Bookwork Learn these carefully!

ALGEBRA

DIFFERENCE OF TWO SQUARES

$x^2 - y^2 = (x - y)(x + y)$

SQUARE OF A SUM

$x^2 + 2xy + y^2 = (x + y)^2$

square the first + twice the product + square the second

SQUARE OF A DIFFERENCE

$x^2 - 2xy + y^2 = (x - y)^2$

square the first – twice the product + square the second

SUM OF TWO CUBES

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

DIFFERENCE OF TWO CUBES

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

THE QUADRATIC FORMULA

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

SUM AND PRODUCT OF ROOTS OF A QUADRATIC

If α and β are roots of a quadratic then

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Learn these identities:

- $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
- $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$
- $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$
- $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$
- $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

SIMULTANEOUS EQUATIONS

Type I: *x*, *y*, *z*: Look out for a short cut e.g. one of the equations might have only two letters.

Type II: one of the equations has an x^2.

Use the simple equation to get one letter in terms of another. Change the quadratic to a single letter version and solve.

Bookwork Learn these carefully!

ABSOLUTE VALUE INEQUALITIES

e.g. Solve $|2x-3|<4$

ANSWER:

$|2x-3|<4$

$\Rightarrow -4<2x-3<4$ then use rule of opposites

$\Rightarrow -4+3<2x<4+3$

$\Rightarrow -1<2x<7$

$\Rightarrow -\frac{1}{2}<x<\frac{7}{2}$

SURD EQUATIONS

Isolate the surd.
Square both sides and solve the quadratic.
Test both answers.

QUADRATIC INEQUALITIES

quadratic ≤ 0

If quadratic has roots a,b then $a \leq x \leq b$.

quadratic ≥ 0

If quadratic has roots a,b then $x \leq a$ or $x \geq b$.

FACTOR THEOREM (FOR QUADRATICS OR CUBICS)

To Prove: if $f(k)=0$ then $x-k$ is a factor of $f(x)$.

Method: Work out $f(x)=f(x)-f(k)$.

Get $x-k$ as a common factor using the difference of two squares / cubes.

LAWS OF INDICES

MOST BASIC LAW: $a^x a^y = a^{x+y}$

CONSEQUENCES

$a^0=1$

$\frac{1}{a}=a^{-1}$

Bookwork Learn these carefully!

LAWS OF LOGARITHMS

A log is a power e.g. $\log_3 81 = x$ means that x is the power. Here $x = 4$ since $3^4 = 81$.

LAW ONE: Log of a product = Sum of logs

$$\log_a xy = \log_a x + \log_a y$$

LAW TWO: Log of a ratio = Difference of logs

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

LAW THREE: Log of a power = power times the log

$$\log_a x^n = n \log_a x$$

LAW FOUR: Change of base rule

$$\log_{old}(x) = \frac{\log_{new}(x)}{\log_{new}(old)} \text{ e.g. } \log_3(5) = \frac{\log_{10}(5)}{\log_{10}(3)}$$