Rightarrow (\sqrt{x+3})^2 = (x-3)^2$$

$$x+3 = x^2 - 6x + 9$$

$$0 = x^2 - 6x + 9 - x - 3$$

$$0 = x^2 - 7x + 6 = x^2 - x - 6x - 3$$

$$x(x-1) - 6(x-1) = 0$$

$$(x-6)(x-1) = 0 \Rightarrow x = 6 \text{ or } x = 1$$

CHECK: $\sqrt{x+3} = x-3$

$$\sqrt{(6)+3} = (6)-3 \Rightarrow 3 = 3 \text{ TRUE}$$

$$\sqrt{(1)+3} = (1)-3 \Rightarrow 2 = -2 \text{ FALSE}$$

ANSWER: $x = 6$

2.

$$\sqrt{2x+1} = x-1 \Rightarrow (\sqrt{2x+1})^2 = (x-1)^2$$

$$2x+1 = x^2 - 2x + 1$$

$$0 = x^2 - 2x + 1 - 2x - 1$$

$$0 = x^2 - 4x = x(x-4)$$

$$\Rightarrow x = 0 \text{ or } x = 4$$

CHECK: $\sqrt{2x+1} = x-1$

$$\sqrt{2(0)+1} = 0-1 \Rightarrow 1 = -1 \text{ FALSE}$$

$$\sqrt{2(4)+1} = 4-1 \Rightarrow 3 = 3 \text{ TRUE}$$

ANSWER: $x = 4$

3.

$$x - \sqrt{x-3} = 5 \Rightarrow x - 5 = \sqrt{x-3}$$

$$(x-5)^2 = (\sqrt{x-3})^2$$

$$x^2 - 10x + 25 = x - 3$$

$$x^2 - 10x + 25 - x + 3 = 0$$

$$x^2 - 11x + 28 = x^2 - 4x - 7x + 28 = 0$$

$$x(x-4) - 7(x-4) = 0$$

$$(x-7)(x-4) = 0 \Rightarrow x = 7 \text{ or } x = 4$$

CHECK: $x - \sqrt{x-3} = 5$

$$(7) - \sqrt{(7)-3} = 5 \Rightarrow 5 = 5 \text{ TRUE}$$

$$(4) - \sqrt{(4)-3} = 5 \Rightarrow 3 = 5 \text{ FALSE}$$

ANSWER: $x = 7$

4.

$$x + \sqrt{x} = 2 \Rightarrow x - 2 = -\sqrt{x}$$

$$(x - 2)^2 = (-\sqrt{x})^2$$

$$x^2 - 4x + 4 = x$$

$$x^2 - 4x + 4 - x = 0$$

$$x^2 - 5x + 4 = x^2 - 1x - 4x + 4 = 0$$

$$x(x - 1) - 4(x - 1) = 0$$

$$(x - 4)(x - 1) = 0 \Rightarrow x = 4 \text{ or } x = 1$$

CHECK: $x + \sqrt{x} = 2$

$$(4) + \sqrt{(4)} = 2 \Rightarrow 6 = 2 \text{ FALSE}$$

$$(1) + \sqrt{(1)} = 2 \Rightarrow 2 = 2 \text{ TRUE}$$

ANSWER: $x = 1$

5.

$$\sqrt{7x+1} - x = 1 \Rightarrow \sqrt{7x+1} = x + 1$$

$$\Rightarrow (\sqrt{7x+1})^2 = (x+1)^2$$

$$7x + 1 = x^2 + 2x + 1$$

$$0 = x^2 + 2x + 1 - 7x - 1$$

$$0 = x^2 - 5x = x(x - 5)$$

$$\Rightarrow x = 0 \text{ or } x = 5$$

CHECK: $\sqrt{7x+1} - x = 1$

$$\sqrt{7(0)+1} - (0) = 1 \Rightarrow 1 = 1 \text{ TRUE}$$

$$\sqrt{7(5)+1} - (5) = 1 \Rightarrow 1 = 1 \text{ TRUE}$$

ANSWER: $x = 0$ or $x = 5$

6.

$$\sqrt{x+4} = \sqrt{x-1} + 1$$

$$(\sqrt{x+4})^2 = (\sqrt{x-1} + 1)^2$$

$$x + 4 = (\sqrt{x-1})^2 + 2(\sqrt{x-1})(1) + 1^2$$

$$x + 4 = x - 1 + 2\sqrt{x-1} + 1$$

$$\Rightarrow 2 = \sqrt{x-1} \Rightarrow 2^2 = (\sqrt{x-1})^2$$

$$\Rightarrow 4 = x - 1 \Rightarrow 5 = x$$

CHECK: $\sqrt{x+4} = \sqrt{x-1} + 1$

$$\sqrt{(5)+4} = \sqrt{(5)-1} + 1 \Rightarrow 3 = 3 \text{ TRUE}$$

ANSWER: $x = 5$

7.

$$\sqrt{3x+1} = \sqrt{x-1} + 2$$

$$(\sqrt{3x+1})^2 = (\sqrt{x-1} + 2)^2$$

$$3x+1 = (\sqrt{x-1})^2 + 2(\sqrt{x-1})(2) + 2^2$$

$$3x+1 = x-1 + 4\sqrt{x-1} + 4$$

$$3x+1 - x + 1 - 4 = 4\sqrt{x-1}$$

$$2x - 2 = 4\sqrt{x-1} \Rightarrow x - 1 = 2\sqrt{x-1}$$

$$(x-1)^2 = (2\sqrt{x-1})^2$$

$$x^2 - 2x + 1 = 4(x-1) = 4x - 4$$

$$x^2 - 2x + 1 - 4x + 4 = 0$$

$$x^2 - 6x + 5 = 0 = x^2 - x - 5x + 5$$

$$x(x-1) - 5(x-1) = 0 = (x-5)(x-1)$$

$$\Rightarrow x = 5 \text{ or } x = 1$$

CHECK: $\sqrt{3x+1} = \sqrt{x-1} + 2$

$$\sqrt{3(5)+1} = \sqrt{(5)-1} + 2 \Rightarrow 4 = 4 \text{ TRUE}$$

$$\sqrt{3(1)+1} = \sqrt{(1)-1} + 2 \Rightarrow 3 = 3 \text{ TRUE}$$

ANSWER: $x = 5$ or $x = 1$

8.

$$\sqrt{2x-1} = \sqrt{x-1} + 1$$

$$(\sqrt{2x-1})^2 = (\sqrt{x-1} + 1)^2$$

$$2x-1 = (\sqrt{x-1})^2 + 2(\sqrt{x-1})(1) + 1^2$$

$$2x-1 = x-1 + 2\sqrt{x-1} + 1$$

$$2x-1 - x + 1 - 1 = 2\sqrt{x-1}$$

$$x-1 = 2\sqrt{x-1} \Rightarrow (x-1)^2 = (2\sqrt{x-1})^2$$

$$x^2 - 2x + 1 = 4(x-1) = 4x - 4$$

$$x^2 - 2x + 1 - 4x + 4 = 0$$

$$x^2 - 6x + 5 = 0 = x^2 - x - 5x + 5$$

$$x(x-1) - 5(x-1) = 0 = (x-5)(x-1)$$

$$\Rightarrow x = 5 \text{ or } x = 1$$

CHECK: $\sqrt{2x-1} = \sqrt{x-1} + 1$

$$\sqrt{2(5)-1} = \sqrt{(5)-1} + 1 \Rightarrow 3 = 3 \text{ TRUE}$$

$$\sqrt{2(1)-1} = \sqrt{(1)-1} + 1 \Rightarrow 1 = 1 \text{ TRUE}$$

ANSWER: $x = 5$ or $x = 1$

9.

$$\sqrt{5x+1} + \sqrt{x+1} = 6 \Rightarrow \sqrt{5x+1} = 6 - \sqrt{x+1}$$

$$(\sqrt{5x+1})^2 = (6 - \sqrt{x+1})^2$$

$$5x+1 = 6^2 - 2(6)(\sqrt{x+1}) + (\sqrt{x+1})^2$$

$$5x+1 = 36 - 12\sqrt{x+1} + x+1$$

$$5x+1 - 36 - x - 1 = -12\sqrt{x+1}$$

$$4x - 36 = -12\sqrt{x+1} \Rightarrow x - 9 = -3\sqrt{x+1}$$

$$(x-9)^2 = (-3\sqrt{x+1})^2$$

$$x^2 - 18x + 81 = 9(x+1) = 9x + 9$$

$$x^2 - 18x + 81 - 9x - 9 = 0$$

$$x^2 - 27x + 72 = 0 = x^2 - 3x - 24x + 72$$

$$x(x-3) - 24(x-3) = 0 = (x-3)(x-24)$$

$$\Rightarrow x = 3 \text{ or } x = 24$$

$$\text{CHECK: } \sqrt{5x+1} + \sqrt{x+1} = 6$$

$$\sqrt{5(3)+1} + \sqrt{(3)+1} = 6 \Rightarrow 4 + 2 = 6 \text{ TRUE}$$

$$\sqrt{5(24)+1} + \sqrt{(24)+1} = 6 \Rightarrow 11 + 5 = 6 \text{ FALSE}$$

$$\text{ANSWER: } x = 3$$

10.

$$\sqrt{x} + \sqrt{x-3} = \sqrt{x+5} \Rightarrow (\sqrt{x} + \sqrt{x-3})^2 = (\sqrt{x+5})^2$$

$$(\sqrt{x})^2 + 2(\sqrt{x})(\sqrt{x-3}) + (\sqrt{x-3})^2 = x+5$$

$$x + 2\sqrt{x}\sqrt{x-3} + x - 3 = x + 5$$

$$2\sqrt{x}\sqrt{x-3} = x + 5 - x - x + 3 = 8 - x$$

$$(2\sqrt{x}\sqrt{x-3})^2 = (8-x)^2$$

$$4x(x-3) = 64 - 16x + x^2$$

$$4x^2 - 12x = 64 - 16x + x^2$$

$$4x^2 - 12x - 64 + 16x - x^2 = 0$$

$$3x^2 + 4x - 64 = 0 = 3x^2 - 12x + 16x - 64$$

$$3x(x-4) + 16(x-4) = 0 = (3x+16)(x-4)$$

$$\Rightarrow 3x+16=0 \Rightarrow x = -\frac{16}{3} \text{ or } x=4$$

CHECK: $\sqrt{x} + \sqrt{x-3} = \sqrt{x+5}$

$$\sqrt{\left(-\frac{16}{3}\right)} + \sqrt{\left(-\frac{16}{3}\right) - 3} = \sqrt{\left(-\frac{16}{3}\right) + 5}$$

\Rightarrow CALCULATOR ERROR: FALSE

$$\sqrt{(4)} + \sqrt{(4)-3} = \sqrt{(4)+5} \Rightarrow 2+1=3 \text{ TRUE}$$

ANSWER: $x=4$