

Oliver Murphy: Discovering Maths 4: EXERCISE 1A

1.(i) $x^2 - 2x = x(x - 2)$

(ii) $2a^2b - 8ab^2 = 2ab(a - 4b)$

(iii) $x^3 + x^2 = x^2(x + 1)$

(iv) $25x^2y^3 + 35x^3y^2 = 5x^2y^2(5y + 7x)$

(v) $xyz^2 - wx^2z = xz(yz - wx)$

2.(i)

$$\begin{aligned}ax + bx + ay + by &= x(a + b) + y(a + b) \\ &= (x + y)(a + b)\end{aligned}$$

(ii)

$$\begin{aligned}3ax - 9ay + bx - 3by &= 3a(x - 3y) + b(x - 3y) \\ &= (3a + b)(x - 3y)\end{aligned}$$

(iii)

$$\begin{aligned}ps - pq - rs + rq &= p(s - q) - r(s - q) \\ &= (p - r)(s - q)\end{aligned}$$

(iv)

$$\begin{aligned}2x^2 + 3x - 10xy - 15y &= x(2x + 3) - 5y(2x + 3) \\ &= (x - 5y)(2x + 3)\end{aligned}$$

(v)

$$\begin{aligned}ac - 3bc - 2ad + 6bd &= c(a - 3b) - 2d(a - 3b) \\ &= (c - 2d)(a - 3b)\end{aligned}$$

3.(i)

$$\begin{aligned}x^2 - x - 90 &= x^2 - 10x + 9x - 90 \\ &= x(x - 10) + 9(x - 10) \\ &= (x + 9)(x - 10)\end{aligned}$$

(ii)

$$\begin{aligned}4x^2 + 4x + 1 &= 4x^2 + 2x + 2x + 1 \\ &= 2x(2x + 1) + 1(2x + 1) \\ &= (2x + 1)(2x + 1) = (2x + 1)^2\end{aligned}$$

(iii)

$$\begin{aligned}10x^2 - x - 2 &= 10x^2 - 5x + 4x - 2 \\ &= 5x(2x - 1) + 2(2x - 1) \\ &= (5x + 2)(2x - 1)\end{aligned}$$

(iv)

$$\begin{aligned}9x^2 - 12xy + 4y^2 &= 9x^2 - 6xy - 6xy + 4y^2 \\ &= 3x(3x - 2y) - 2y(3x - 2y) \\ &= (3x - 2y)(3x - 2y) = (3x - 2y)^2\end{aligned}$$

(v)

$$\begin{aligned}14x^2 - 15x + 4 &= 14x^2 - 7x - 8x + 4 \\ &= 7x(2x - 1) - 4(2x - 1) \\ &= (7x - 4)(2x - 1)\end{aligned}$$

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4.(i)

$$x^2 - 100 = (x - 10)(x + 10)$$

(ii)

$$4x^2 - 81 = (2x - 9)(2x + 9)$$

(iii)

$$25x^2 - 49y^2 = (5x - 7y)(5x + 7y)$$

(iv)

$$121a^2 - 144b^2 = (11a - 12b)(11a + 12b)$$

(v)

$$\begin{aligned} & (5x + 6y)^2 - (x + y)^2 \\ &= [(5x + 6y) - (x + y)][(5x + 6y) + (x + y)] \\ &= [5x - x + 6y - y][5x + x + 6y + y] \\ &= [4x + 5y][6x + 7y] \end{aligned}$$

5.(i)

$$\begin{aligned} x^3 + 27 &= x^3 + 3^3 \\ &= (x + 3)(x^2 - 3x + 9) \end{aligned}$$

(ii)

$$\begin{aligned} a^3 + 8 &= a^3 + 2^3 \\ &= (a + 2)(a^2 - 2a + 4) \end{aligned}$$

(iii)

$$\begin{aligned} b^3 + 1000 &= b^3 + 10^3 \\ &= (b + 10)(b^2 - 10b + 100) \end{aligned}$$

(iv)

$$\begin{aligned} 8x^3 + 125 &= (2x)^3 + 5^3 \\ &= (2x + 5)((2x)^2 - (2x)5 + 25) \\ &= (2x + 5)(4x^2 - 10x + 25) \end{aligned}$$

(v)

$$\begin{aligned} 125x^3 + 27y^3 &= (5x)^3 + (3y)^3 \\ &= (5x + 3y)((5x)^2 - (5x)(3y) + (3y)^2) \\ &= (5x + 3y)(25x^2 - 15xy + 9y^2) \end{aligned}$$

6.(i)

$$y^3 - 1^3 = (y - 1)(y^2 + y + 1)$$

(ii)

$$\begin{aligned} 8y^3 - 1 &= (2y)^3 - 1 \\ &= (2y - 1)((2y)^2 + (2y) \times 1 + 1^2) \\ &= (2y - 1)(4y^2 + 2y + 1) \end{aligned}$$

(iii)

$$27a^3 - 8b^3 = (3a - 2b)(9a^2 + 6ab + 4b^2)$$

(iv)

$$x^3 - 216 = (x - 6)(x^2 + 6x + 36)$$

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(v)

$$\begin{aligned}1000a^3 - 343b^3 &= (10a)^3 - (7b)^3 \\ &= (10a - 7b)(100a^2 + 70ab + 49b^2)\end{aligned}$$

7.(i)

$$\begin{aligned}2x^2 - 50 &= 2(x^2 - 25) \\ &= 2(x - 5)(x + 5)\end{aligned}$$

(ii)

$$\begin{aligned}3x^2 - 12 &= 3(x^2 - 4) \\ &= 3(x - 2)(x + 2)\end{aligned}$$

(iii)

$$(x + y)^2 - z^2 = (x + y - z)(x + y + z)$$

(iv)

$$\begin{aligned}4x^3 - 9x &= x(4x^2 - 9) \\ &= x(2x - 3)(2x + 3)\end{aligned}$$

(v)

$$\begin{aligned}x^4 + x &= x(x^3 + 1) \\ &= x(x + 1)(x^2 - x + 1)\end{aligned}$$

8.(i)

$$\begin{aligned}x^2 + 20x + 51 &= x^2 + 17x + 3x + 51 \\ &= x(x + 17) + 3(x + 17) \\ &= (x + 3)(x + 17)\end{aligned}$$

(ii)

$$169 - x^2 = (13 - x)(13 + x)$$

(iii)

$$\begin{aligned}ax + by + ay + bx \\ &= ax + ay + bx + by \\ &= a(x + y) + b(x + y) \\ &= (a + b)(x + y)\end{aligned}$$

iv

$$\begin{aligned}a^2 - 2ab + b^2 \\ &= a^2 - ab - ab + b^2 \\ &= a(a - b) - b(a - b) \\ &= (a - b)(a - b) = (a - b)^2\end{aligned}$$

(v)

$$\begin{aligned}a^2 - 2ab + b^2 - c^2 &= (a - b)^2 - c^2 \\ &= (a - b - c)(a - b + c)\end{aligned}$$

9.(i)

$$\begin{aligned}3x^2 - 75 &= 3(x^2 - 25) \\ &= 3(x - 5)(x + 5)\end{aligned}$$

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(ii)

$$\begin{aligned}9x^3 - 25x &= x(9x^2 - 25) \\ &= x(3x - 5)(3x + 5)\end{aligned}$$

(iii)

$$\begin{aligned}x^4 - y^4 &= (x^2)^2 - (y^2)^2 \\ &= [(x^2) - (y^2)][(x^2) + (y^2)] \\ &= (x - y)(x + y)(x^2 + y^2)\end{aligned}$$

(iv)

$$\begin{aligned}x^4 - 81 &= x^4 - 3^4 \\ &= (x^2 - 3^2)(x^2 + 3^2) \\ &= (x - 3)(x + 3)(x^2 + 9)\end{aligned}$$

(v)

$$\begin{aligned}ax^2 - bx^2 - ay^2 + by^2 \\ &= x^2(a - b) - y^2(a - b) \\ &= (x^2 - y^2)(a - b) \\ &= (x - y)(x + y)(a - b)\end{aligned}$$

10.(i)

$$\begin{aligned}x^3 + y^3 + 3x + 3y \\ &= (x + y)(x^2 - xy + y^2) + 3(x + y) \\ &= (x + y)(x^2 - xy + y^2 + 3)\end{aligned}$$

(ii)

$$\begin{aligned}x^2 - y^2 + 5x + 5y \\ &= (x - y)(x + y) + 5(x + y) \\ &= (x + y)(x - y + 5)\end{aligned}$$

(iii)

$$\begin{aligned}x^2 - 2xy + y^2 - 4z^2 \\ &= (x - y)^2 - 4z^2 \\ &= (x - y - 2z)(x - y + 2z)\end{aligned}$$

(iv)

$$\begin{aligned}x^3 - y^3 + x^2 - y^2 \\ &= (x - y)(x^2 + xy + y^2) + (x - y)(x + y) \\ &= (x - y)(x^2 + xy + y^2 + x + y)\end{aligned}$$

(v)

$$\begin{aligned}a^2 - (b + c)^2 &= [a - (b + c)][a + (b + c)] \\ &= [a - b - c][a + b + c]\end{aligned}$$

11.(i)

$$\frac{x^2}{100} - \frac{y^2}{49} = \left(\frac{x}{10} - \frac{y}{7}\right)\left(\frac{x}{10} + \frac{y}{7}\right)$$

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(ii)

$$\begin{aligned}x^3 + \frac{1}{x^3} &= \left(x + \frac{1}{x}\right) \left(x^2 - x\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2\right) \\ &= \left(x + \frac{1}{x}\right) \left(x^2 - 1 + \frac{1}{x^2}\right)\end{aligned}$$

(iii)

$$\begin{aligned}x^4 - 16 &= (x^2 - 4)(x^2 + 4) \\ &= (x - 2)(x + 2)(x^2 + 4)\end{aligned}$$

(iv)

$$a^2b^2 - 64 = (ab - 8)(ab + 8)$$

(v)

$$\begin{aligned}x^2 - y^2 + 2x + 1 &= x^2 + 2x + 1 - y^2 \\ &= (x + 1)^2 - y^2 \\ &= [x + 1 - y][x + 1 + y]\end{aligned}$$

12.(i)

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

(ii)

$$\begin{aligned}x^3 + y^3 + 3xy(x + y) &= (x + y)(x^2 - xy + y^2) + 3xy(x + y) \\ &= (x + y)(x^2 - xy + y^2 + 3xy) \\ &= (x + y)(x^2 + 2xy + y^2) = (x + y)(x + y)^2 \\ &= (x + y)^3\end{aligned}$$

(iii)

$$\begin{aligned}x^3 + y^3 + z^3 + 3xy(x + y) &= (x + y)^3 + z^3 \\ &= (x + y + z)((x + y)^2 - (x + y)z + z^2) \\ &= (x + y + z)(x^2 + 2xy + y^2 - xz - yz + z^2)\end{aligned}$$