1.

$$\frac{3}{x-3}+\frac{5}{x+4}=\frac{3(x+4)+5(x-3)}{(x-3)(x+4)}$$
$$=\frac{3x+12+5x-15}{(x-3)(x+4)}$$
$$=\frac{8x-3}{(x-3)(x+4)}$$

2.

$$\frac{7}{2y+1}-\frac{6}{2y-1}=\frac{7(2y-1)-6(2y+1)}{(2y+1)(2y-1)}$$
$$=\frac{14y-7-12y-6}{(2y+1)(2y-1)}$$
$$=\frac{2y-13}{(2y+1)(2y-1)}$$

3.

$$\frac{2}{x+1}-\frac{x}{x-1}=\frac{2(x-1)-x(x+1)}{(x+1)(x-1)}$$
$$=\frac{2x-2-x^2-x}{(x+1)(x-1)}$$
$$=\frac{x-2-x^2}{(x+1)(x-1)}$$

4.

$$\frac{x+1}{x-1}+\frac{x+2}{x-2}=\frac{(x+1)(x-2)+(x+2)(x-1)}{(x-1)(x-2)}$$
$$=\frac{x(x-2)+1(x-2)+x(x-1)+2(x-1)}{(x-1)(x-2)}$$
$$=\frac{x^2-2x+x-2+x^2-x+2x-2}{(x-1)(x-2)}$$
$$=\frac{2x^2-4}{(x-1)(x-2)}$$

5.

$$\frac{x+1}{x-1}-\frac{x+2}{x-2}=\frac{(x+1)(x-2)-(x+2)(x-1)}{(x-1)(x-2)}$$

$$=\frac{x(x-2)+1(x-2)-x(x-1)-2(x-1)}{(x-1)(x-2)}$$

$$=\frac{x^2-2x+x-2-x^2+x-2x+2}{(x-1)(x-2)}$$

$$=\frac{x^2-x^2-2x+x+x-2x-2+2}{(x-1)(x-2)}$$

$$=\frac{-2x}{(x-1)(x-2)}$$

6.(i)

$$x^2+4x+3=x^2+1x+3x+3$$

$$=x(x+1)+3(x+1)$$

$$=(x+1)(x+3)$$

(ii)

$$\frac{7}{x+3}+\frac{4}{x^2+4x+3}=\frac{7}{x+3}+\frac{4}{(x+3)(x+1)}$$

$$=\frac{7(x+1)+4}{(x+3)(x+1)}$$

$$=\frac{7x+7+4}{(x+3)(x+1)}$$

$$=\frac{7x+11}{(x+3)(x+1)}$$

7.(i)

$$\frac{5}{x-2}+\frac{1}{2-x}=\frac{5}{x-2}-\frac{1}{x-2}=\frac{4}{x-2}$$

(ii)

$$\frac{7}{2y-1}+\frac{5}{1-2y}=\frac{7}{2y-1}-\frac{5}{2y-1}=\frac{2}{2y-1}$$

(iii)

$$\frac{2}{b-a}+\frac{2}{a-b}=\frac{2}{b-a}-\frac{2}{b-a}=0$$

(iv)

$$\frac{9}{2x-1}-\frac{4}{1-2x}=\frac{9}{2x-1}+\frac{4}{2x-1}=\frac{13}{2x-1}$$

8.

$$\frac{7}{2x-4}+\frac{5}{3x-6}=\frac{7(3x-6)+5(2x-4)}{(2x-4)(3x-6)}$$
$$=\frac{21x-42+10x-20}{(2x-4)(3x-6)}$$
$$=\frac{31x-62}{(2x-4)(3x-6)}$$
$$=\frac{31\cancel{(x-2)}}{2\cancel{(x-2)}\,3(x-2)}$$
$$=\frac{31}{6(x-2)}$$

9.(i)

$$\frac{6x}{x^2-9}-\frac{1}{x+3}=\frac{6x}{(x-3)(x+3)}-\frac{1}{(x+3)}\frac{(x-3)}{(x-3)}$$
$$=\frac{6x-1(x-3)}{(x-3)(x+3)}$$
$$=\frac{6x-x+3}{(x-3)(x+3)}$$
$$=\frac{5x+3}{(x-3)(x+3)}$$

(ii)

$$\frac{4}{3y-2}-\frac{2y}{9y^2-4}=\frac{4}{3y-2}-\frac{2y}{(3y-2)(3y+2)}$$
$$=\frac{4(3y+2)-2y}{(3y-2)(3y+2)}$$
$$=\frac{12y+8-2y}{(3y-2)(3y+2)}$$
$$=\frac{10y+8}{(3y-2)(3y+2)}$$

10.

$$\frac{2}{x-4}+\frac{2(9-2x)}{4-x}=\frac{2}{x-4}-\frac{2(9-2x)}{x-4}$$
$$=\frac{2-2(9-2x)}{x-4}$$
$$=\frac{2-18+4x}{x-4}$$
$$=\frac{4x-16}{x-4}$$
$$=\frac{4\cancel{(x-4)}}{\cancel{x-4}}=4$$

11.

$$\frac{2x}{x+3}+\frac{3x}{x-3}-\frac{5x^2+9}{x^2-9}$$

$$=\frac{2x}{x+3}+\frac{3x}{x-3}-\frac{(5x^2+9)}{(x-3)(x+3)}$$

$$=\frac{2x(x-3)+3x(x+3)-5x^2-9}{(x-3)(x+3)}$$

$$=\frac{2x^2-6x+3x^2+9x-5x^2-9}{(x-3)(x+3)}$$

$$=\frac{2x^2+3x^2-5x^2-6x+9x-9}{(x-3)(x+3)}$$

$$=\frac{3x-9}{(x-3)(x+3)}=\frac{3\cancel{(x-3)}}{\cancel{(x-3)}(x+3)}$$

$$=\frac{3}{x+3}$$

12.(i) $1+\frac{x}{y}=\frac{y}{y}+\frac{x}{y}=\frac{y+x}{y}$

(ii)

$$\left(1+\frac{x}{y}\right)\left(\frac{y^2}{y^2-x^2}\right)=\left(\frac{\cancel{y+x}}{y}\right)\left(\frac{y^2}{(y-x)\cancel{(y+x)}}\right)$$

$$=\frac{y}{y-x}$$

13(a)(i) $y-\frac{1}{y}=\frac{yy-1}{y}=\frac{y^2-1}{y}$

(ii)

$$2+\frac{2}{y-1}=\frac{2(y-1)+2}{y-1}$$

$$=\frac{2y-2+2}{y-1}$$

$$=\frac{2y}{y-1}$$

(b)

$$\left(y-\frac{1}{y}\right)\left(2+\frac{2}{y-1}\right)$$

$$=\left(\frac{y^2-1}{y}\right)\left(\frac{2y}{y-1}\right)$$

$$=\left(\frac{(y-1)(y+1)}{y}\right)\left(\frac{2y}{y-1}\right)$$

$$=2(y+1)$$

14.

$$\frac{\left(\frac{z}{z-1}\right)+\left(\frac{z}{z+1}\right)}{\left(\frac{z}{z-1}\right)-\left(\frac{z}{z+1}\right)}$$

$$=\frac{\left(\frac{z}{z-1}\right)(z-1)(z+1)+\left(\frac{z}{z+1}\right)(z-1)(z+1)}{\left(\frac{z}{z-1}\right)(z-1)(z+1)-\left(\frac{z}{z+1}\right)(z-1)(z+1)}$$

$$=\frac{z(z+1)+z(z-1)}{z(z+1)-z(z-1)}$$

$$=\frac{z^2+z+z^2-z}{z^2+z-z^2+z}=\frac{2z^2}{2z}=z$$

15.

$$u^2(u^2+4)=\left(x-\frac{1}{x}\right)^2\left(\left(x-\frac{1}{x}\right)^2+4\right)$$

$$=\left(x-\frac{1}{x}\right)^2\left(x^2-2x\left(\frac{1}{x}\right)+\left(\frac{1}{x}\right)^2+4\right)$$

$$=\left(x-\frac{1}{x}\right)^2\left(x^2-2+\frac{1}{x^2}+4\right)$$

$$=\left(x-\frac{1}{x}\right)^2\left(x^2+2+\frac{1}{x^2}\right)$$

$$=\left(x-\frac{1}{x}\right)^2\left(x+\frac{1}{x}\right)^2=\left(x-\frac{1}{x}\right)\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}\right)$$

$$=\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)$$

$$=\left(x^2-\frac{1}{x^2}\right)\left(x^2-\frac{1}{x^2}\right)=\left(x^2-\frac{1}{x^2}\right)^2=v^2$$

15. (Alternative, longer method)

$$u^2(u^2+4)=\left(x-\frac{1}{x}\right)^2\left(\left(x-\frac{1}{x}\right)^2+4\right)$$

$$=\left(x^2-2x\left(\frac{1}{x}\right)+\left(\frac{1}{x}\right)^2\right)\left(x^2-2x\left(\frac{1}{x}\right)+\left(\frac{1}{x}\right)^2+4\right)$$

$$=\left(x^2-2+\frac{1}{x^2}\right)\left(x^2-2+\frac{1}{x^2}+4\right)$$

$$==\left(x^2-2+\frac{1}{x^2}\right)\left(x^2+2+\frac{1}{x^2}\right)$$

$$=x^2\left(x^2+2+\frac{1}{x^2}\right)-2\left(x^2+2+\frac{1}{x^2}\right)+\frac{1}{x^2}\left(x^2+2+\frac{1}{x^2}\right)$$

$$=x^4+2x^2+1-2x^2-4-\frac{2}{x^2}+1+\frac{2}{x^2}+\frac{1}{x^4}$$

$$==x^4+\cancel{2x^2}\cancel{-2x^2}+1-4+1\cancel{-\frac{2}{x^2}}\cancel{+\frac{2}{x^2}}+\frac{1}{x^4}$$

$$=x^4-2+\frac{1}{x^4}$$

$$v^2=\left(x^2-\frac{1}{x^2}\right)^2=\left(x^2-\frac{1}{x^2}\right)\left(x^2-\frac{1}{x^2}\right)$$

$$=x^2\left(x^2-\frac{1}{x^2}\right)-\frac{1}{x^2}\left(x^2-\frac{1}{x^2}\right)$$

$$=x^4-1-1+\frac{1}{x^4}=x^4-2+\frac{1}{x^4}$$

$$=u^2(u^2+4)$$

16.

$$\frac{x^2}{(x-y)(x-z)}+\frac{y^2}{(y-z)(y-x)}+\frac{yz}{(z-x)(z-y)}$$

$$=\frac{x^2}{(x-y)(x-z)}-\frac{y^2}{(y-z)(x-y)}+\frac{yz}{(x-z)(y-z)}$$

$$=\frac{x^2(y-z)-y^2(x-z)+yz(x-y)}{(x-y)(x-z)(y-z)}$$

$$=\frac{x^2y-x^2z-y^2x+\cancel{y^2z}+yzx-\cancel{y^2z}}{(x-y)(x-z)(y-z)}$$

$$=\frac{x(xy-xz-y^2+yz)}{(x-y)(x-z)(y-z)}$$

$$=\frac{x[x(y-z)-y(y-z)]}{(x-y)(x-z)(y-z)}$$

$$=\frac{x[\cancel{(x-y)}\,\cancel{(y-z)}]}{\cancel{(x-y)}(x-z)\cancel{(y-z)}}=\frac{x}{x-z}$$