

Number of Students: 19

Time: 1 ½ hours

ANSWER ALL QUESTIONS

Question One

$$\begin{aligned} \frac{6y}{x(x+4y)} - \frac{3}{2x} &= \frac{2(6y) - 3(x+4y)}{2x(x+4y)} \\ &= \frac{12y - 3x - 12y}{2x(x+4y)} \\ &= \frac{-3x}{2x(x+4y)} \\ &= \frac{-3}{2(x+4y)} \end{aligned}$$

(10 marks)

(03-I-1a)

Question Two

(i) $x + 2y + 4z = 7$

(ii) $x + 3y + 2z = 1$

(iii) $-y + 3z = 8$

(i) $x + 2y + 4z = 7$

(ii) $x + 3y + 2z = 1$

$-y + 2z = 6 \dots(\text{iv})$

(iv) $-y + 2z = 6$

(iii) $-y + 3z = 8$

$-z = -2$

$z = 2$

(iii) $-y + 3z = 8$

$-y + 6 = 8$

$-y = 2$

$y = -2$

(i) $x + 2y + 4z = 7$

$x - 4 + 8 = 7$

$x = 3$

$x = 3$

$y = -2$

$z = 2$

(5 marks eliminating x, 5 marks finish, A2, A2)

(02-I-2a)

Question Three

(i) : $x - y = 0$

(ii) : $(x+2)^2 + y^2 = 10$

(i) : $x = y$

(ii) : $(x+2)^2 + x^2 = 10$

$$x^2 + 4x + 4 + x^2 - 10 = 0$$

$$2x^2 + 4x - 6 = 0$$

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

$$x = -3 \quad \text{or} \quad x = 1$$

$$y = -3 \quad \text{or} \quad y = 1$$

(10 marks, A3)

(01-I-2a)

Question Four

$$|3x+5| < 4$$

$$\Rightarrow -4 < 3x+5 < 4$$

$$-4 < 3x+5$$

$$-9 < 3x$$

$$-3 < x$$

$$3x+5 < 4$$

$$3x < -1$$

$$x < -\frac{1}{3}$$

$$\Rightarrow -3 < x < -\frac{1}{3}$$

OR
2(b)

$$|3x+5| < 4$$

$$(3x+5)^2 < (4)^2$$

$$9x^2 + 30x + 25 - 16 < 0$$

$$9x^2 + 30x + 9 < 0$$

$$3x^2 + 10x + 3 < 0$$

Note: $+x^2 \Rightarrow$ Min PtNote: when $3x^2 + 10x + 3 = 0$

$$\Rightarrow (3x+1)(x+3) = 0$$

$$\Rightarrow x = -\frac{1}{3} \text{ or } x = -3$$

$$-3 < x < -\frac{1}{3}$$



(10 marks, A3)(01-I-2bi)

Question Five

5(a)(i)

$$\begin{aligned}
 x-2 &= \sqrt{7x-6} \\
 (x-2)^2 &= 7x-6 \\
 x^2 - 4x + 4 &= 7x-6 \\
 x^2 - 11x + 10 &= 0 \\
 (x-1)(x-10) &= 0 \\
 \Rightarrow x-1 &= 0 & \text{or} & x-10 = 0 \\
 x &= 1 & \text{or} & x = 10
 \end{aligned}$$

Test $x = 1$ LHS = -1 RHS = $\sqrt{7x-6} = \sqrt{1} = 1$
 LHS \neq RHS

$x = 10$ LHS = 8 RHS = $\sqrt{70-6} = \sqrt{64} = 8$
 LHS = RHS

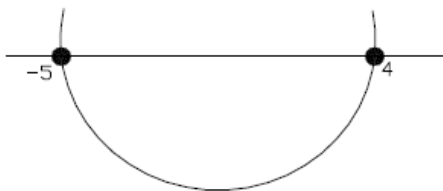
Solution: $x = 10$

(Quadratic 5 A2, finish 5 A2) (03-I-5a)

Question Six

2(b)(i)

$$\begin{aligned}
 x^2 + x - 20 &\leq 0 \\
 (x+5)(x-4) &= 0 \\
 x = -5 & \text{ or } x = 4
 \end{aligned}$$



$$-5 \leq x \leq 4$$

(Roots 5 A2, finish 5 A2) (02-I-2bi)

Question Seven

1(b)(i)

$$\begin{aligned}
 f(x) &= ax^2 + bx + c \\
 f(k) &= ak^2 + bk + c \\
 \hline
 f(x) - f(k) &= a(x^2 - k^2) + b(x - k) \\
 f(k) = 0 &\Rightarrow f(x) = a(x-k)(x+k) + b(x-k) \\
 &= (x-k)[a(x+k) + b] \\
 &\Rightarrow (x-k) \text{ is factor}
 \end{aligned}$$

(10 marks A3) (03-I-1bi)

Question Eight

2(c)(i) Given: $\alpha + \beta = -7$ and $\alpha\beta = 11$

$$\alpha^2 + \beta^2 = \alpha^2 + 2\alpha\beta + \beta^2 - 2\alpha\beta$$

$$= (\alpha + \beta)^2 - 2(\alpha\beta)$$

$$= (-7)^2 - 2(11)$$

$$= 49 - 22$$

$$= 27$$

(ii) Roots of new equation: $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

$$\text{Sum} = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{27}{11}$$

$$\text{Product} = \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right) = 1$$

$$\text{Equation: } x^2 - (\text{Sum Roots})x + (\text{Product Roots}) = 0$$

$$x^2 - \left(\frac{27}{11}\right)x + (1) = 0$$

$$11x^2 - 27x + 11 = 0$$

(i) 5 marks (ii) 15 marks (01-I-2c)

Question Nine

2(b) $f(x) = 4x^3 + 10x^2 - 7x - 3$

Integral root must be $\pm 1, \pm 3$

$$f(1): 4 + 10 - 7 - 3 \neq 0$$

$$f(-1): \quad \quad \quad \neq 0$$

$$f(3): 108 + 90 - 21 - 3 \neq 0$$

$$f(-3): -108 + 90 + 21 - 3 = 0$$

$\Rightarrow x = -3$ is a root $\Rightarrow (x + 3)$ is a factor

$$\begin{array}{r} 4x^2 - 2x - 1 \\ x+3 \overline{) 4x^3 + 10x^2 - 7x - 3} \\ \underline{4x^3 + 12x^2} \\ -2x^2 - 7x \\ \underline{-2x^2 - 6x} \\ -x - 3 \\ \underline{-x - 3} \\ 0 \end{array}$$

So, need to solve: $4x^2 - 2x - 1 = 0$

$$x = \frac{2 \pm \sqrt{4 + 16}}{2(4)} = \frac{2 \pm \sqrt{20}}{8} = \frac{2 \pm 2\sqrt{5}}{8} = \frac{1 \pm \sqrt{5}}{4}$$

Irrational roots: $\frac{1 + \sqrt{5}}{4}, \frac{1 - \sqrt{5}}{4}$

Test 5 marks
 Linear Factor 5 marks
 Other Factor 5 marks
 Roots 5 marks

Att 2
 Att 2
 Att 2
 Att 2 (05-I-2b)

Question Ten

5(a)(i)

$$\begin{aligned}\frac{8}{2^x} &= 32 \\ 2^{3-x} &= 2^5 \\ \Rightarrow 3-x &= 5 \\ x &= -2\end{aligned}$$

5(a)(ii)

$$\begin{aligned}\log_9 x &= \frac{3}{2} \\ x &= (9)^{\frac{3}{2}} = 27\end{aligned}$$

(i) 5 A2, (ii) 5 A2 (02-I-5a)

Happy Christmas!

