Skills Required for Probability Etc.

FUNDAMENTAL PRINCIPLE OF COUNTING.

Be able to apply this to menus etc.

FACTORIALS

- Use calculator to get e.g. 6!
- Recall that 0! = 1
- Recall and recognise formula: $n! = n(n-1)(n-2)...3\times 2\times 1$
- Apply above pattern to e.g. $(n+3)!$ or $(n-5)!$
- Evaluate e.g. 4! without using a calculator
- Recall that in the expression $n!$, the variable n is a natural number e.g. it *cannot* be negative.
- Understand that $n!$ is the number of ways that n distinct things can be arranged, where order is important.
- Write out all the ways, e.g. three letters, can be arranged.
- Find the number of ways a group people can be seated in a row with or without restrictions e.g. some must sit together. (2009 II 6 b, 2007 II 6a,)
- Understand that n^n is the number of ways n things can be arranged *with repetition*.
- Apply above to mobile phone passwords (2007 II7 a)
- Find number of ways e.g. 6 things can be arranged in a row if there are repeats e.g. the number of way all six letters B,A,N,A,N,A can be arranged in a row.
- Deduce that e.g. 9! can be re-written as $9! = 9\times 8\times 7!$
- Deduce that e.g. 11! can be re-written as $\frac{12!}{12}$ i.e. express on factorial in terms of another
- Be able to simplify ratios of factorials, using numbers and variables. (NB: calculator may not necessarily be able to work these out as there are too many digits.)
  e.g. $\frac{4000!}{3999!} = 4000$, $\frac{(n+2)!}{(n-1)!} = (n+2)(n+1)n$
- Be able to add or subtract ratios involving factorials.
  e.g. $\frac{1}{(n+1)!} + \frac{1}{n!} = \frac{1}{(n+1)!} + \frac{(n+1)}{(n+1)!} = \frac{n+2}{(n+1)!}$
- Solve factorial equations that reduce to quadratics
  e.g. $\frac{(n+1)!\times 3!}{(n+3)!} = \frac{1}{35}$ (2009 II 6b iii)

PERMUTATIONS

- Be able to calculate e.g. ${}^5P_3$ using the calculator.
- Understand that e.g. ${}^5P_3$ gives the number of ways of arranging 5 distinct things in groups of 3.
- Write out the number of ways e.g. 3 things can be arranged 2 at a time.

- Be able to express ${}^5P_3$ in all three forms

  i.e. ${}^5P_3 = \dfrac{5!}{(5-3)!} = 5\times 4\times 3$
- Evaluate e.g. ${}^{750}P_2$ without using a calculator.
- Understand that ${}^nP_r$ gives the number of ways of arranging *n* distinct things in groups of size *r*.
- Find the number of 3 digit numbers using 1 – 5 with and without repetitions (2005 II 6a)
- Recall the formula ${}^nP_r = \dfrac{n!}{(n-r)!} = n(n-1)(n-2)...(n-r+1)$
- Be able to apply above formula to e.g. ${}^{n+1}P_r$, ${}^nP_{r+1}$ etc.

COMBINATIONS

- Evaluate e.g. ${}^{10}C_3$ with a calculator.
- Understand that e.g. ${}^{10}C_3$ is the number of ways to choose 3 distinct things from a group of 10 distinct things.
- Understand that e.g. ${}^{10}C_3 = \dfrac{{}^{10}P_3}{3!}$
- Write e.g. ${}^{10}C_3 = \dfrac{10!}{(10-3)!3!} = \dfrac{10\times 9\times 8}{1\times 2\times 3}$
- Recall both notations i.e. ${}^{10}C_3$ is the same as $\binom{10}{3}$.
- Understand why ${}^{10}C_3 = {}^{10}C_7$ and ${}^{200}C_{180} = {}^{200}C_{20}$ etc.
- Evaluate e.g. ${}^{100}C_{98}$ without using the calculator

  (NOTE: ${}^{100}C_{98} = {}^{100}C_2 = \dfrac{100\times 99}{2\times 1} = 4950$).
- Evaluate number of ways of choosing *r* distinct things from *n* with or without restrictions
  e.g. choosing books by different authors (2009 II 6a),
  choosing subjects where one is compulsory (2008 II 7a),
  choosing letters from a word, one must be a vowel (2007 II 7b)
  choosing teams from a mixed panel (2006 II 6a, 2005 II 7a)

- Understand "at least one" can be done as "total minus none" (2007 II 7a ii)
- Recall the formula

  ${}^nC_r = \dfrac{n!}{(n-r)!r!} = \dfrac{n(n-1)(n-2)...(n-r+1)}{r(r-1)(r-2)...3\times 2\times 1}$
- Write an expression for
  e.g. ${}^{6+x}C_3$ (2008 II 6c), ${}^{r+w}C_2$ (2007 II 6c)

Skills Required for Probability Etc.

PROBABILITY

- Recall $\mathrm{P} = \dfrac{\text{total favourable}}{\text{total possible}}$.
  (Note finite nature)
- Deduce that $0 \le \mathrm{P} \le 1$.
- Recall that something is certain if and only if P = 1, and something is impossible if and only if P = 0.
- Recall that $\mathrm{P}(\text{it happens}) = 1 - \mathrm{P}(\text{it doesn't happen})$ and vice versa.
- Understand "at least" as "total minus" (2006 II 6c iii, 2006 II 7b iii)
- Apply to playing cards. (2008 II 7b)
- Apply to a single die.
- Define sample space and event.
- List sample spaces.
- Calculate probabilities for toss of two six-sided dice (2007 II 7b).
- Calculate probabilities using combination notation.
- Calculate probabilities for arrangements, including backwards questions. (2009 II 6b)
- Apply to coordinate geometry formulae, using a table to find $\#S$ and $\#E$. (2009 II 6c)
- Apply to bag with different coloured discs (2008 II 6 c, 2007 II 6c, 2005 II 7b)
- Apply to students with birthday in same month (2006 II 6c)
- Apply to numbered cards (2005 II 6c)

MEAN AND STANDARD DEVIATION

- Recall that the mean of N values is $\mu = \dfrac{\text{Sum}}{N}$
- Calculate the mean of a list of values
- Backwards questions e.g. find sum given mean and N
- Be able to give a list where all the values are close to the mean or all the values are far from the mean
- Recall and apply $\sigma = \sqrt{\dfrac{\Sigma(x-\mu)^2}{N}}$
- Concept of σ as a measure of dispersion.
- Understand why the brackets are squared in formula for σ i.e. appreciate σ as a root-mean-square of differences from mean.
- Relation between σ and sum of differences from mean (2008 II 7c)
- Recall that the mean of a frequency table is $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$
- Recall and apply $\sigma = \sqrt{\dfrac{\Sigma f\left(x-\bar{x}\right)^2}{\Sigma f}}$

- Given two lists with same mean different σ find a value for standard deviation of union of two lists. (2009 II 7c)
- Find $\bar{x}$ and σ for a number of terms of an arithmetic sequence (2007 II 7c)
- Find σ for integers between $-n$ and n. (2006 II 7c)
- Effect of aging a years on $\bar{x}$ and σ for a year group. (2005 II 7c)

WEIGHTED MEAN

- Recall and use formula $\bar{x} = \frac{\Sigma wx}{\Sigma w}$.
- Apply to list of prices and weights (2009 II 7a)
- Backwards questions e.g. find a weight. Apply to list of exam results as percentages with weights (2008 II 6b)

DIFFERENCE EQUATIONS

- Re-write $au_{n+2} + bu_{n+1} + cu_n = 0$ as a quadratic and find roots α and β
- Find $u_n = p\alpha^n + q\beta^n$ using initial conditions and simultaneous equations. (2009 II 7b, 2008 II 6b, 2006 II 6b, 2005 II 6b)
- Proof

  α and β are the roots of the quadratic equation $px^2 + qx + r = 0$.

  $u_n = l\alpha^n + m\beta^n$, for all $n \in \mathrm{N}$.

  Show that $pu_{n+2} + qu_{n+1} + ru_n = 0$, for all $n \in \mathrm{N}$.

  (2007 II 6b)
- Be able to check simplified solution by substitution (2006 II 6b, 2005 II 6b)
- Use g.p. formula to find $u_1 + u_2 + ... + u_n$ (2009 II 7b)