

Oliver Murphy: Discovering Maths 4: EXERCISE 1D

1(i) $\sqrt{441} = 21$ (ii) $3\sqrt{289} = 3 \times 17 = 51$

(iii) $2\sqrt{1024} = 64$ (iv) $\sqrt{\frac{1}{4}} = \frac{1}{2}$

(v) $\sqrt{\frac{9}{16}} = \frac{3}{4}$ (vi) $\sqrt{\frac{100}{121}} = \frac{10}{11}$

(vii) $\sqrt{6\frac{1}{4}} = \sqrt{\frac{25}{4}} = \frac{5}{2} = 2\frac{1}{2}$

(viii) $\sqrt{5\frac{4}{9}} = 2\frac{1}{3}$ (ix) $\sqrt{0.0004} = 0.02$

(x) $\sqrt{0.0625} = 0.25$

2(i) $\sqrt{28} = \sqrt{4}\sqrt{7} = 2\sqrt{7}$

(ii) $\sqrt{18} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$

(iii) $\sqrt{200} = \sqrt{100}\sqrt{2} = 10\sqrt{2}$

(iv) $\sqrt{108} = \sqrt{36}\sqrt{3} = 6\sqrt{3}$

(v) $\sqrt{98} = \sqrt{49}\sqrt{2} = 7\sqrt{2}$

(vi) $\sqrt{75} = \sqrt{25}\sqrt{3} = 5\sqrt{3}$

(vii) $\sqrt{300} = \sqrt{100}\sqrt{3} = 10\sqrt{3}$

(viii) $\sqrt{27} = \sqrt{9}\sqrt{3} = 3\sqrt{3}$

(ix) $\sqrt{40} = \sqrt{4}\sqrt{10} = 2\sqrt{10}$

(x) $\sqrt{32} = \sqrt{16}\sqrt{2} = 4\sqrt{2}$

3(i)

$$\begin{aligned}\sqrt{40} + \sqrt{90} &= \sqrt{4}\sqrt{10} + \sqrt{9}\sqrt{10} \\ &= 2\sqrt{10} + 3\sqrt{10} = 5\sqrt{10}\end{aligned}$$

(ii)

$$\begin{aligned}\sqrt{28} + \sqrt{63} &= \sqrt{4}\sqrt{7} + \sqrt{9}\sqrt{7} \\ &= 2\sqrt{7} + 3\sqrt{7} = 5\sqrt{7}\end{aligned}$$

(iii)

$$\begin{aligned}\sqrt{500} + \sqrt{125} - \sqrt{5} &= \sqrt{100}\sqrt{5} + \sqrt{25}\sqrt{5} - \sqrt{5} \\ &= 10\sqrt{5} + 5\sqrt{5} - \sqrt{5} = 14\sqrt{5}\end{aligned}$$

(iv)

$$\begin{aligned}\sqrt{24} + \sqrt{54} - 4\sqrt{6} &= \sqrt{4}\sqrt{6} + \sqrt{9}\sqrt{6} - 4\sqrt{6} \\ &= 2\sqrt{6} + 3\sqrt{6} - 4\sqrt{6} = \sqrt{6}\end{aligned}$$

(v)

$$\begin{aligned}\sqrt{75} - \sqrt{27} &= \sqrt{25}\sqrt{3} - \sqrt{9}\sqrt{3} \\ &= 5\sqrt{3} - 3\sqrt{3} = 2\sqrt{3}\end{aligned}$$

4.

$$\begin{aligned}5\sqrt{6} - \sqrt{150} + 2\sqrt{24} &= 5\sqrt{6} - \sqrt{25}\sqrt{6} + 2\sqrt{4}\sqrt{6} \\ &= 5\sqrt{6} - 5\sqrt{6} + 4\sqrt{6} = 4\sqrt{6}\end{aligned}$$

$$5.(i) \quad \frac{6}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$$

$$(ii) \quad \frac{7}{\sqrt{7}} \frac{\sqrt{7}}{\sqrt{7}} = \frac{7\sqrt{7}}{7} = \sqrt{7}$$

$$(iii) \quad \frac{1}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$(iv) \quad \frac{3}{\sqrt{5}} \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

(v)

$$\begin{aligned}\frac{1}{(\sqrt{5}-2)} \frac{(\sqrt{5}+2)}{(\sqrt{5}+2)} &= \frac{(\sqrt{5}+2)}{(\sqrt{5})^2 - 2^2} \\ &= \frac{\sqrt{5}+2}{5-4} = \sqrt{5}+2\end{aligned}$$

(vi)

$$\begin{aligned}\frac{1}{(3+\sqrt{3})} \frac{(3-\sqrt{3})}{(3-\sqrt{3})} &= \frac{3-\sqrt{3}}{3^2 - (\sqrt{3})^2} \\ &= \frac{3-\sqrt{3}}{9-3} = \frac{3-\sqrt{3}}{6}\end{aligned}$$

(vii)

$$\begin{aligned} \frac{5\sqrt{2}}{(\sqrt{2}-1)} \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)} &= \frac{5\sqrt{2}\sqrt{2}+5\sqrt{2}}{(\sqrt{2})^2-1} \\ &= \frac{10+5\sqrt{2}}{2-1} = 10+5\sqrt{2} \end{aligned}$$

(viii)

$$\begin{aligned} \frac{(2-\sqrt{3})(5+\sqrt{3})}{(5-\sqrt{3})(5+\sqrt{3})} &= \frac{2(5+\sqrt{3})-\sqrt{3}(5+\sqrt{3})}{5^2-(\sqrt{3})^2} \\ &= \frac{10+2\sqrt{3}-5\sqrt{3}-3}{25-3} \\ &= \frac{7-3\sqrt{3}}{25-3} = \frac{13-3\sqrt{3}}{22} \end{aligned}$$

(ix)

$$\begin{aligned} \frac{1}{(\sqrt{3}-\sqrt{2})} \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})} &= \frac{\sqrt{3}+\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} \\ &= \frac{\sqrt{3}+\sqrt{2}}{3-2} = \sqrt{3}+\sqrt{2} \end{aligned}$$

(x)

$$\begin{aligned} \frac{(\sqrt{3}-7)(\sqrt{3}-7)}{(\sqrt{3}+7)(\sqrt{3}-7)} &= \frac{\sqrt{3}(\sqrt{3}-7)-7(\sqrt{3}-7)}{(\sqrt{3})^2-7^2} \\ &= \frac{3-7\sqrt{3}-7\sqrt{3}+49}{3-49} \\ &= \frac{52-14\sqrt{3}}{-46} = \frac{7\sqrt{3}-26}{23} \end{aligned}$$

(Divide above and below by -2)

6.

$$\begin{aligned} \frac{1}{a} &= \frac{1}{(\sqrt{10}-3)(\sqrt{10}+3)} \\ &= \frac{\sqrt{10}+3}{(\sqrt{10})^2-3^2} = \frac{\sqrt{10}+3}{10-9} \\ &= \sqrt{10}+3 \end{aligned}$$

$$a+6 = \sqrt{10}-3+6 = \sqrt{10}+3$$

$$= \frac{1}{a}$$

7.

$$\begin{aligned} \frac{11(\sqrt{2}-1)+2}{2(\sqrt{2}-1)+1} &= \frac{11\sqrt{2}-11+2}{2\sqrt{2}-2+1} \\ &= \frac{(11\sqrt{2}-9)(2\sqrt{2}+1)}{(2\sqrt{2}-1)(2\sqrt{2}+1)} \\ &= \frac{11\sqrt{2}(2\sqrt{2}+1)-9(2\sqrt{2}+1)}{(2\sqrt{2})^2-1^2} \\ &= \frac{22\sqrt{2}\sqrt{2}+11\sqrt{2}-18\sqrt{2}-9}{(2\sqrt{2})(2\sqrt{2})-1} \\ &= \frac{22 \times 2 - 7\sqrt{2} - 9}{8-1} = \frac{35-7\sqrt{2}}{7} \\ &= 5 - \sqrt{2} \quad (\text{Divide above and below by 7}) \end{aligned}$$

8(i)

$$\begin{cases} ax^2 + bx + c = 0 \\ x^2 - 4x - 2 = 0 \end{cases} \quad a = 1, b = -4, c = -2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{24}}{2} = \frac{4 \pm \sqrt{4}\sqrt{6}}{2} = \frac{4 \pm 2\sqrt{6}}{2} = 2 \pm \sqrt{6}$$

(ii)

$$\begin{cases} ax^2 + bx + c = 0 \\ x^2 - 6x + 4 = 0 \end{cases} \quad a = 1, b = -6, c = 4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{20}}{2} = \frac{6 \pm \sqrt{4}\sqrt{5}}{2} = \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5}$$

(iii)

$$\left\{ \begin{array}{l} ax^2 + bx + c = 0 \\ x^2 + 2x - 1 = 0 \end{array} \right\} \quad a = 1, \quad b = 2, \quad c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm \sqrt{4}\sqrt{2}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

(iv)

$$\left\{ \begin{array}{l} ax^2 + bx + c = 0 \\ x^2 - 8x + 9 = 0 \end{array} \right\} \quad a = 1, \quad b = -8, \quad c = 9$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{28}}{2} = \frac{8 \pm \sqrt{4}\sqrt{7}}{2} = \frac{8 \pm 2\sqrt{7}}{2} = 4 \pm \sqrt{7}$$

9.

$$\begin{aligned} & 3 \frac{\sqrt{2}}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} + \sqrt{24} \\ &= \frac{3\sqrt{6}}{3} - \frac{\sqrt{6}}{2} + \sqrt{4}\sqrt{6} \\ &= \left(1 - \frac{1}{2} + 2\right)\sqrt{6} = \frac{5\sqrt{6}}{2} \end{aligned}$$

10.

$$\begin{aligned} \frac{\sqrt{2}}{\sqrt{5}} \frac{\sqrt{5}}{\sqrt{5}} + \frac{\sqrt{5}}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} &= \frac{\sqrt{10}}{5} + \frac{\sqrt{10}}{2} \\ &= \left(\frac{1}{5} + \frac{1}{2}\right)\sqrt{10} = \frac{7\sqrt{10}}{10} \end{aligned}$$